Relational STE and Theorem Proving for Formal Verification of Industrial Circuit Designs

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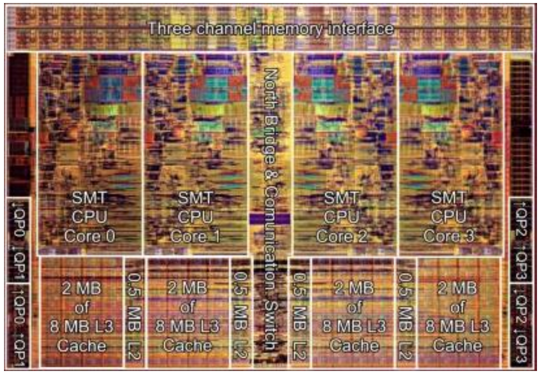
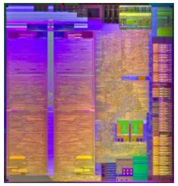
CPU datapath verification at Intel

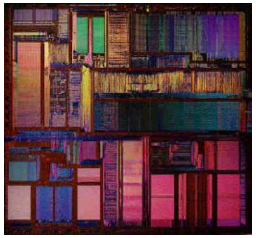
- Thousands of operations
  - Integer, FP, SSE, AVX, …
  - “Miscellaneous”
  - Various operating modes, flags, faults
- Live RTL, changing frequently until a few weeks before tapeout

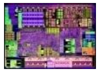


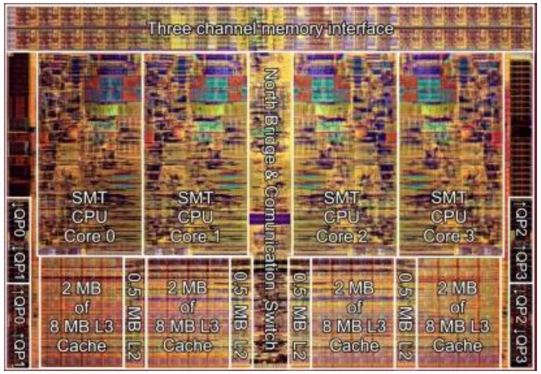
Scaling up

- Tens of designs
- Different optimization points
- Different teams
- Different countries
- Not only CPUs
- Not all have FV experts on staff



