

SWAPPER: A Framework for Automatic Generation of Formula Simplifiers based on Conditional Rewrite Rules

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Massachusetts Institute of Technology

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Mountain View, CA, USA

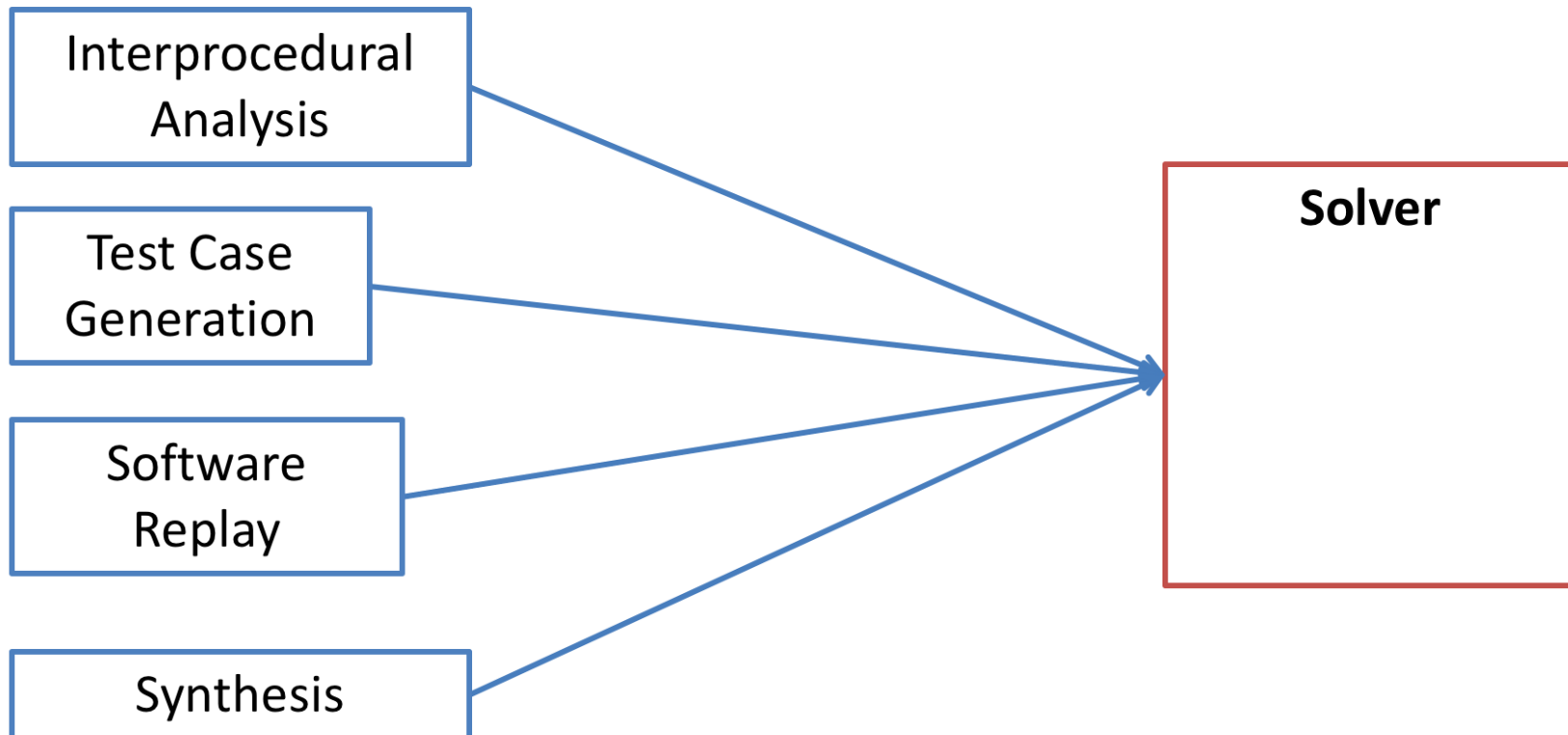
General Constraint Solvers

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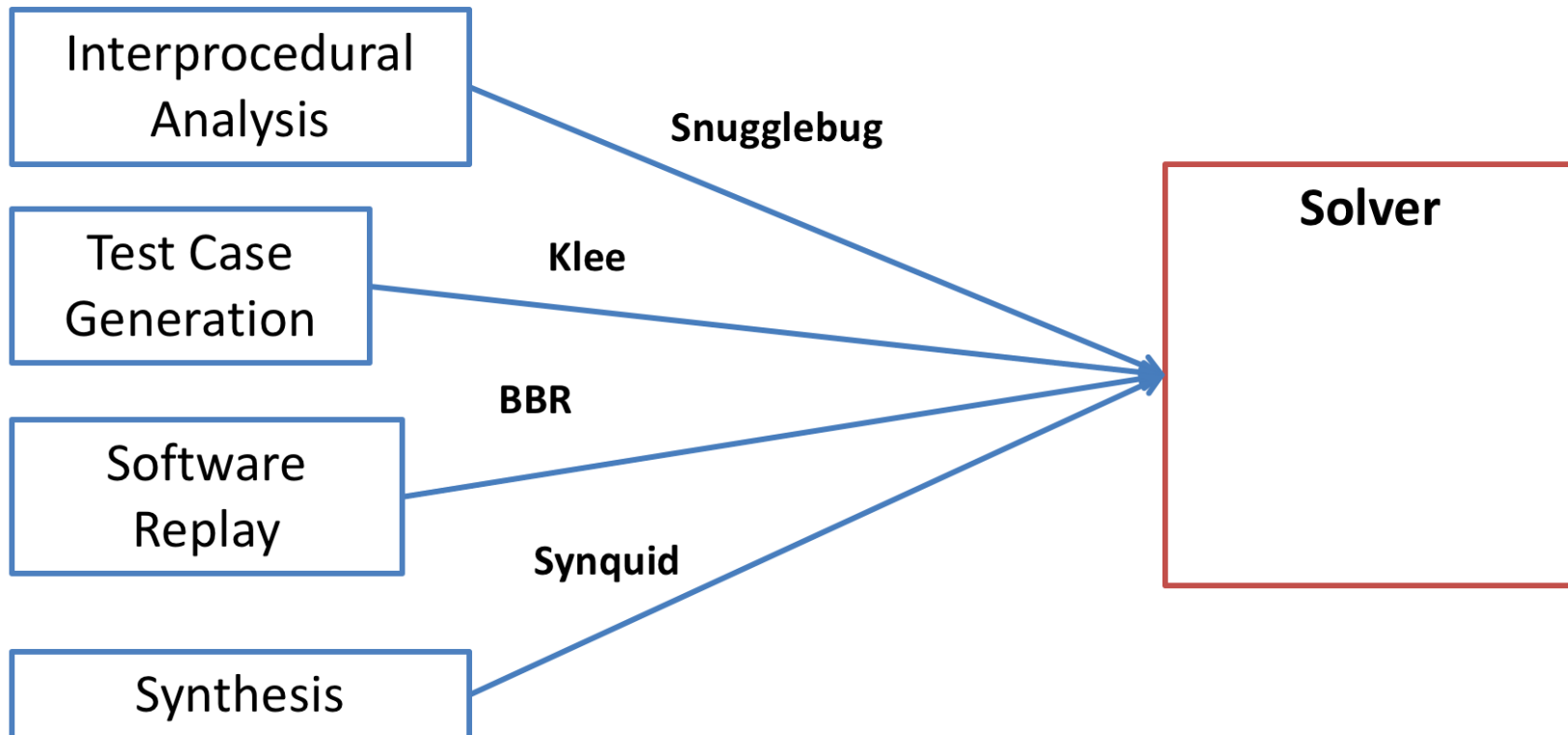


