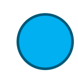
Review: Linear and Vector Algebra

Points in Euclidean Space $\mathbb{R}^n$

Location in space

Tuple of n **coordinates** x, y, z, etc

$$p = (p_x, p_y, p_z)$$

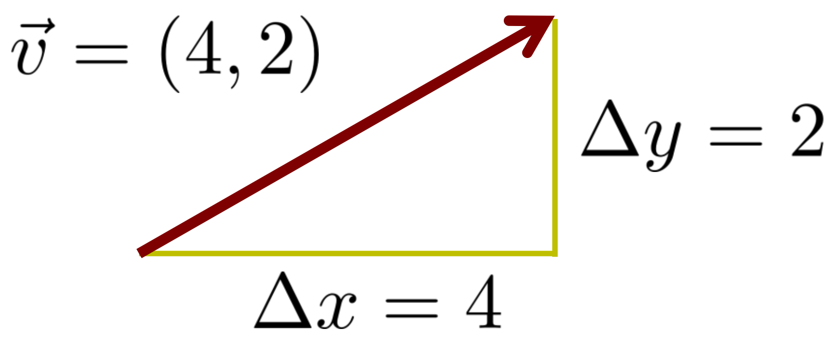


Cannot be added or multiplied together

Vectors: “Arrows in Space”

Vectors are **point changes**

Also number tuple: coordinate changes

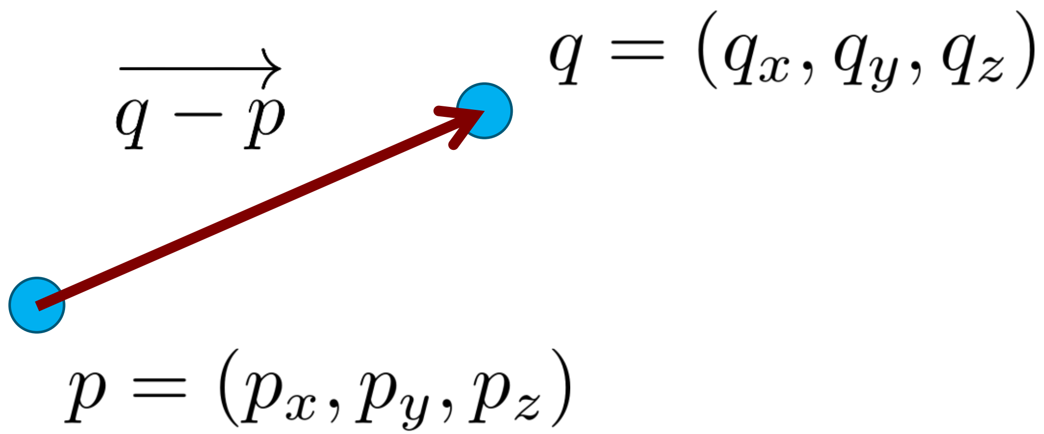


Exist **independent** of any reference point

Vector Arithmetic

Subtracting points gives vectors

- Vector between p and q: q - p

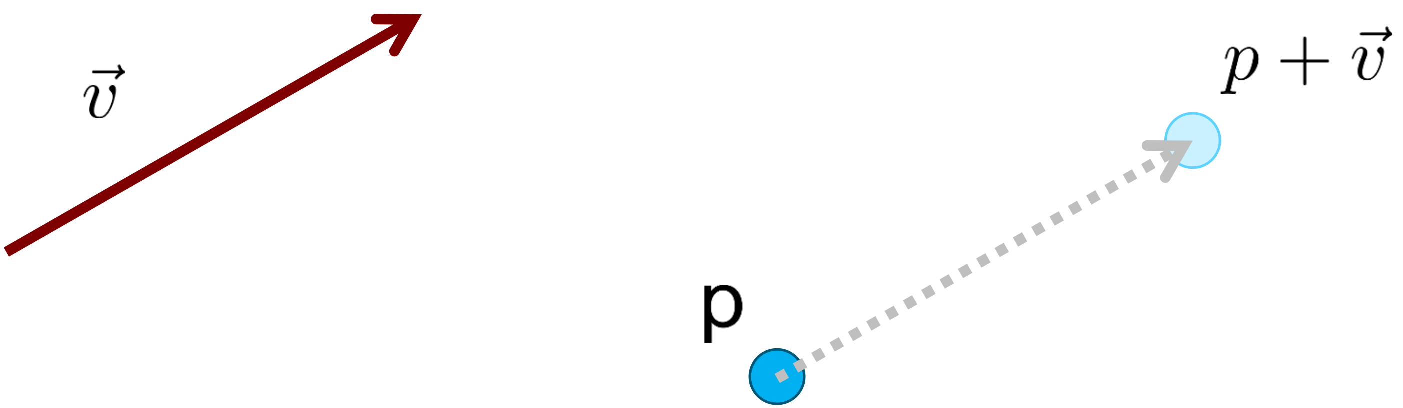


Vector Arithmetic

Subtracting points gives vectors

- Vector between p and q: q – p

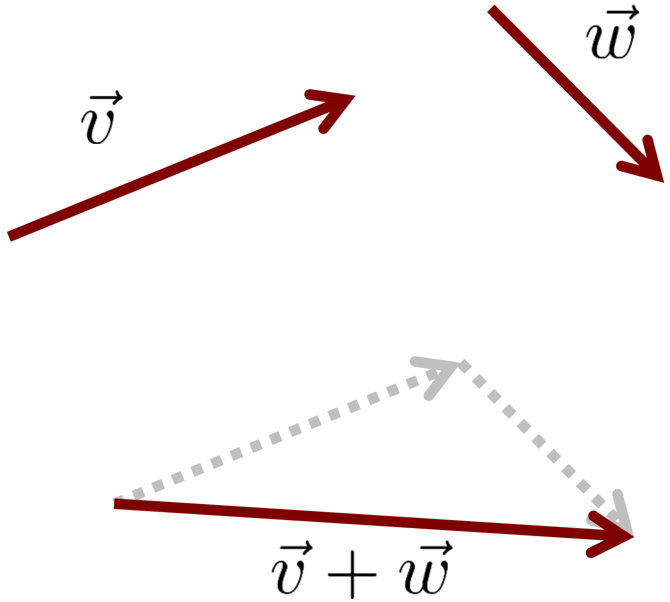
Add vector to point to get new point



Vector Arithmetic

Vectors can be

- added (tip to tail)