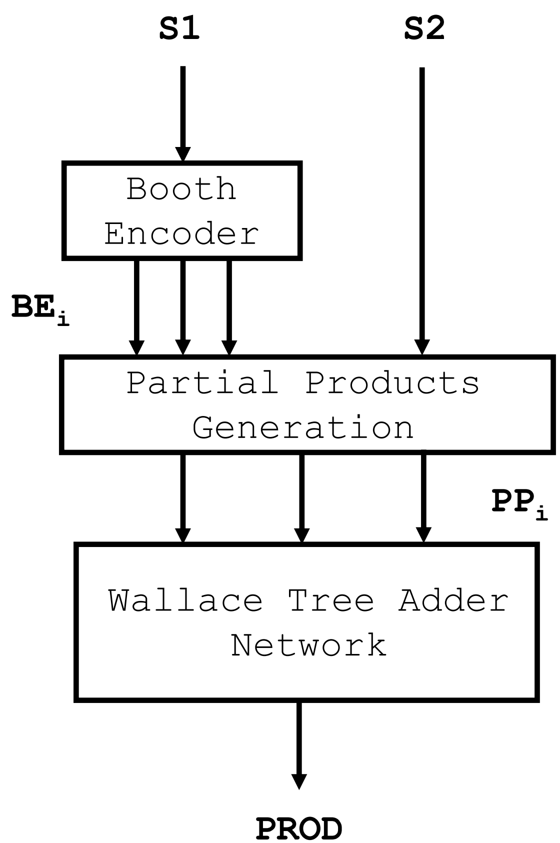






Integer multiplier

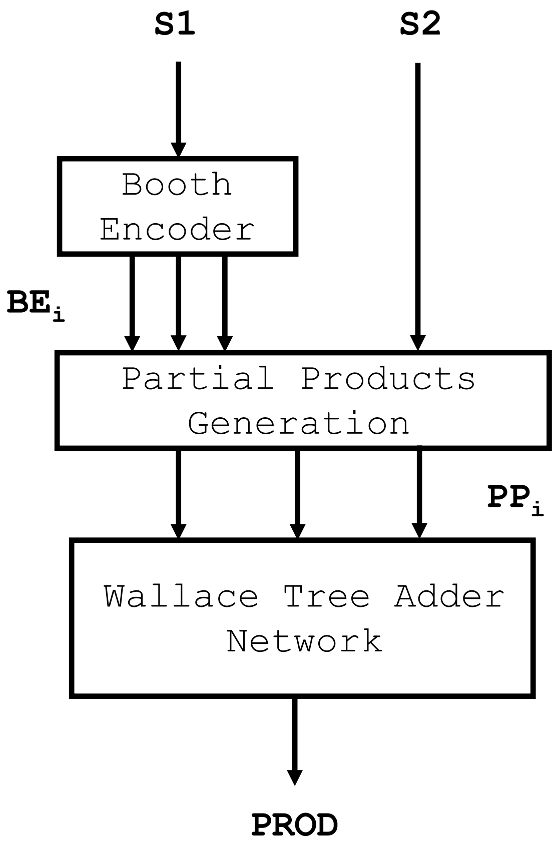
$S1 = \sum BE_i * 2^{ki}$



$PP_i = S2 * BE_i$

$P = \sum PP_i * 2^{ki}$

The multiplier zoo



- 10-20 multipliers
- Hand designed
- Hand optimized
- All different

FV challenges

- Varying specs and verification strategies
  - Implementation changes from design to design
  - Multiplier always requires decomposition
- Ten designers but not ten multiplier FV experts

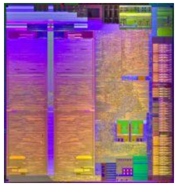
- Same story for integer, MMX, FP, SSE, GPU flavors of multiplication, addition, division, …
  - Some operations require even more intricate decomposition

The solution

Regression

Development

Parameters → CVE → Per-design specs
Verification runs



Deduction

$$\vdash \text{specs} + \text{runs} \Rightarrow \text{correct}$$

The solution *done right*

- An executable logic for writing the specs and verification scripts: reFLect
- A symbolic simulator that admits relational specifications written in logic: rSTE
- A tightly integrated theorem prover for executing the deductive proofs: Goaled

The solution *done right*

- An executable logic for writing the specs and verification scripts: applicative common lisp
- A symbolic simulator that admits relational specifications written in logic: ESIM+GL
- A tightly integrated theorem prover for executing the deductive proofs: ACL2

[Slobodová *et al*, MEMOCODE’11]

The reFLect Language

- Core syntax:

$$n,o,p ::= \mathrm{k} \mid v \mid n\ o \mid \underbrace{\lambda p.\, n \,\square\, o}_{\textit{pattern matching}} \mid \underbrace{\langle n \rangle \mid \hat{}\,n{:}\sigma}_{\textit{reflection}}$$

- … plus extensions driven by necessity
  - BDDs built in as a primitive type
  - Quotient types
  - Overloading
  - Named function parameters
  - Records
  - Possibly unsafe features: references, I/O, recursion

Higher Order Logic of reFLect

- HOL, following Church:

$$\text{Logic} = \begin{cases} \lambda\text{-calculus} \\ \quad + \\ \text{logical constants} \\ \quad + \\ \text{rules} \end{cases}$$

- The reFLect logic:

$$\text{Logic} = \begin{cases} \text{reFLect} \\ \quad + \\ \text{logical constants} \\ \quad + \\ \text{rules} \end{cases}$$

- Basic idea in both systems:

  $n \to p$ means $\vdash n = p$

  Define $\forall$, $\exists$, etc by axioms

  Add rules for function equality

  Proof by evaluation

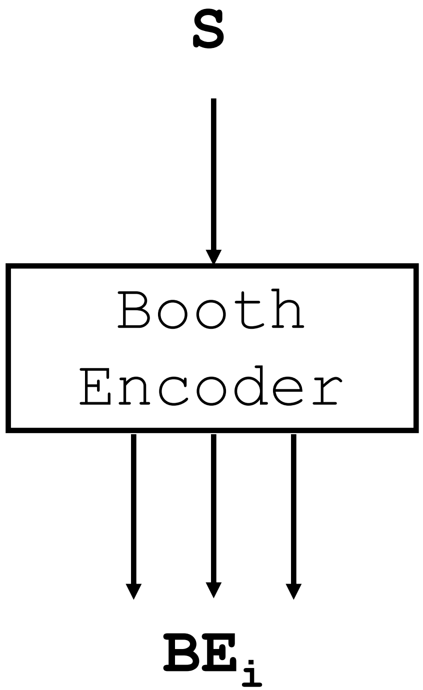
Goaled Theorem Prover

- LCF-style implementation, following in the footsteps of HOL and HOL Light
  - Thm is a protected data type, constructible only through a small set of trusted function calls (a.k.a. inference rules)
- Features driven by necessity
  - Theories: of reFLect data types, natural numbers, integers, rationals, lists, pairs, reFLect ADTs
  - Proof automation: rewriting, first order solving, linear arithmetic
  - Bitstring arithmetic
  - Support for the reflect language extensions

The last bit

- An executable logic for writing the specs and verification scripts: reFLect
- A symbolic simulator that admits relational specifications written in logic: rSTE
- A tightly integrated theorem prover for executing the deductive proofs: Goaled

Limitations of STE



$$S = \sum_{i=0}^{N-1} BE_i * 2^{ki}$$

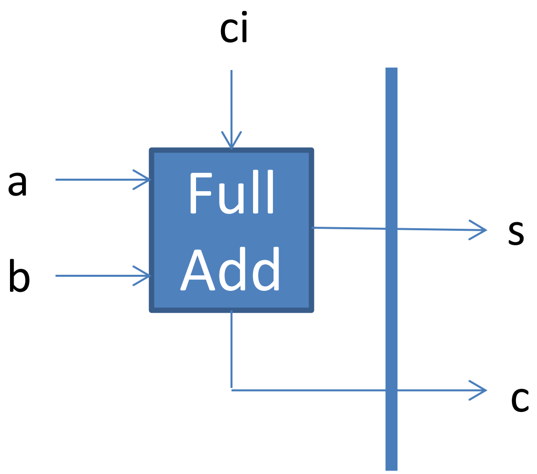
- Trajectory assertion:
  - ckt |= [[S is v ==>> (BE_i is f_i(v))]]

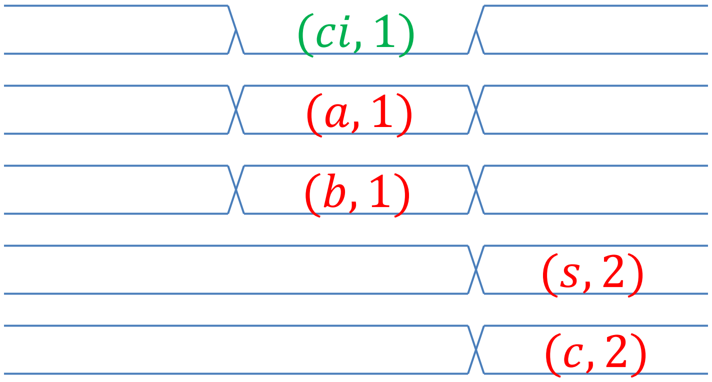
- But,
  - You need a special purpose reasoning system for this special purpose logic
  - Relational specifications cannot be expressed directly