Solver

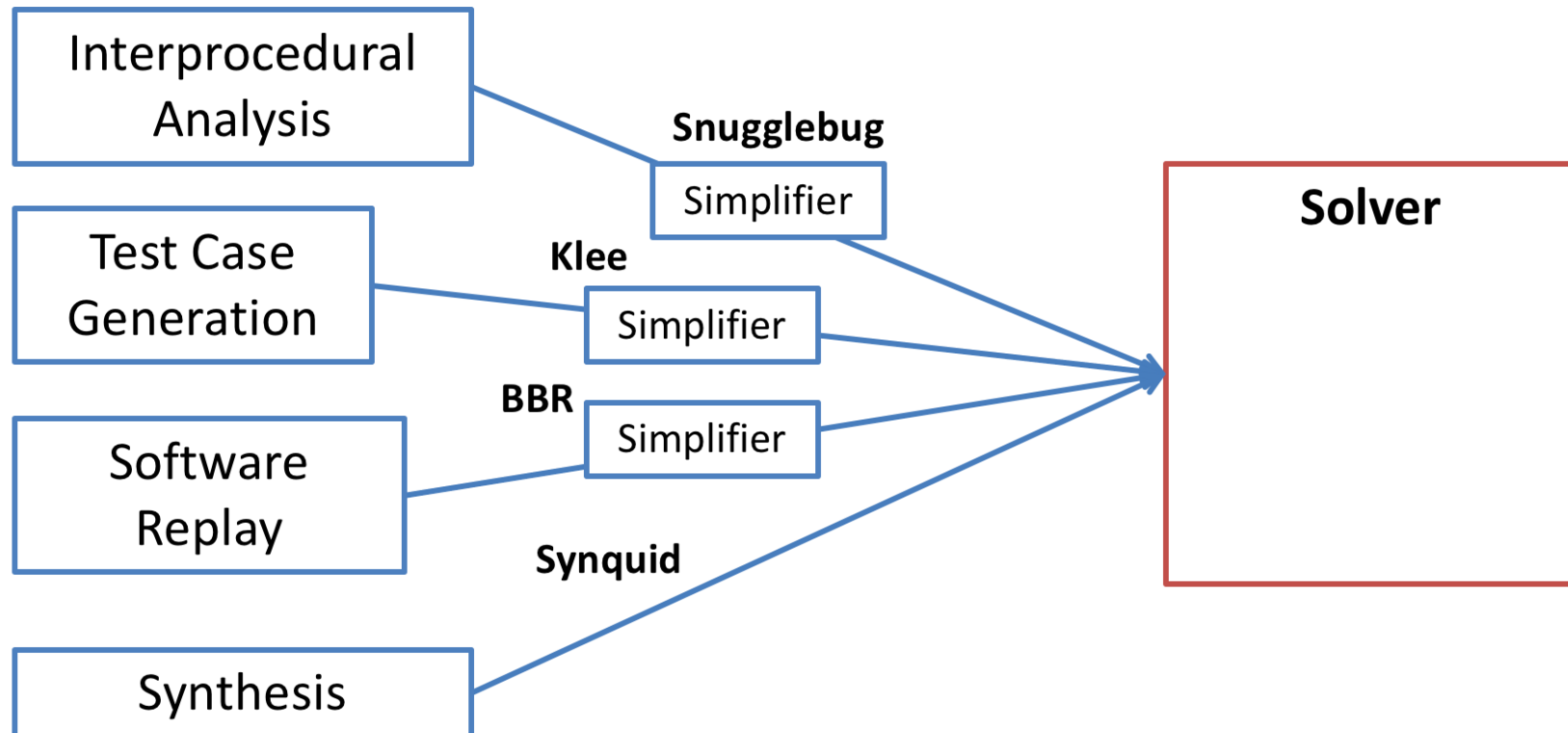
General Constraint Solvers



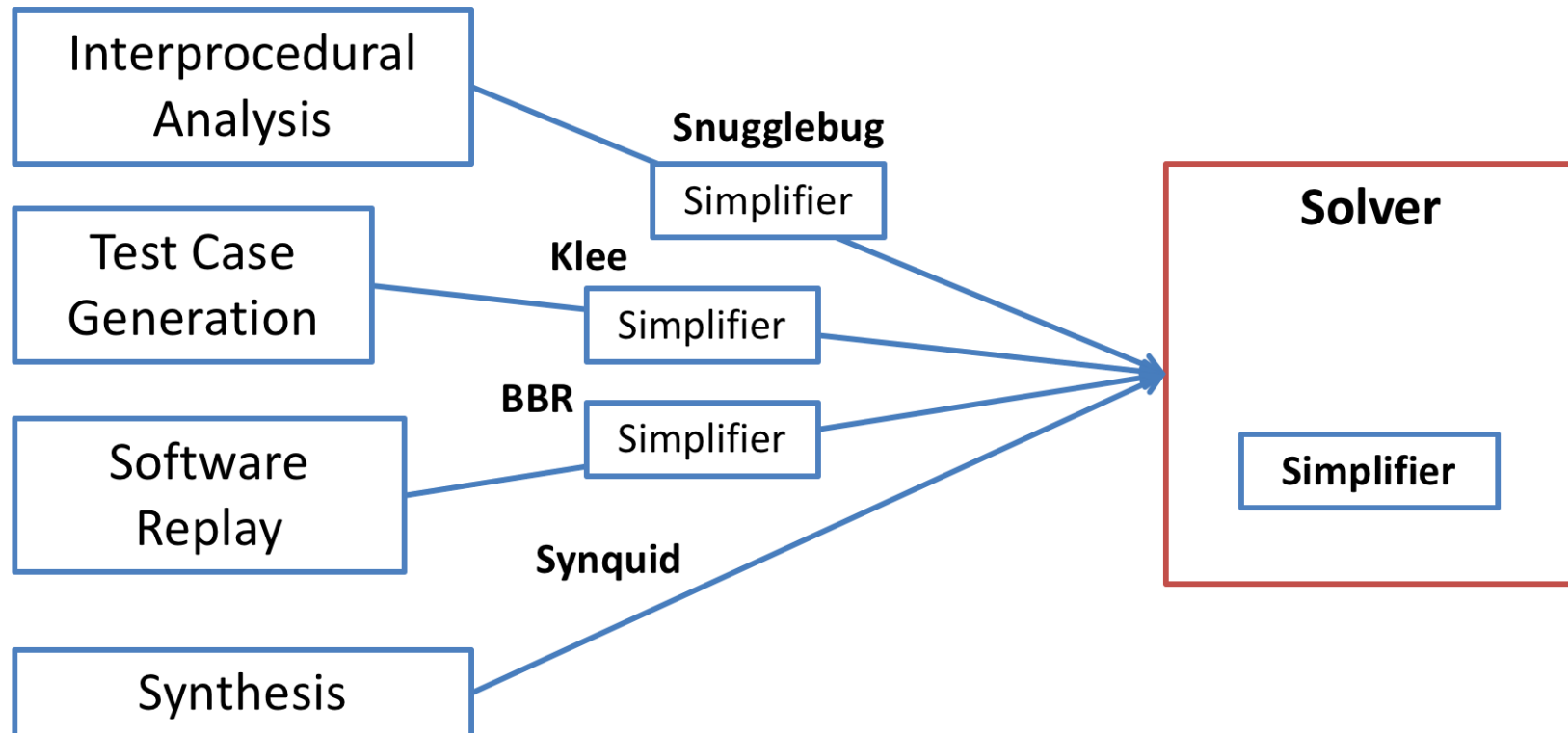