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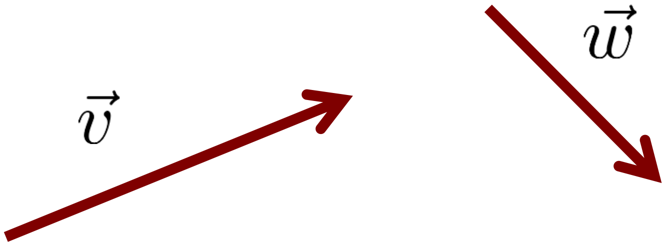
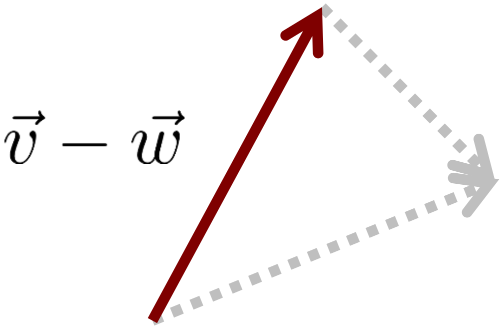


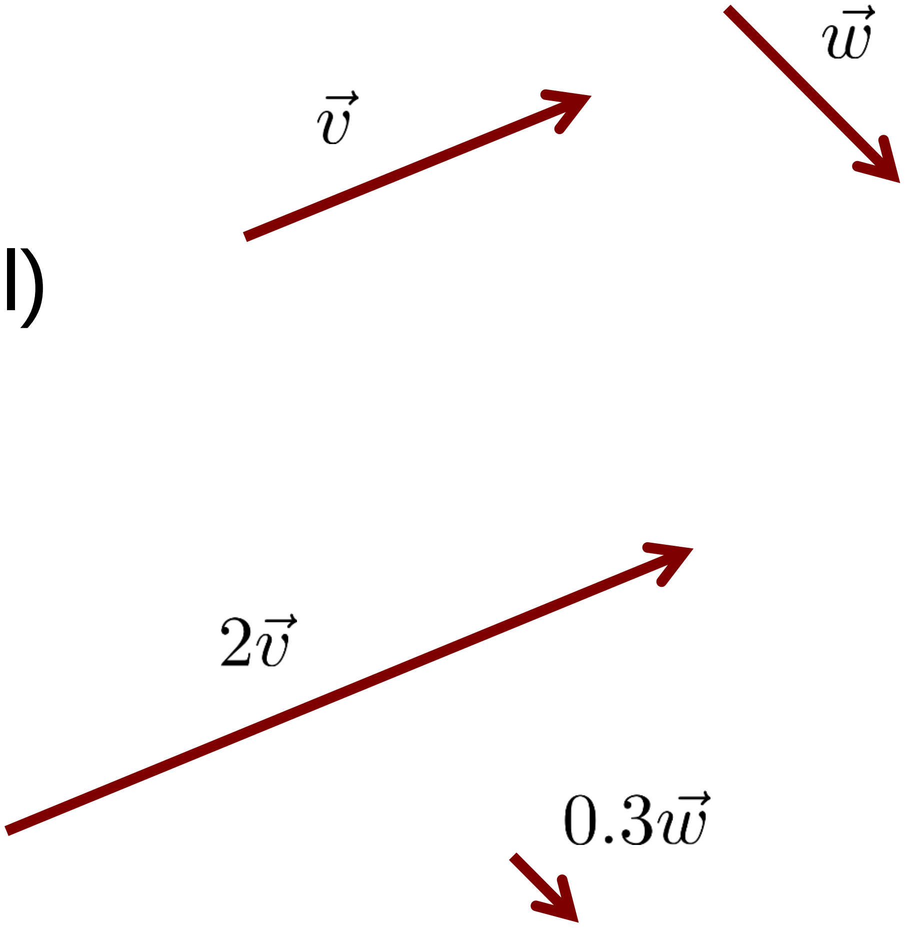
- subtracted



Vector Arithmetic

Vectors can be

- added (tip to tail)
- subtracted
- scaled

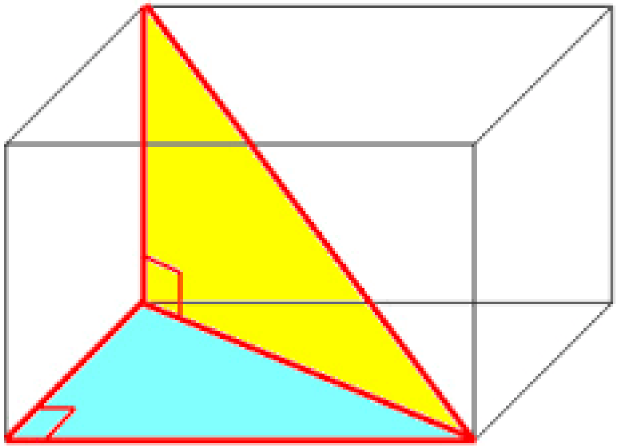


Vector Norm

Vectors have magnitude (length or **norm**)

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2 + \cdots}$$

- n-dimensional Pythagorean theorem

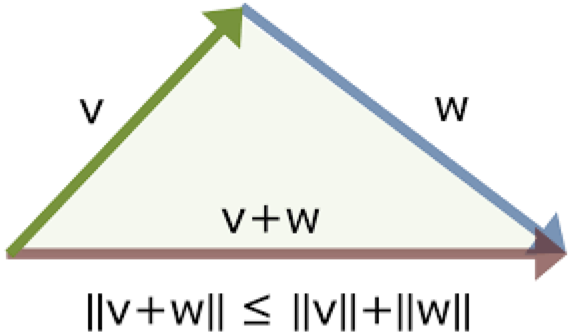


Vector Norm

Vectors have magnitude (length or **norm**)

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2 + \cdots}$$

Triangle inequality: $\|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$



Unit Vectors

Vectors with $||\vec{v}|| = 1$ **unit** or **normalized**

- encode pure direction

Borrowed from physics: “hat notation” $\hat{v}$

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Vectors with $\|\vec{v}\| = 1$ **unit** or **normalized**

- encode pure direction

Borrowed from physics: “hat notation” $\hat{v}$

Any non-zero vector can be normalized:

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

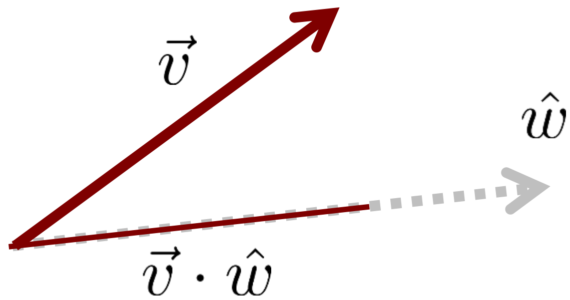
Dot Product

Takes two vectors, returns scalar

$$\vec{v} \cdot \vec{w} = v_x w_x + v_y w_y + v_z w_z + \cdots$$

- (works in any dimension)

$\vec{v} \cdot \hat{w}$ is length of $\vec{v}$ "in the $\hat{w}$ direction"



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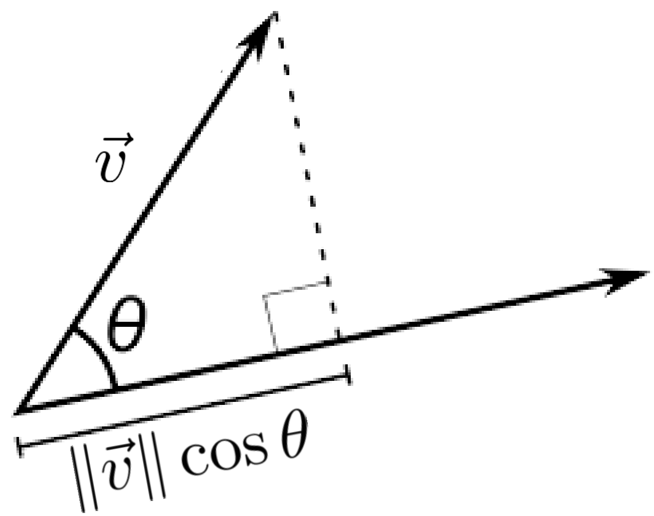
Alternate formula:

$$\vec{v} \cdot \hat{w} = \|\vec{v}\| \cos\theta$$



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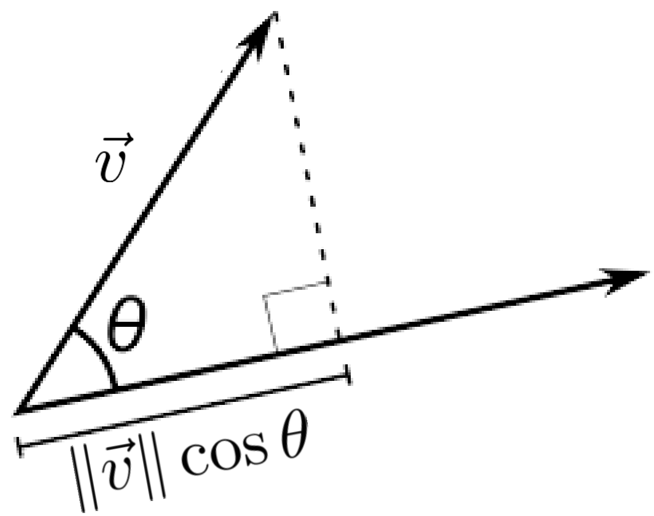


Dot Product Properties

Symmetry: $\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$

Linearity: $\vec{v} \cdot (\vec{w} + \vec{u}) = \vec{v} \cdot \vec{w} + \vec{v} \cdot \vec{u}$

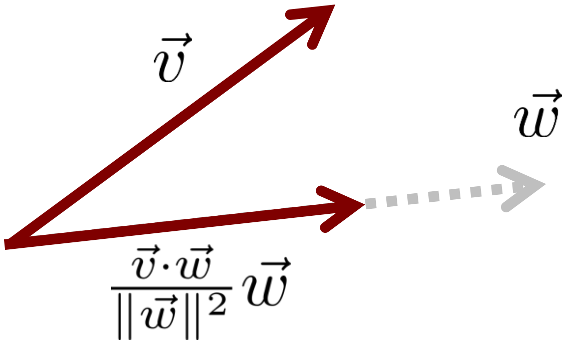
$$\vec{v} \cdot (\alpha \vec{w}) = \alpha(\vec{v} \cdot \vec{w})$$

Perpendicular vectors: $\vec{v} \cdot \vec{w} = 0$

Also note: $\vec{v} \cdot \vec{v} = \|v\|^2$

Projection

Projection of $\vec{v}$ onto $\vec{w}$



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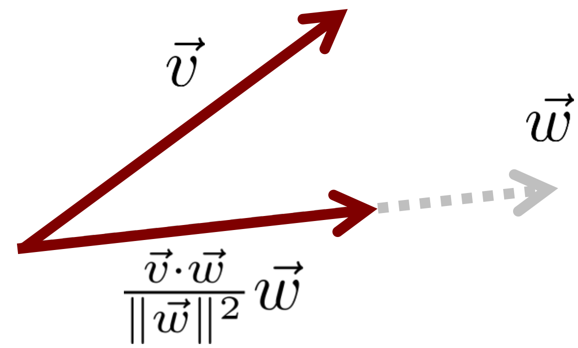
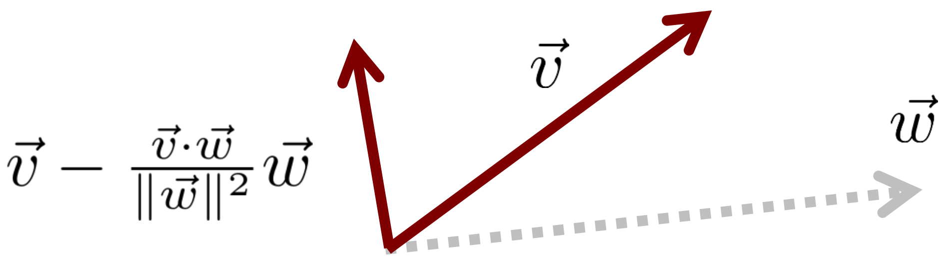


Can also project **out** component along $\vec{w}$



Dot Product and Angles

Note $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||\,||\vec{w}||}$ requires only multiplications and sqrts

