#### **Mesh Data Structures and CSG**





triangle soup



manifold mesh



watertight mesh





#### List of points

may or may not include normals



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Raw data from depth sensors



### **Triangle Soup**

- List of triangles
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No notion of triangle neighbors

• (but can find nearby triangles)

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Why do we need neighbors anyway?



Must satisfy three properties:

1. Every edge shared by one/two faces



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not manifold: has T junction



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Intuitively: mesh locally "looks like a single surface"



Must satisfy three properties:

- 1. Every edge shared by one/two faces
- 2. Faces around verts are triangle fans
- 3. Faces have consistent orientation



### Watertight Mesh

Manifold mesh that

- is single piece
- has no boundary



### Watertight Mesh

Manifold mesh that

- is single piece
- has no boundary

Splits space into well-defined inside/out

• can be "filled with water" without leaks





point cloud



triangle soup

"mesh repair" algorithms



manifold mesh



watertight mesh





watertight mesh



#### point cloud



triangle soup

manifold mesh

Poisson surface reconstruction



watertight mesh-

#### **Poisson Surface Reconstruction**

#### Interpolates point cloud and normals



#### **Triangle Mesh Data Structures**

#### List of points & triangle indices



Vertex List			
X	Y	Z	
0.0	0.0	1.0	
1.0	0.0	1.0	
0.0	1.0	1.0	
1.0	1.0	1.0	
0.0	0.0	0.0	
1.0	0.0	0.0	
0.0	1.0	0.0	
1.0	1.0	0.0	

Triangle List		
i	j	k
0	1	2
1	3	2
2	3	7
2	7	6
1	7	3
1	5	7
6	7	4
7	5	4
0	4	1
1	4	5
2	6	4
0	2	4

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List of points & triangle indices Pros:

- lightweight, compact
- native GPU data structure
- very common file data structure
  Cons:

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  Cons:
- neighbor queries **slow**
- finding boundaries slow

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- easy to "walk around" faces, vertices
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- large memory footprint
- tricky to implement (tons of pointers!)
  - use existing libraries (e.g. OpenMesh)

### **Types of Manifold Meshes**



#### **Triangles vs Quads**



#### Triangles simpler Quads more natural for flat & cylindrical geometry

#### **Arbitrary Quads Are Not Planar!**



#### Arbitrarily triangulate to render them...

#### Subdivision

#### Input: coarse control mesh



Output: finer mesh with smoother details

#### **Linear Subdivision**

Split faces 1:4

Insert verts at edge midpts

Adds faces, but doesn't change shape



#### **Nonlinear Subdivision**

**Everybody** has pet subdivision method Most popular:

• triangle meshes: Loop subdivision



#### **Nonlinear Subdivision**

**Everybody** has pet subdivision code Most popular:

- triangle meshes: Loop subdivision
- quad meshes: Catmull-Clark



#### **Catmull-Clark Subdivision**

# Rules for adding new points and replacing old points



Works best for regular quad meshes

#### **Regular vs Irregular Vertices**

#### Regular vertices have four edges



Irregular vertices have 3 or 5+ edges

• also called **extraordinary** vertices



#### **Catmull-Clark Subdivision**

# Rules for adding new points and replacing old points



Many schemes for handling irregular verts

### **Subdivision: Complications**

Dealing with irregular vertices Dealing with creases

some edges shouldn't be smoothed...



### **Subdivision: Complications**

Dealing with irregular vertices Dealing with creases

some edges shouldn't be smoothed...
 Dealing with boundaries

In general, allowing finer-grained control of subdivision process

#### **Subdivision Surface**

#### Smooth surface at limit of subdivision



Fundamental building block of Pixar's Renderman engine

#### **Other Mesh Operations**



#### Remeshing



#### Quadrangulation



#### **Other Mesh Operations**

Graphics subfield: geometry processing

• uses sophisticated theory from linear algebra, differential geometry, etc.

In practice: several good packages

- CGAL general purpose
- OpenMesh halfedge, subdivision

Who cares about volumes?

• just render outer skin?



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#### Translucent object (colored glass, fog,...)

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Translucent object (colored glass, fog,...) Physical simulations & deformations

• fracture

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Modeling primitive for cutting & sculpting

#### Voxelization: 3D rasterization





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really easy... but boundary chunky

- Tetrahedral (tet) mesh: 3D analogue of triangle mesh
- each tet "hyperface" has 4 faces, 6 edges, 4 verts









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 bdry is watertight triangle mesh





#### **Generating 2D Mesh From Curve**

#### Can be done by triangulation



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Greedy "ear-cutting" always works

#### **Generating 3D Mesh From Surface**

#### "Tetrahedralization" not always possible!

Shoenhardt Polytope



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"Tetrahedralization" not always possible!

Shoenhardt Polytope

Must add extra inner points

• "Steiner pts"



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Algorithm is **complex** and **bug-prone** 

• popular library: TetGen

- Hexahedral mesh: 3D version of quad mesh
- "hexahedron" = 6 sides (cube)



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No general hexahedralization algorithm exists (!)

#### **Constructive Solid Geometry**

Start with simple buildings blocks

• sphere, cubes, cylinders, ...

Build complicated objects using tree of operations



#### **CSG Operations**



# $\begin{array}{c} \text{union} \\ A \cup B \end{array}$

#### **CSG Operations**



 $\begin{array}{c} \text{union} \\ A \cup B \end{array}$ 

 $intersection \\ A \cap B$ 

#### **CSG Operations**



union  $A\cup B$ 

intersection  $A \cap B$ 

set difference



#### **CSG Tree**

Pros:

- geometry is exact
- simple representation for visually-complex shapes



### **CSG Tree**

Pros:

- geometry is exact
- simple representation for visually-complex shapes

Cons:

 some shapes difficult to model

Used in many game engines (e.g. Unreal)

