My mother's contribution to Honsberger's collection.

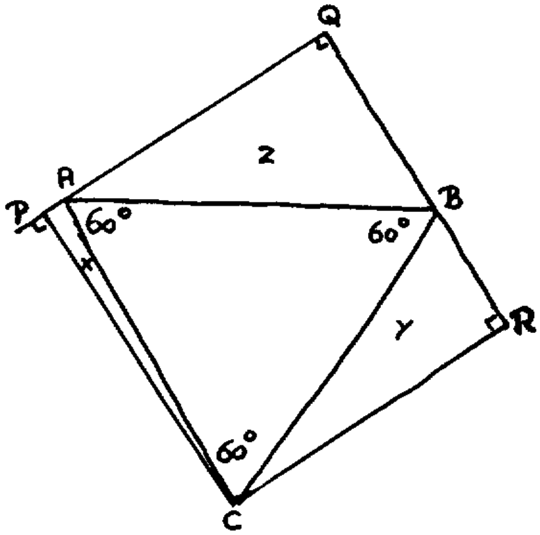

fig.1

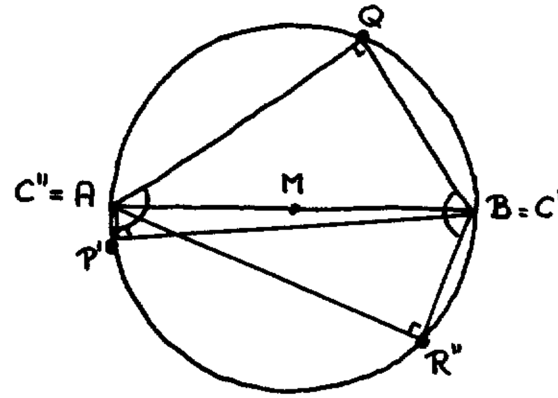

fig.2

In fig.1 we have an equilateral triangle ABC with a circumscribed rectangle PQRC. Prove that the areas x, y, and z of the three rectangular triangles in fig.1 satisfy $x + y = z$.

Subject triangle APC to an anti-clockwise rotation of 60° around A ; in fig.2 , P' and C' are the images of P and C. Subject triangle BRC to a clockwise rotation of 60° around B ; in fig.2 , R" and C" are the images of R and C. Because all three triangles are rectangular, the circle with diameter AB is the circumscribed circle of all three. Because our rotations were over 60°, $\angle P'AQ = \angle R''BQ = 120°$. Hence, arc P'AQ = arc R"BQ = 1/3 of the circle's circumference ; hence, so is arc P'R", in other words: in fig.2 triangle P'QR" is equilateral.

Because in an equilateral triangle the centroid coincides with the circumcentre, the centroid of P'QR", called M, lies on the diameter AB. Therefore, with equal weights in its vertices, triangle P'QR" is in balance when supported by a horizontal axis AB. From the fact that the torque caused by the weight at Q is compensated by the sum of the torques caused by the weights at P' and R" respectively, $z = x + y$ immediately follows.

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The problem stated in the first paragraph occurred in a series of geometrical problems I received from Ross A. Honsberger. After having sent a copy to my mother, mrs. B.C. Dijkstra - Kluyver, I received (among others) by returning mail the above argument, which I think too beautiful not to be recorded.

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