Useful because trig calls are **slow**

Also in a pinch (slow): $\theta = \arccos\frac{\vec{v}\cdot\vec{w}}{||\vec{v}||\,||\vec{w}||}$

Cross Product

Takes two vectors, returns vector

$$\vec{v} \times \vec{w} = (v_y w_z - v_z w_y, v_z w_x - v_x w_z, v_x w_y - v_y w_x)$$

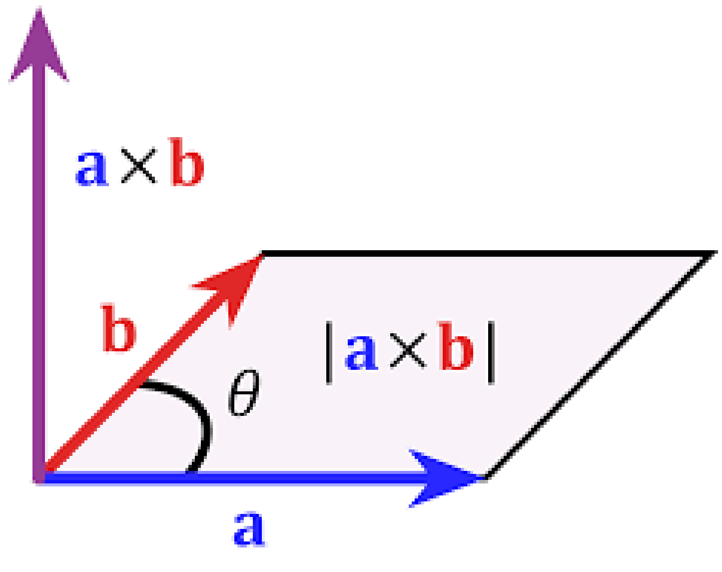
- works **only** in 3D

Direction: perpendicular to both $\vec{v}, \vec{w}$

Magnitude: $\|\vec{v} \times \vec{w}\| = \|\vec{v}\| \|\vec{w}\| \sin\theta$

Cross Product Intuition

Magnitude is **area of parallelogram** formed by vectors



Cross Product Intuition

There are two perpendicular directions.
Which direction is $\vec{v} \times \vec{w}$?

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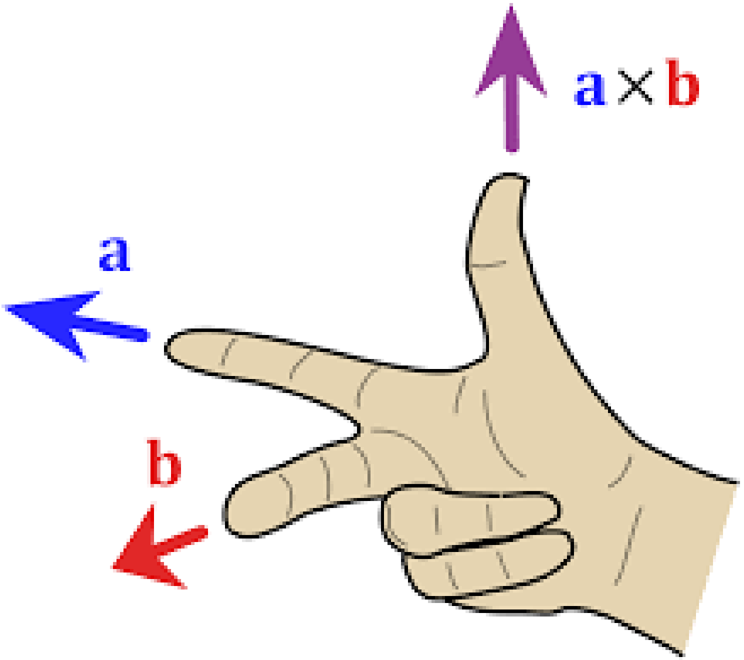
Right-hand rule



Cross Product Properties

Anti-symmetry: $\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}$ (why?)

Linearity: $\vec{v} \times (\vec{w} + \vec{u}) = \vec{v} \times \vec{w} + \vec{v} \times \vec{u}$

$$\vec{v} \times (\alpha\vec{w}) = \alpha(\vec{v} \times \vec{w})$$

Also note: $\vec{v} \times \vec{v} = 0$ (why?)

Cross Product Uses

Easily computes unit vector perpendicular to two given vectors

$$\hat{n} = \frac{u \times v}{\|u \times v\|}$$

(which one? right-hand rule)

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Easily computes unit vector perpendicular to two given vectors

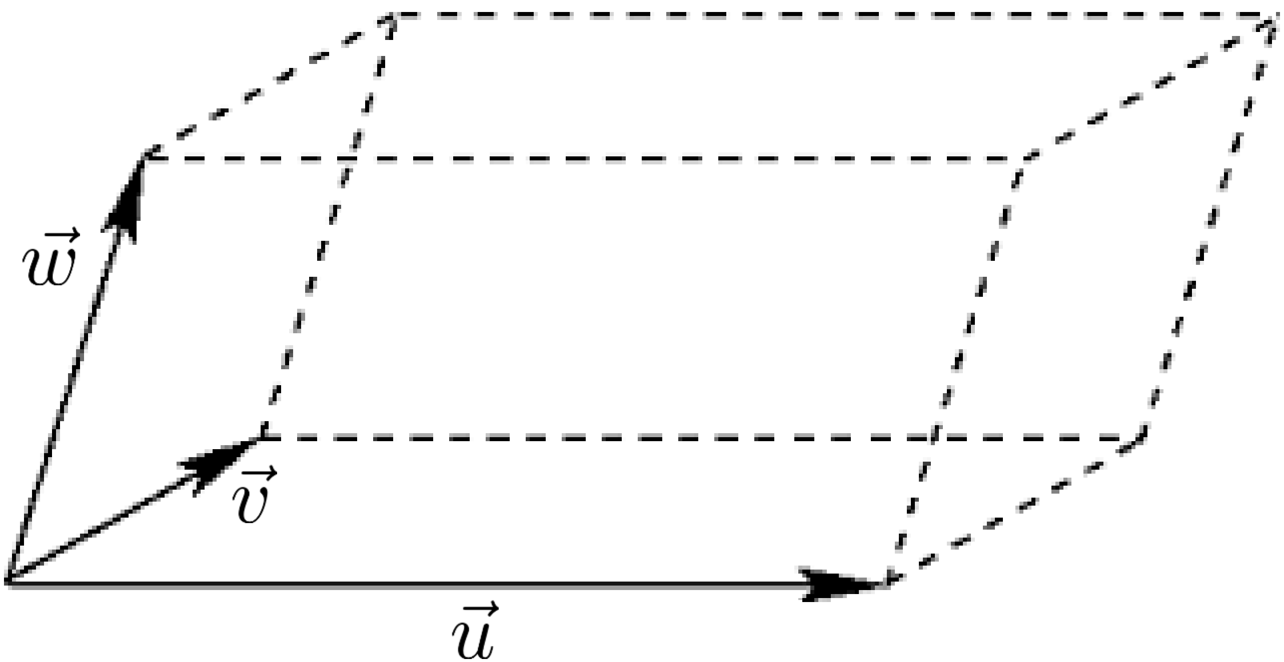
$$\hat{n} = \frac{u \times v}{\|u \times v\|}$$ (which one? right-hand rule)