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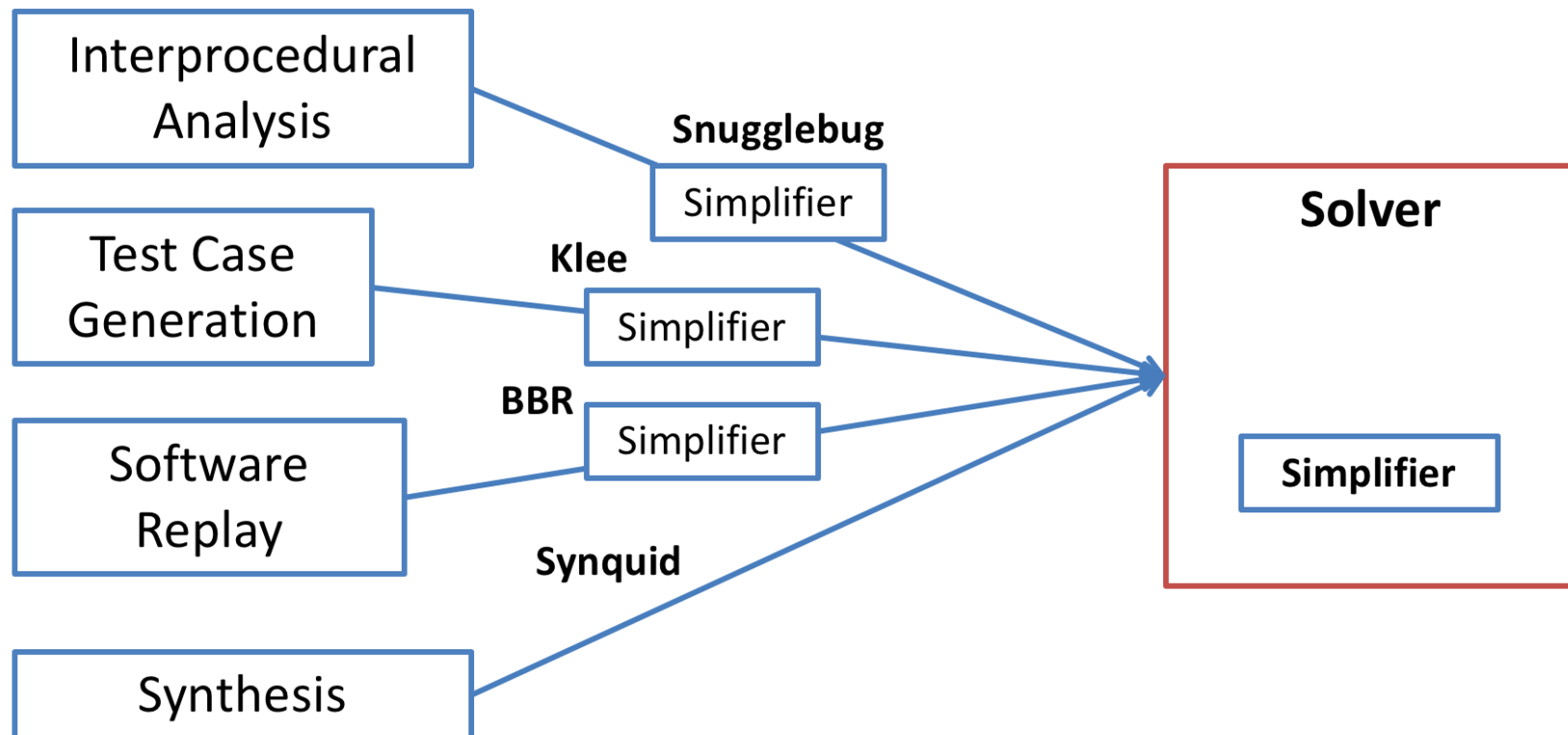
General Constraint Solvers