Relational STE

- STE’s antecedent and consequent are replaced with lists of constraints
  - A constraint is a relationship between a finite set of circuit nodes at specified points in time
- Idea:
  - *rSTE ckt cin cout* means “In any behavior of *ckt* in which all of the constraints *cin* hold, all of the constraints *cout* hold”

Relational STE Intuition





rSTE ckt

$$["!\,(ci, 1)"]\ ["(a, 1) + (b, 1) = (s, 2) + 2 \times (c, 2)"]$$

Constraints

- A constraint c has three components:
  - name(c) : $string$
  - sig(c) : $(string \times num)\ list$
  - pred(c) : $\big((string \times num) \rightarrow bool\big) \rightarrow bool$

- The behavior of the circuit is also formulated as a constraint:

$$[\![ckt]\!] : \ ((string \times num) \rightarrow bool) \rightarrow bool$$

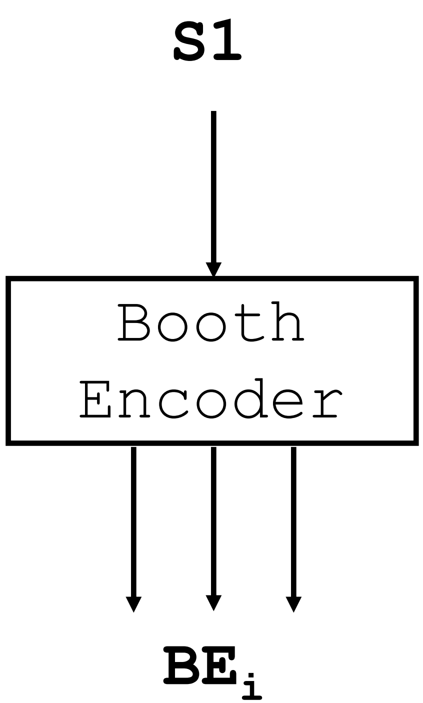
From Relational STE to Logic

- Theorem:

$$\forall ckt\ cin\ cout.$$
$$\quad rSTE\ ckt\ cin\ cout \Rightarrow$$
$$\quad\quad \forall e. [\![ckt]\!] e \Rightarrow$$
$$\quad\quad\quad predl\ cin\ e \Rightarrow predl\ cout\ c$$

- For lists of constraints,
  - $predl\ []\ e\ \triangleq T$
  - $predl\ (c::cs)\ e\ \triangleq\ pred(c)\ e\ \wedge\ predl(cs)\ e$

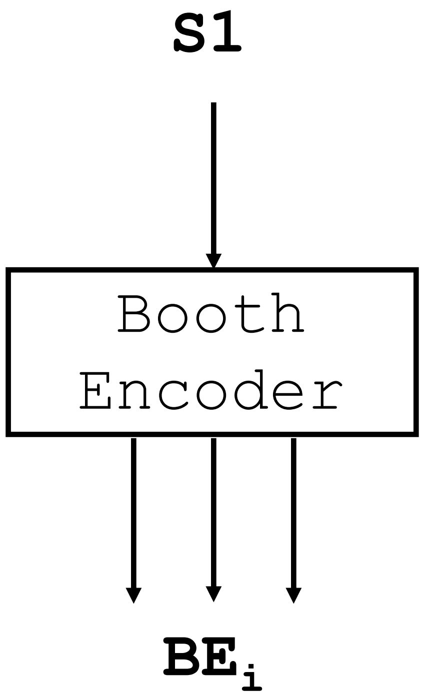
Relational STE in Action



- Define *boothc* such that
  - $pred(boothc) = \lambda e.\, eqn1(s2i\ e\ s1)$
  - $eqn1(x) \triangleq (x = \sum_{i=0}^{N-1} BE_i(x) \times 2^{ki})$
- Then, *rSTE ckt* $[]\ [boothc] \rightarrow T$ implies