Relation to angles: $\sin\theta = \frac{\|u \times v\|}{\|u\| \|v\|}$

Even better: $\tan\theta = \frac{\|u \times v\|}{u \cdot v}$ no sqrts!

Vector Triple Product

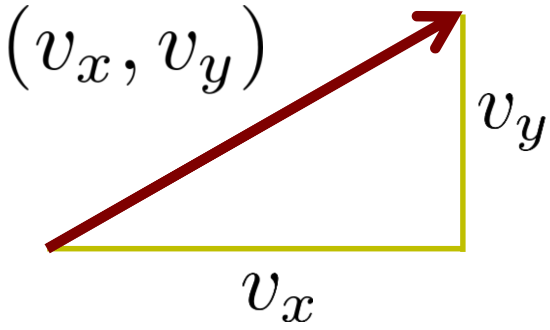
$\vec{w} \cdot (\vec{u} \times \vec{v})$ signed volume of **parallelepiped**



Euclidean Coordinates

A vector in 2D (v_x, v_y) can be interpreted as instructions

"move to the right v_x *and up* v_y*"*



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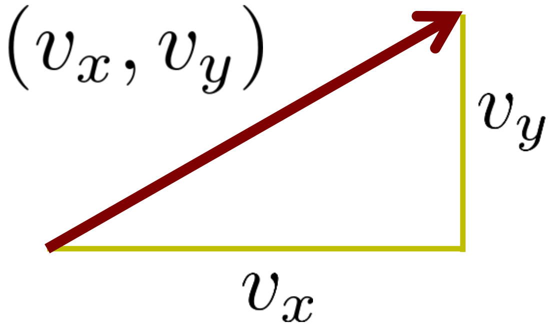
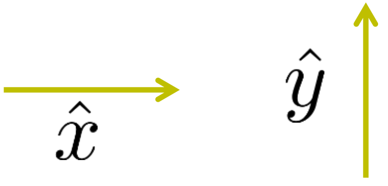


In other words: $(v_x, v_y) = v_x\hat{x} + v_y\hat{y}$



(Finite) Vector Spaces

We say: 2D vectors are **vector space** of vectors **spanned** by **basis vectors** $\{\hat{x}, \hat{y}\}$

- basis vectors: “directions” to travel
- span: all linear combinations

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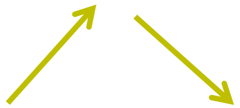

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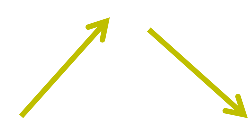
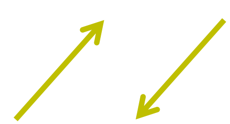


Of $\{(1,1),(-1,-1)\}$?

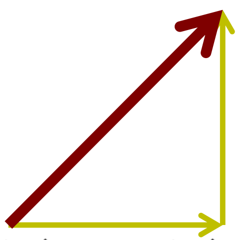


Vectors and Bases

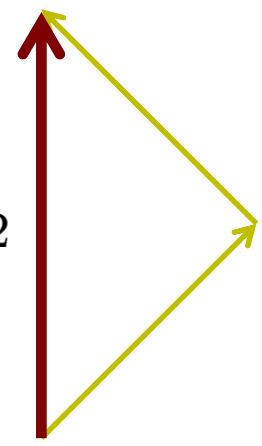
Consider V spanned by

$$\vec{v}_1 = (1, 1)$$
$$\vec{v}_2 = (1, -1)$$

Coordinates (1,1) can represent:



$1\hat{x} + 1\hat{y}$



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Key Point

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The same coordinates can represent different vectors

Vectors and Bases

- Consider V spanned by $\vec{v}_1 = (1,1)$, $\vec{v}_2 = (1,-1)$
- Coordinates (1,1) can represent: $1\hat{x} + 1\hat{y}$, $1\vec{v}_1 + 1\vec{v}_2$

Key Point

A vector (arrow in space) can have different coordinates in different bases

The same coordinates can represent different vectors

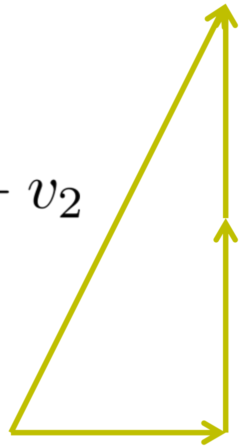
Default basis: Euclidean

Linear Dependence

Informal: vectors linearly dependent if they are redundant

$$v_1 = (0, 1, 0)$$
$$v_2 = (1, 0, 0)$$
$$v_3 = (2, 1, 0)$$

$$v_3 = 2v_1 + v_2$$



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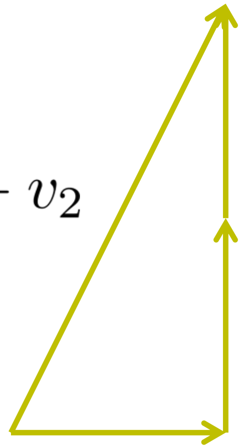

$$\text{span}\{v_1, v_2, v_3\} = \text{span}\{v_1, v_2\}$$

Linear Dependence

Informal: vectors linearly dependent if they are redundant

Formal: basis $\{\vec{b}_i\}$ linearly independent if

$$\sum_i \alpha_i \vec{b}_i = 0 \quad \Rightarrow \quad \alpha_i = 0$$

Dimension

Dimension is size of biggest set of linearly independent basis vectors

Examples:

$$\dim\{\hat{x}, \hat{y}, \hat{z}\} = 3$$

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Dimension

Dimension is size of biggest set of linearly independent basis vectors

Examples:

$$\dim\{\hat{x}, \hat{y}, \hat{z}\} = 3$$

$$\dim\{(1,1,0),(1,-1,0),(0,0,1)\} = 3$$

$$\dim\{(1,1),(0,0),(-3,-3)\} = 1$$

Geometry of Dimension

Adding all vectors of vector space V to p

If $\dim V =$:

- 0:

Geometry of Dimension

Adding all vectors of vector space V to p

If $\dim V =$:

- 0: just p
- 1:

Geometry of Dimension

Adding all vectors of vector space V to p

If $\dim V =$:

- 0: just p
- 1: line through p
- 2: plane through p
- 3+: hyperplane through p

Matrix

Matrix $A_{n \times m}$ array with n rows, m columns

$$A_{2\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Notes:

- 1) first row, then column

Matrix

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Notes:

- 1) first row, then column
- 2) one-indexed

Matrix Addition and Scaling

Can add two matrices of same size:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \end{bmatrix}$$

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Scalar multiplication works as expected:

$$\alpha\left[\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{array}\right]=\left[\begin{array}{ccc} \alpha a_{11} & \alpha a_{12} & \alpha a_{13} \\ \alpha a_{21} & \alpha a_{22} & \alpha a_{23} \end{array}\right]$$

Matrix Multiplication

Can multiply matrices $A_{n \times m}, B_{m \times p}$; get $(AB)_{n \times p}$

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$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3\times 3} = \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}_{2\times 3}$$

Term $(ab)_{ij}$ is dot product of i-th **row** of A with j-th **column** of B

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Term $(ab)_{ij}$ is dot product of i-th **row** of A with j-th **column** of B

Is associative: $(AB)C = A(BC)$

Multiplication Not Commutative!

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} =$$

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Multiplication Not Commutative!

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix}$$

Has tripped up even professors…

Special Case: Vectors

Vectors are represented using **column** matrices:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a_{11}v_1 + a_{12}v_2 + a_{13}v_3 \\ a_{21}v_1 + a_{22}v_2 + a_{23}v_3 \\ a_{31}v_1 + a_{32}v_2 + a_{33}v_3 \end{bmatrix}$$

Can treat vectors like $n \times 1$ matrix

Special Case: Vectors

Mathematically,

$$(AB)v = A(Bv)$$

Special Case: Vectors

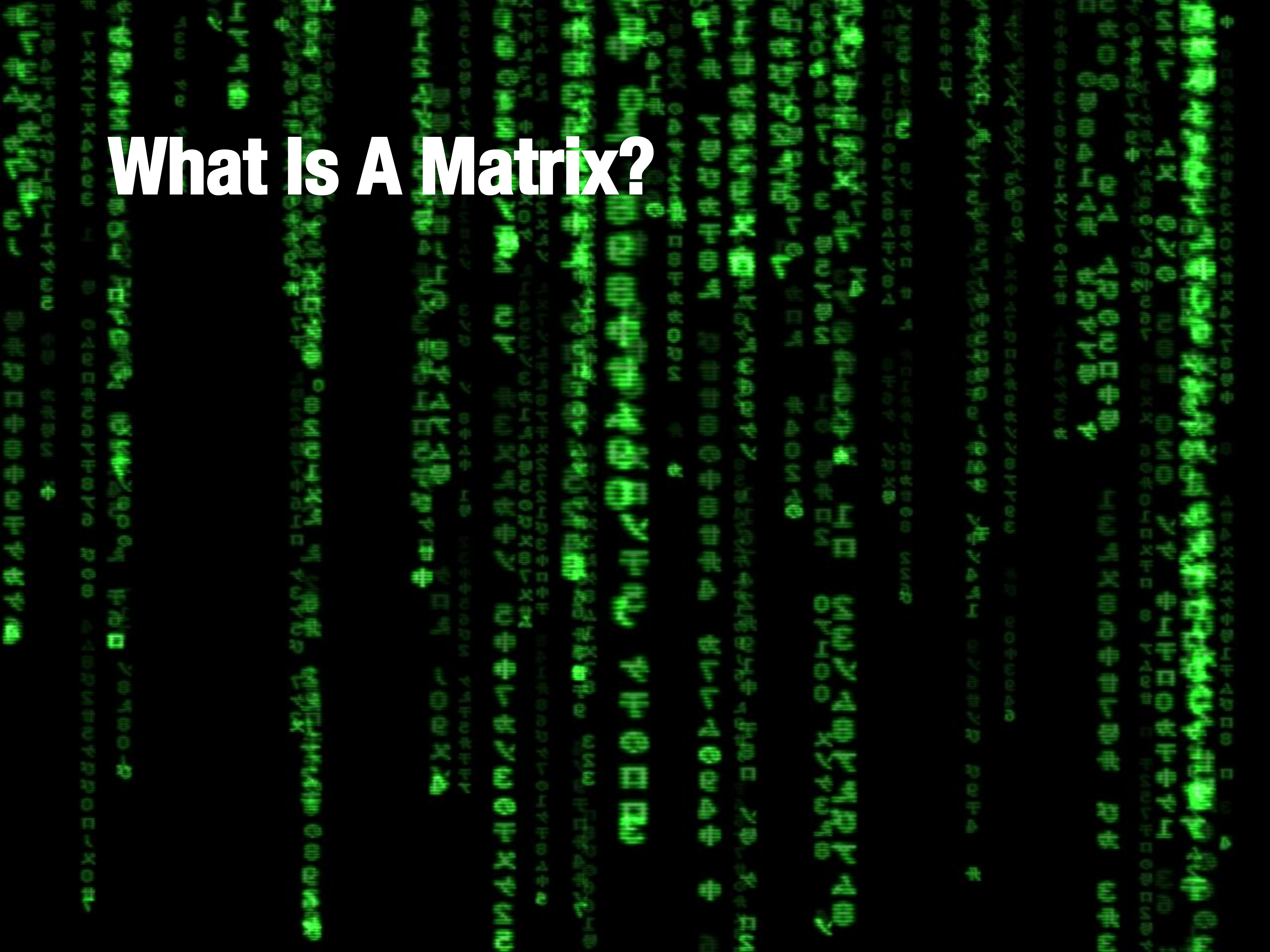
Mathematically,

$$(AB)v = A(Bv)$$

In practice: **slow** $O(n^3)$ OK $O(n^2)$

Avoid matrix-matrix multiplies

What Is A Matrix?



