A theorem about infinite sequences of numbers

<u>Theorem</u>. An infinite sequence of (real) numbers contains an infinite monotonic subsequence. (End of Theorem.)

<u>Proof</u>. Let the sequence be $x(i: 0 \leq i)$. Element $x(i)$ is "dominant" means $(\underline{A} j: i+1 \leq j: x(i) \geq x(j))$. We distinguish two cases.

The number_of_dominant_elements_is_infinite. In this case the dominant elements form an infinite subsequence, which is monotonic (viz. descending).

The_number_of dominant_elements_is_finite. In this case an N exists such that from $x(N)$ onwards, no dominant element occurs, i.e. $(\underline{A} i: i \geq N: (\underline{E} j: i+1 \leq j: x(i) < x(j)))$. Hence each $x(i)$ with $i \geq N$ is the starting element of an infinite subsequence that is increasing, and therefore monotonic.

(End of Proof.)

    I found this proof last Sunday evening; the day before I had heard the theorem together with the rumour that it might require a fancy proof. Quod non.

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