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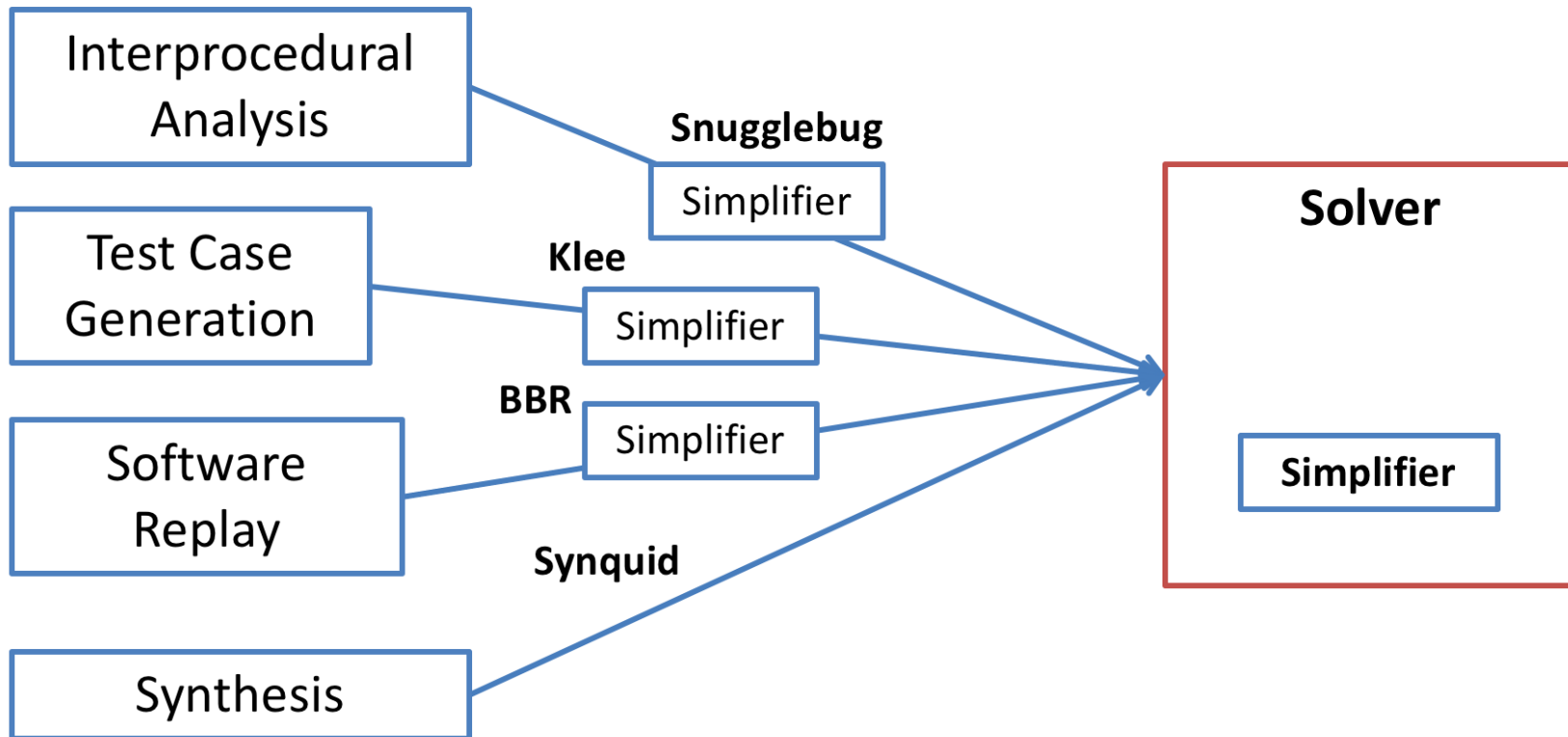