Interpretation 1: Row Products

Matrix multiplication is dot product of vector with **rows**

$$\begin{bmatrix} \vec{a}_{1\star} \\ \hline \vec{a}_{2\star} \\ \hline \vec{a}_{3\star} \end{bmatrix} \begin{bmatrix} \\ \vec{v} \\ \\ \end{bmatrix} = \begin{bmatrix} \vec{a}_{1\star} \cdot \vec{v} \\ \vec{a}_{2\star} \cdot \vec{v} \\ \vec{a}_{3\star} \cdot \vec{v} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a_{11}v_1 + a_{12}v_2 + a_{13}v_3 \\ a_{21}v_1 + a_{22}v_2 + a_{23}v_3 \\ a_{31}v_1 + a_{32}v_2 + a_{33}v_3 \end{bmatrix}$$

Interpretation 2: Column Sums

Matrix multiplication is linear combination of **columns**

$$\left[\begin{array}{c|c|c} \vec{a}_{\star 1} & \vec{a}_{\star 2} & \vec{a}_{\star 3} \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1 \vec{a}_{\star 1} + v_2 \vec{a}_{\star 2} + v_3 \vec{a}_{\star 3} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a_{11}v_1 + a_{12}v_2 + a_{13}v_3 \\ a_{21}v_1 + a_{22}v_2 + a_{23}v_3 \\ a_{31}v_1 + a_{32}v_2 + a_{33}v_3 \end{bmatrix}$$

Interpretation 3: Change of Coord

Matrix A transforms vector:

- **from** basis $\{\vec{a}_{\star 1}, \vec{a}_{\star 2}, \vec{a}_{\star 3}\}$
- **to** Euclidean basis

$$\left[\begin{array}{c|c|c} \vec{a}_{\star 1} & \vec{a}_{\star 2} & \vec{a}_{\star 3} \end{array}\right] \left[\begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array}\right] = \left[\begin{array}{c} v_1 \vec{a}_{\star 1} + v_2 \vec{a}_{\star 2} + v_3 \vec{a}_{\star 3} \end{array}\right]$$

Identity Matrix

Square diagonal matrix

$$I_{n \times n} = \begin{bmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Satisfies

- $I_{n \times n} A_{n \times m} = A_{n \times m}$
- $A_{n \times m} I_{m \times m} = A_{n \times m}$
- $I\vec{v} = \vec{v}$

Inverse Matrix

For some square matrices, inverse exists

$$A^{-1}A = AA^{-1} = I$$

$$Ax = b \Rightarrow x = A^{-1}b$$

Useful identity:

$$(AB)^{-1} = B^{-1}A^{-1}$$

Inverse Matrix

For some square matrices, inverse exists

$$A^{-1}A = AA^{-1} = I$$

$$Ax = b \Rightarrow x = A^{-1}b$$

Matrix A^{-1} transforms vector:

- **from** Euclidean basis
- **to** basis $\{\vec{a}_{\star 1}, \vec{a}_{\star 2}, \vec{a}_{\star 3}\}$

Inverse Matrix

For some square matrices, inverse exists

$$A^{-1}A = AA^{-1} = I$$

$$Ax = b \Rightarrow x = A^{-1}b$$

When does inverse exist?

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}^{-1} = ?$$

Inverse Matrix

For some square matrices, inverse exists

$$A^{-1}A = AA^{-1} = I$$

$$Ax = b \Rightarrow x = A^{-1}b$$

Note:

- Computing A^{-1} is **slow**: $O(n^3)$

Inverse Matrix

For some square matrices, inverse exists

$$A^{-1}A = AA^{-1} = I$$

$$Ax = b \Rightarrow x = A^{-1}b$$

Note:

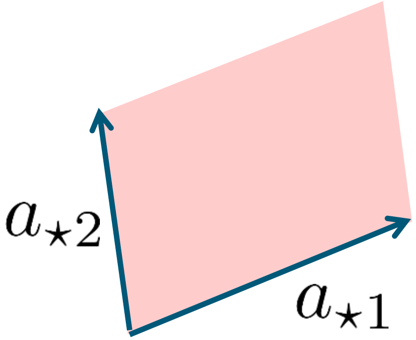
- Computing A^{-1} is **slow**: $O(n^3)$
- For small matrices (4 x 4), not too bad
- For big matrices, inverse is never computed explicitly

Determinant

Maps square matrix to real number

In 2D: $$\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Measures signed volume of parallelogram

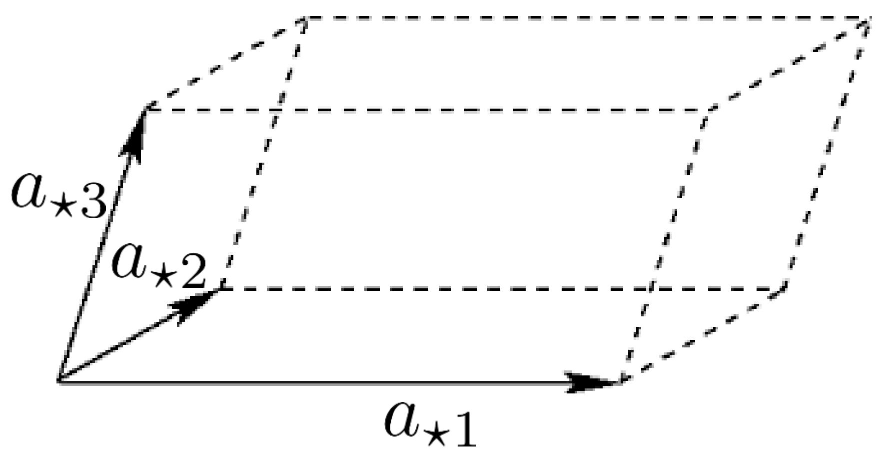


Determinant

Maps matrix to real number

In 3D: $\det \left[\vec{a}_{\star 1} \middle| \vec{a}_{\star 2} \middle| \vec{a}_{\star 3} \right]$ vol. of parallelepiped

(Why do we care?)



