transcription

Why Johnny can't understand

Years ago I heard a lecture on the structure of proofs. Without hesitation the speaker became very pictorial and proofs became directed graphs with arrows from antecedents to consequents. [Mathematics Inc. would market the product these days as Computer-Aided Understanding by Means of Argument Animation.] After fifteen minutes the speaker draw our attention to the fact that some proofs were planar while others were not. Next he showed how simple transformations of proofs into logically equivalent ones could change the proof's planarity, but instead of concluding that proof planarity therefore was probably not a very relevant concept, he then embarked on a study of intrinsically non-planar arguments, etc.

It was the most nonsensical lecture I had heard in years. (That's why I still remember it.) The poor bloke was a serious victim of his education: he confused the directed graph as subset of the ordered pairs with the pictorial representation of arrows between points. [Had the same mind been educated on incidence matrices, he might have lectured on the eigenvalues of proofs.]

This is what happens to us over and over again. When a new concept is introduced we are given a few examples from a hopefully familiar context, or we are given one or two models in which the new formalism, its objects and its operations can be "understood". And we are actually encouraged to make these interpretations to convince ourselves that the new formalism "makes sense". They fail, however, to warn us that such interpretations tend to be misleading because the models are overspecific, that such habits of understanding are utterly confusing when the accompanying visualizations baffle the imagination and that the mental burden of translating back and forth between formula and interpretation had better be avoided. In fact one can only hope that with a growing familiarity with the formalism the model gently fades away from our consciousness.

It already began when we were taught the natural numbers. We did not learn $2+3=5$, we learned first -pictorially!- that two apples and three apples is five apples, and then for pears, for plums, for cats, trees and elephants. The apple model is woefully inadequate as, in order to accommodate a product, the apple had to be squared, and consequently -and fortunately- it did fade away, but not before it had created a hurdle for the negative integers. It can be argued that we continue to pay the price, viz. if we judge zero's invisibility in the apple model responsible for all the mathematical complication caused by considering 1 the smallest natural number. (Compared with the Greeks we have been fortunate: with their line segments they could multiply a little bit, unfortunately just enough not to kick away their model. And eventually Greek mathematics was killed by conceptual poverty and pictorial complexity: a lesson for us all.)

I seriously doubt that the detour via the apple model is essential for teaching little kids the integers, but even if it were I see no reason why a learning process that might be appropriate for little kids should also be appropriate for the adult mind. And yet this seems the assumption on which most writers and many adult readers operate. My -sad- conclusion is that the most widespread patterns of understanding have not been consciously selected for their effectiveness and had better be described as addictive habits, many of which deserve a General Surgeon's Warning.

My most common observation is seeing people feeling more comfortable with the needlessly specific. When confronted with a partially ordered set, they mentally supply "for instance, the integers". While I have been trained to skip the examples when reading a text -because they should be superfluous and are in any case distracting — I see people getting most uncomfortable when faced with a text without examples. People having difficulty in understanding a construction that contained a natural parameter k have assured me that that parameterization presented an additional obstacle that they could remove by initially substituting for k a small value, 3 say. I have no reason to doubt their word; the strange phenomenon was probably connected to the fact that k did not occur in a very arithmetical context but as length of strings or as number of edges meeting at a vertex, i.e. contexts they were used to deal with in terms of pictures. Halfway the text an "arbitrary" permutation created similar problems: they would have preferred a specific one, possibly followed at the end by a remark that the choice of permutation did not really matter. It is very strange, disconcerting even, to see people disturbed when questions are left open whose answers are irrelevant.

A final observation suggests that indeed our educational system is to blame. I remember very well the introduction of the idea that it was the professor's duty to motivate his students. (I remember it very well because I thought the idea so absurd.) I now encounter young scientists educated under the motivating regime, and they have a quite noticeable handicap:

their ability of absorbing unmotivated information is limited to about 10 lines. The object and its purpose are different things, but they have not learned to distinguish and are now unable to separate the corresponding concerns. It is a frightening example of how education can instill psychological needs that turn out to be a serious handycap.

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