General Constraint Solvers



Simplifiers are very application specific

General Constraint Solvers



Simplifiers are very application specific
Not every tool can afford a custom simplifier

Custom simplifiers are expensive

Custom simplifiers are expensive



Trial and Error

Custom simplifiers are expensive



Trial and Error
Each Try is hard

Target: Sketch Solver

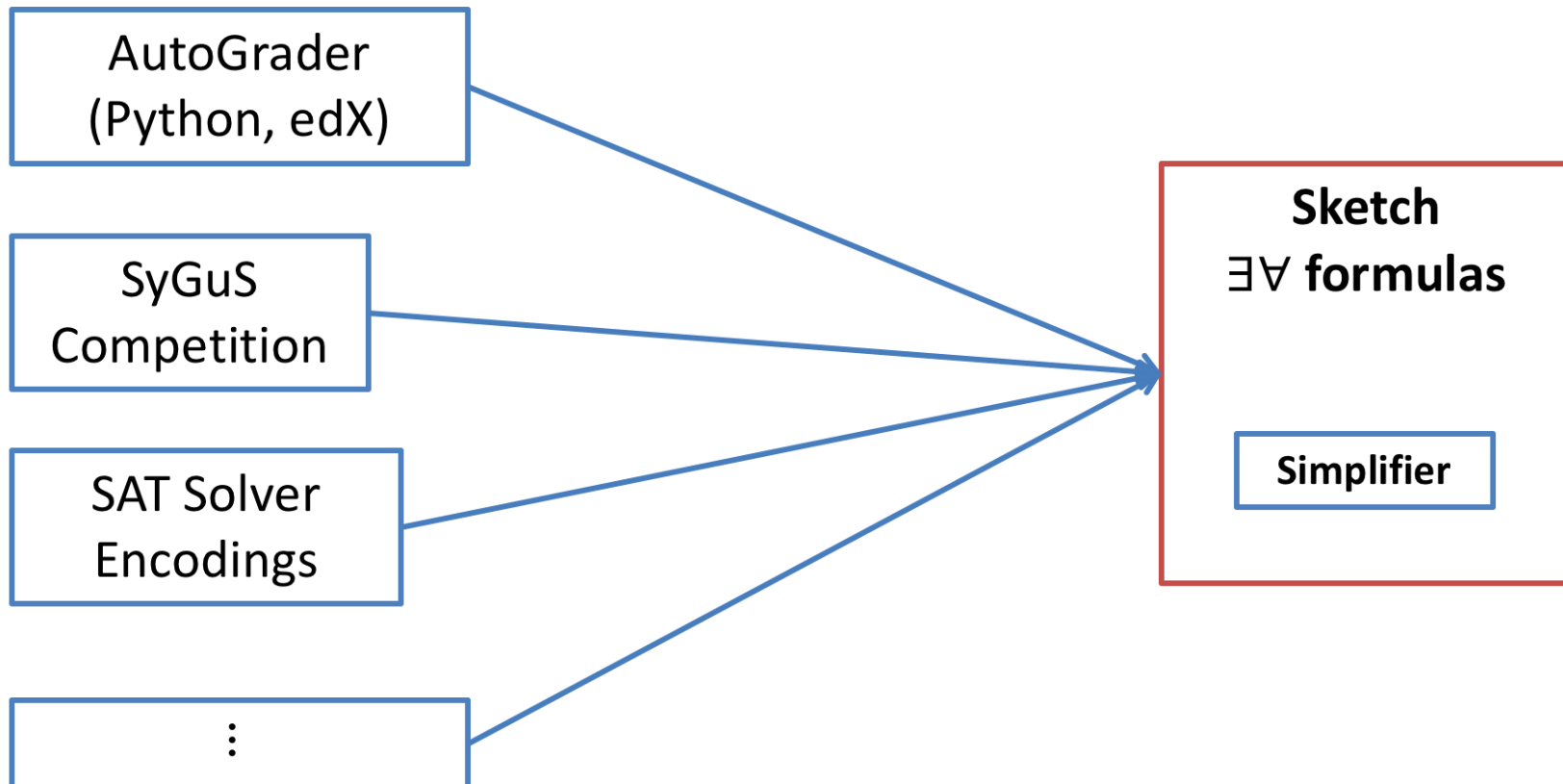
Sketch
 $\exists\forall$ formulas

