From predicate transformers to predicates

Dedicated by the Tuesday Afternoon Club to C.A.R. Hoare at the occasion of his being elected Fellow of the Royal Society.

Lemma 0. For any statement S and any constant predicate C

$$wp(S, C) = wp(S, T) \wedge C \quad .$$

Proof. By substituting for C the two constant predicates T and F, respectively. (End of Proof.)

Lemma 1. For any statement S, any predicate R, and any constant predicate C

$$wp(S, R \vee C) = wp(S, R) \vee wp(S, C) \quad .$$

Proof. By substituting for C the two constant predicates T and F, respectively. (End of Proof.)

In the following, P is a predicate in x and by definition $P' = P^x_{x'}$; variables x and x' range over the same non-empty domain.

Lemma 2. For any predicate P in x we have for all x

$$P = (\underline{A}\, x' :: x \neq x' \vee P') \quad .$$

Proof. $P = (\underline{A}\, x' :: P) = (\underline{A}\, x' : x' = x : P') = (\underline{A}\, x' :: x \neq x' \vee P')$.
(End of Proof.)

Theorem. For any statement S with state space x and any predicate P we have

$$wp(S,P) = wp(S,T) \wedge (\underline{A}\, x' :: wp(S, x \neq x') \vee P') \quad .$$

Proof. $wp(S,P)$
$= \{\text{Lemma 2}\}$
$wp(S, (\underline{A}\, x' :: x \neq x' \vee P'))$
$= \{\text{distributivity of wp over universal quantification}\}$
$(\underline{A}\, x' :: wp(S, x \neq x' \vee P'))$
$= \{\text{Lemma 1}\}$
$(\underline{A}\, x' :: wp(S, x \neq x') \vee wp(S, P'))$
$= \{\text{Lemma 0}\}$
$(\underline{A}\, x' :: wp(S, x \neq x') \vee wp(S,T) \wedge P')$
$= \{wp(S,R) \Rightarrow wp(S,T)\}$
$wp(S,T) \wedge (\underline{A}\, x' :: wp(S, x \neq x') \vee P')$.
(End of Proof.)

Hence, the predicate transformer $wp(S,?)$ is fully characterized by the two predicates $wp(S,T)$ and $wp(S, x \neq x')$.

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