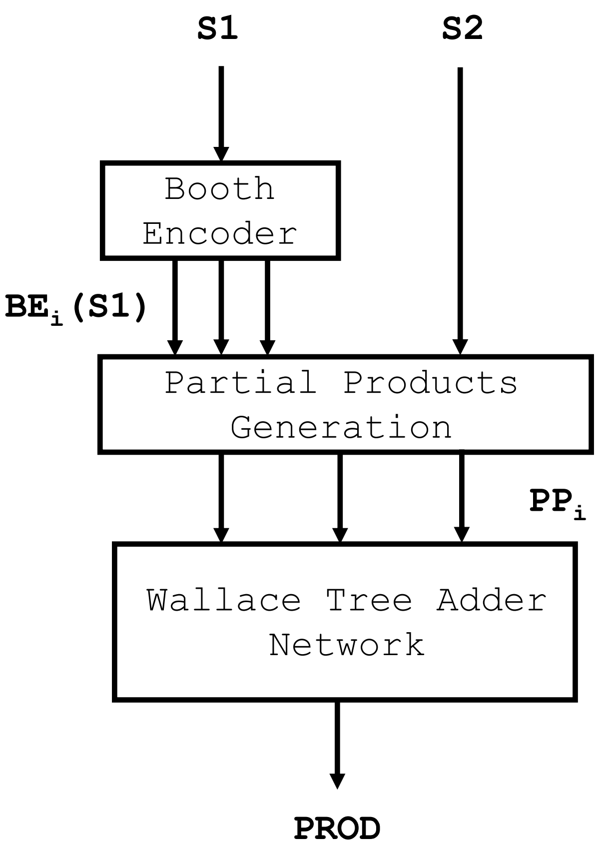
$$\forall e. [\![ckt]\!] e \Rightarrow$$
$$pred l\ []\ e \Rightarrow pred l\ [boothc]\ e$$

Relational STE in Action



- $\forall e. [\![ckt]\!]e \Rightarrow$
  $\quad pred l\ [\,]\ e \Rightarrow pred l\ [boothc]\ e$
- $\forall e. [\![ckt]\!]e \Rightarrow pred(boothc)\ e$
- $\forall e. [\![ckt]\!]e \Rightarrow eqn1(s2i\ e\ s1)$
- $\forall e. [\![ckt]\!]e \Rightarrow$
  $\quad \left(s2i\ e\ s1 = \sum_{i=0}^{N-1} BE_i(s2i\ e\ s1) \times 2^{ki}\right)$

Completing a Multiplier proof



$$\forall e. [\![ckt]\!] e \Rightarrow \left(\bigwedge s2i\ e\ pp_i = BE_i(s2i\ e\ s1) \times s2i\ e\ s2\right)$$

$$\forall e. [\![ckt]\!] e \Rightarrow \left(s2i\ e\ prod = \sum_{i=0}^{N-1} (s2i\ e\ pp_i) \times 2^{ki}\right)$$

$$\forall e. [\![ckt]\!] e \Rightarrow \left(s2i\ e\ s1 = \sum_{i=0}^{N-1} BE_i(s2i\ e\ s1) \times 2^{ki}\right)$$

$$\rightsquigarrow$$

$$\forall e. [\![ckt]\!] e \Rightarrow \left(s2i\ e\ prod = s2i\ e\ s1 \times s2i\ e\ s2\right)$$

Proof engineering

- Additional arguments to rSTE
  - Constant antecedent: clock, reset
  - rSTE options: bdd variable ordering, param, …
  - Not shown here, but see paper
- Analysis of CVE verification scripts
  - N layers of function calls between input parameters and generation of specs
  - Much deductive effort toward exposing the specs
  - Routine rewriting, also not shown here

Status and prospects

- reFLect and rSTE are the main workhorses of datapath verification across Intel
- Frameworks for integer and FP multipliers, FMAs, adders, divide/sqrt are widely deployed
- Goaled checking of integer multipliers is used on a mainline design project and being pushed to others
- We plan to integrate Goaled checking with our other frameworks