Target: Sketch Solver

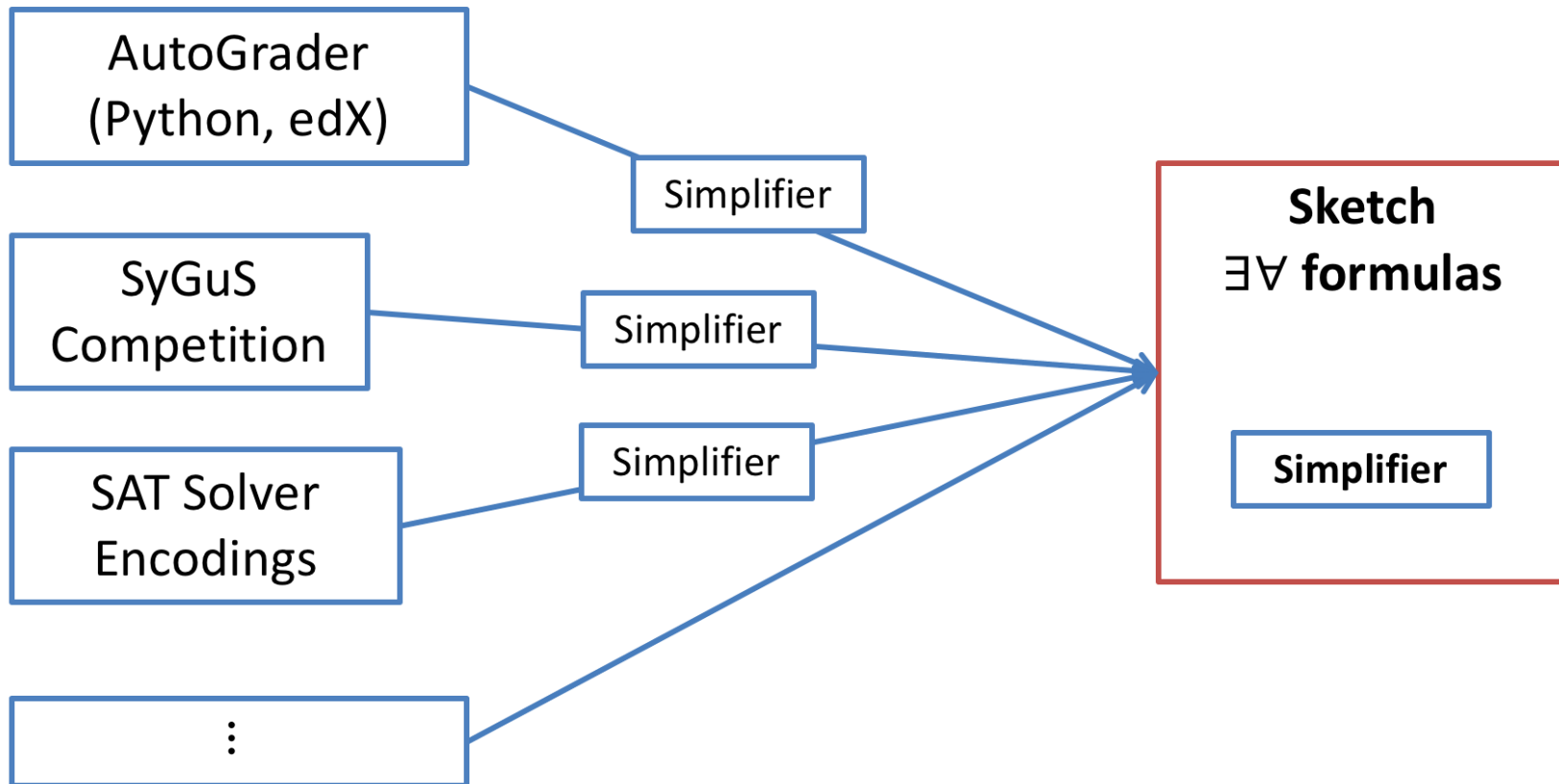
Sketch
 $\exists\forall$ formulas

Simplifier

Target: Sketch Solver



Target: Sketch Solver



Auto-generate efficient domain-specific simplifiers

Sketch Simplifier

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Messy low-level C++ code

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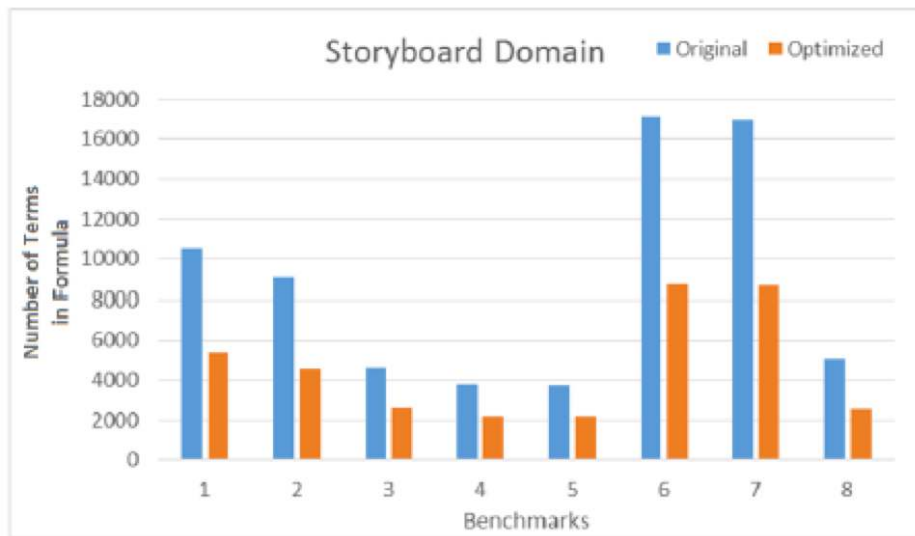
