Some useful formulae

Let f be a propositional expression and R a propositional variable. Using the slash "/" to denote "substituted for", we have the general equivalences

$$f \equiv (f(\text{false}/R) \vee R) \wedge (f(\text{true}/R) \vee \neg R)$$

$$f \equiv (f(\text{true}/R) \wedge R) \vee (f(\text{false}/R) \wedge \neg R) .$$

(We don't prove this; it should probably be proved using induction over the syntax.)

The purpose of this note is to draw the attention to a few special instances.

$$P \equiv Q \vee R \equiv ((P \equiv Q) \vee R) \wedge (P \vee \neg R)$$

$$P \equiv Q \wedge R \equiv (\neg P \vee R) \wedge ((P \equiv Q) \vee \neg R) .$$

(Substituting in the latter formula $\neg P$ for P we get

$$P \not\equiv Q \wedge R \equiv (P \vee R) \wedge ((P \not\equiv Q) \vee \neg R) .$$

We used the last formula recently to prove

$$(y \not\equiv z \wedge a) \wedge (z \not\equiv y \wedge b) \equiv$$
$$(y \vee a) \wedge (z \vee b) \wedge ((y \not\equiv z) \vee \neg(a \vee b)) \quad .)$$

Plataanstraat 5
5671 AL NUENEN
the Netherlands

8 February 1984
drs. A.J.M. van Gasteren
prof.dr. Edsger W. Dijkstra