

Where is Russell's "Paradox"?

Why is "Russell's Paradox" called a paradox? It is supposed to be rendered by the example of the village with cleanshaven men in which the village barber is defined as the man such that the population he shaves is the population of villagers that don't shave themselves. And then comes the question "Who shaves the barber?"

But where is the paradox? If we define in a certain context x to be equal to 3, we define x to satisfy $x=3$, i.e. to be a root of the equation $y:(y=3)$. For quite some time now, roots of equations seems to me the paradigm par excellence for definitions. How else to define $\sqrt{2}$ than as the root of $y:(y^2=2 \wedge y \geq 0)$? After the invention of the reals that equation has indeed one root. Did we only know the rationals, the equation would have no root and $\sqrt{2}$ "would not exist".

For the barber of the village we have the equation
 $y: (\forall i: i \text{ is villager: } i \neq \text{shaver.}i \equiv y = \text{shaver.}i)$
 and that equation has no solution. Conclusion: the village has no barber. Where is the paradox?

Probably I am very naive, but I also think I prefer to remain so, at least for the time being and perhaps for the rest of my life.

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