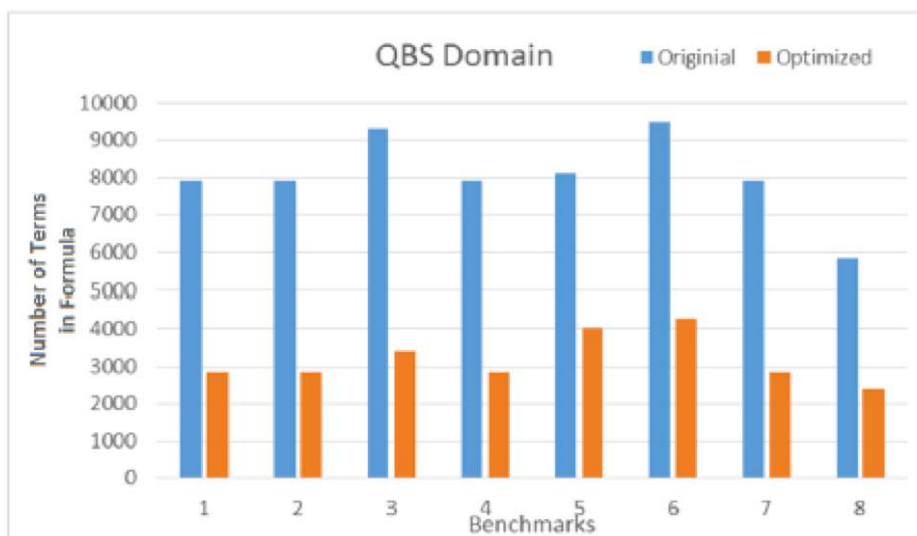
Employs simple declarative *Rewrite rules*

Sketch Simplifier

Messy low-level C++ code

Employs simple declarative *Rewrite rules*

Huge impact on performance

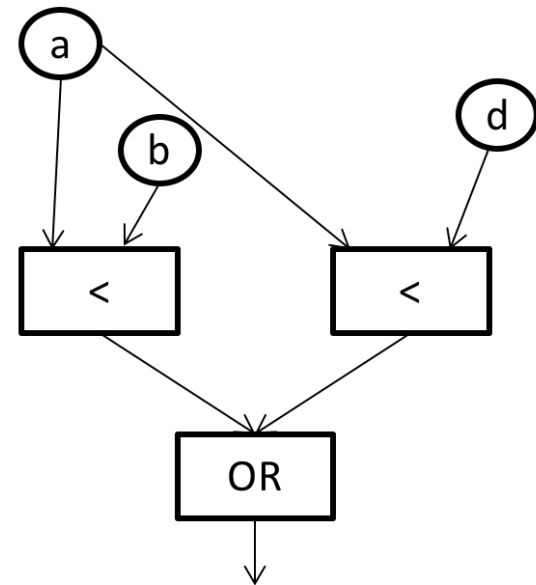


Internal Representation

- Internal language for constraints
- Theory of Arrays, booleans and integer arithmetic

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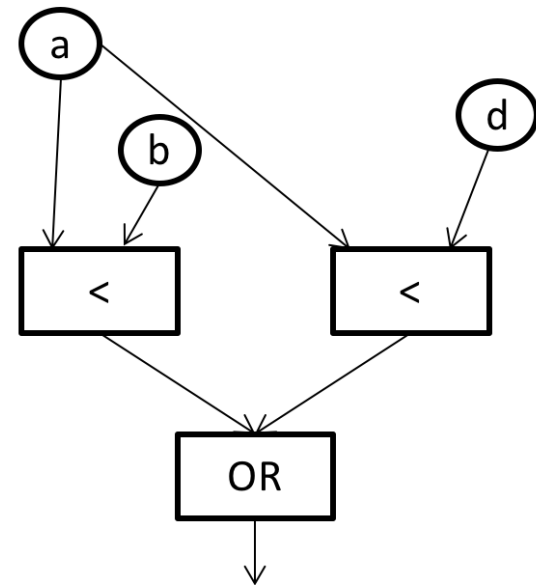


