

A supplement to EWD1140 and EWD1171

In EWD1140 we proved that the arithmetic mean is at least the geometric mean; in EWD1171 we returned to that problem, including most of the heuristics. One rabbit, though, remained: the choice of c between x and y so that

$$(o) \quad x, y := c, x+y-c$$

will decrease the distance between x and y . I know why the rabbit remained: in my first effort I hit upon a case analysis too ugly to include in my text, and so the coward waved his hands. (The case analysis was generated by the value of $x < y$ before and after.)

All this was a little bit stupid (and dishonest, but that is another story) since the whole argument - see the opening sentence of EWD1140 - is built on the identity

$$(x+y)^2 = (x-y)^2 + 4 \cdot x \cdot y$$

and instead of worrying about decreasing the distance between x and y , we should focus our attention to decreasing $(x-y)^2$.

The following simple calculation mas-
sages the initial condition under which
(0) decreases $(x-y)^2$:

$$\begin{aligned}
 & (c - (x+y-c))^2 < (x-y)^2 \\
 = & \quad \{\text{algebra}\} \\
 & (2c - x - y)^2 < (x-y)^2 \\
 = & \quad \{\text{algebra: } a < b+c \equiv a-b < c\} \\
 & (2c - x - y)^2 - (x-y)^2 < 0 \\
 = & \quad \{\text{algebra: } a^2 - b^2 = (a-b)(a+b)\} \\
 & (2c - 2x)(2c - 2y) < 0 \\
 = & \quad \{\text{algebra}\} \\
 & (c-x)(c-y) < 0
 \end{aligned}$$

which compactly and without case
analysis characterizes the situation in
which c lies between x and y .

For the modification chosen in EWD1171:

$$(1) \quad x, y := c, x \cdot y / c$$

the analogous calculation leads to

$$(c^2 - x^2) \cdot (c^2 - y^2) < 0 \quad ,$$

which is only equivalent to $(c-x) \cdot (c-y) < 0$
if $(c+x) \cdot (c+y) > 0$. The whole problem
is in terms of positive numbers, so
this condition is satisfied. The observa-
tion, however, confirms my impression

that the choice of (0) in EWD1140 - i.e. increasing the product under constant sum - is to be preferred over the choice of (1) in EWD1171 - i.e. decreasing the sum under constant product - .

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PS The algebra on the previous page is somewhat elaborate. It just shows how one is affected by lecturing for American undergraduates.

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