Determinant

Maps matrix to real number

In 3D: $\det \left[\begin{array}{c|c|c} \vec{a}_{\star 1} & \vec{a}_{\star 2} & \vec{a}_{\star 3} \end{array} \right]$ vol. of parallelepiped

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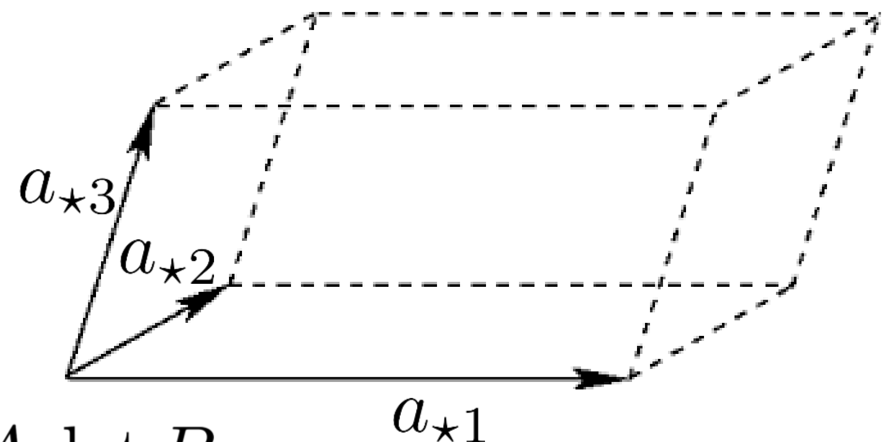

Useful: $\det(AB) = \det A \det B$

Transpose

Flips indices; "reflect about diagonal"

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

Transpose of vector is **row vector**

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}^T = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$

Transpose

Flips indices; “reflect about diagonal”

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

What is $v^T w$?

What is $v^T v$?

Transpose

Flips indices; “reflect about diagonal”

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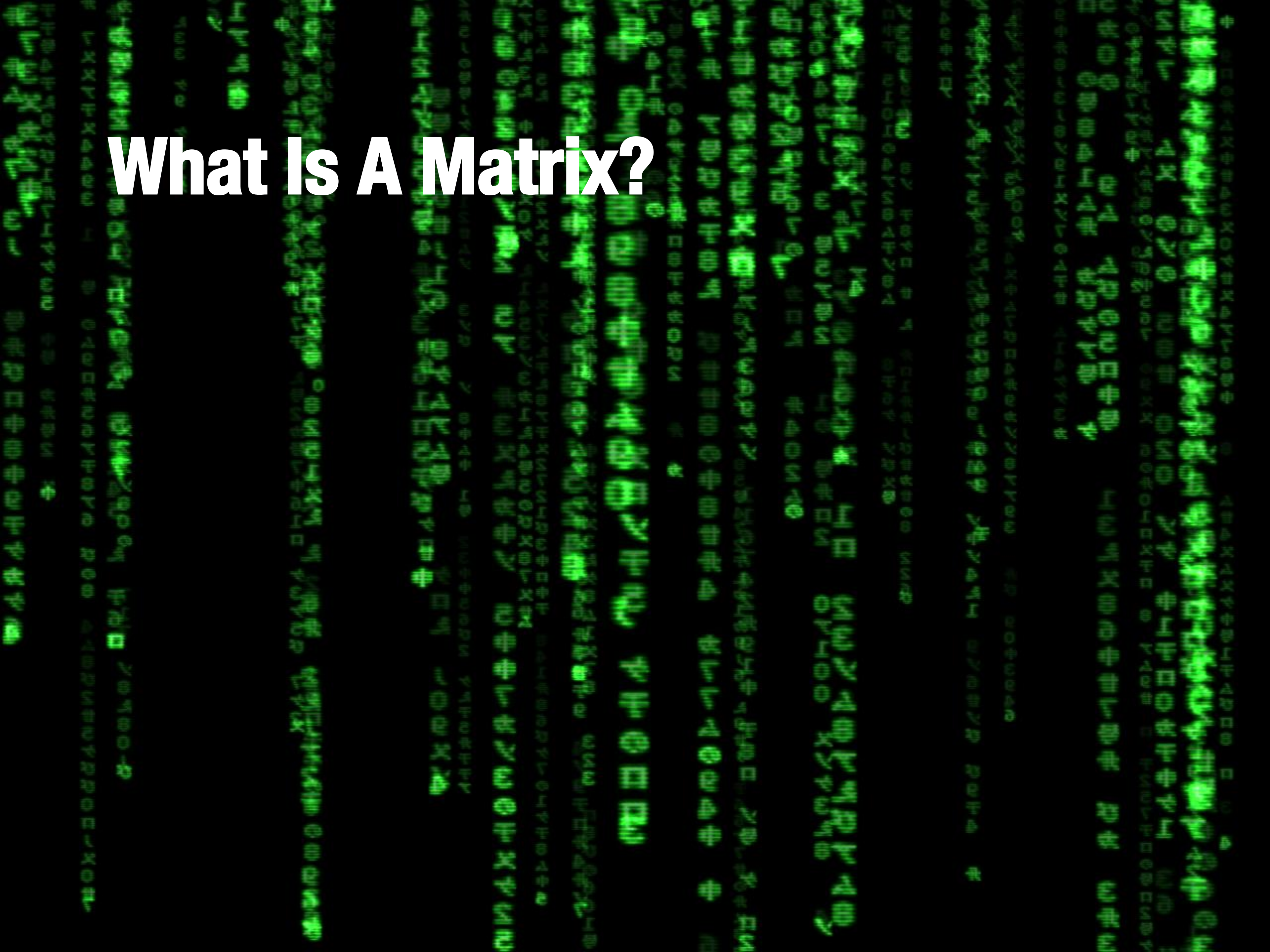
What is $v^T w$?

What is $v^T v$?

Useful identity: $(AB)^T = B^T A^T$

What Is A Matrix?



Interpretation 4: Any Linear Func.

For every **linear** function $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ of vectors to vectors, there exists $A_{m \times n}$ with: $f(v) = Av$

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- Linear: $f(v + w) = f(v) + f(w)$
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- Why true? Look at basis elements