Directed Acyclic Graphs

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$or(lt(a, b), lt(a, d))$



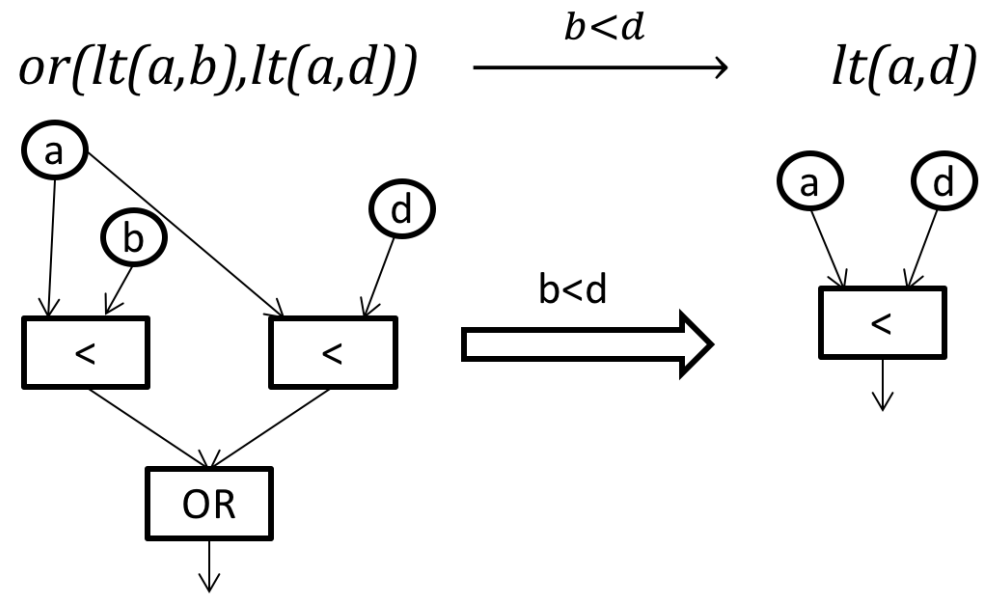
Directed Acyclic Graphs

Conditional Rewrite Rules

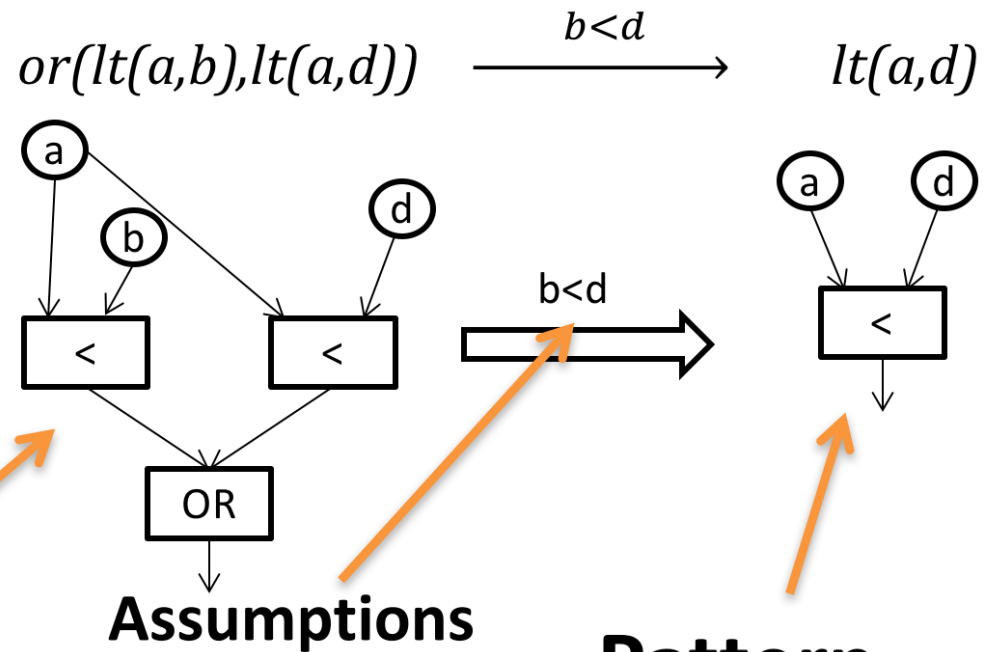
Conditional Rewrite Rules

$$\text{or}(\text{lt}(a,b),\text{lt}(a,d)) \xrightarrow{b < d} \text{lt}(a,d)$$

Conditional Rewrite Rules

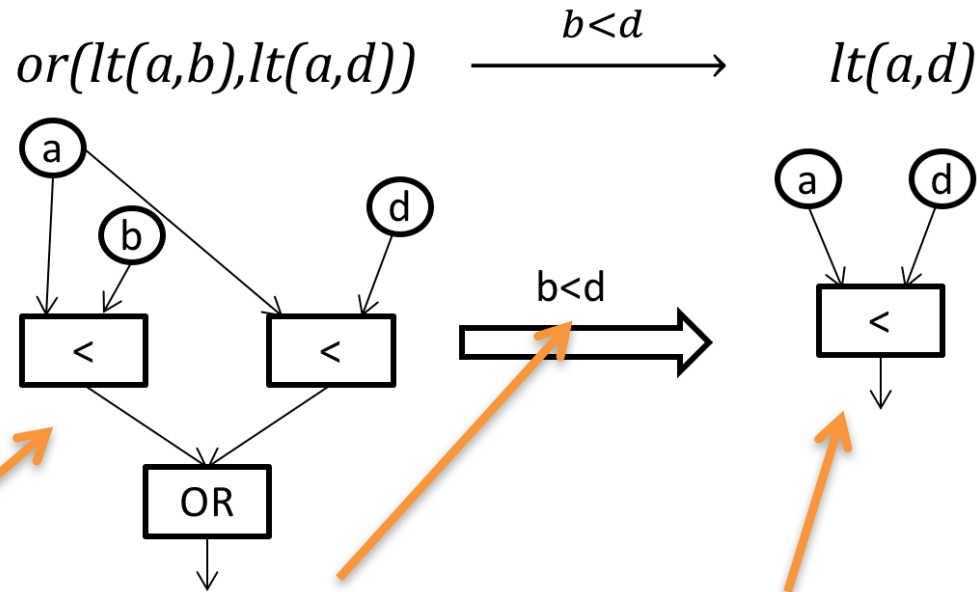


Conditional Rewrite Rules



● **Pattern** → **Pattern**

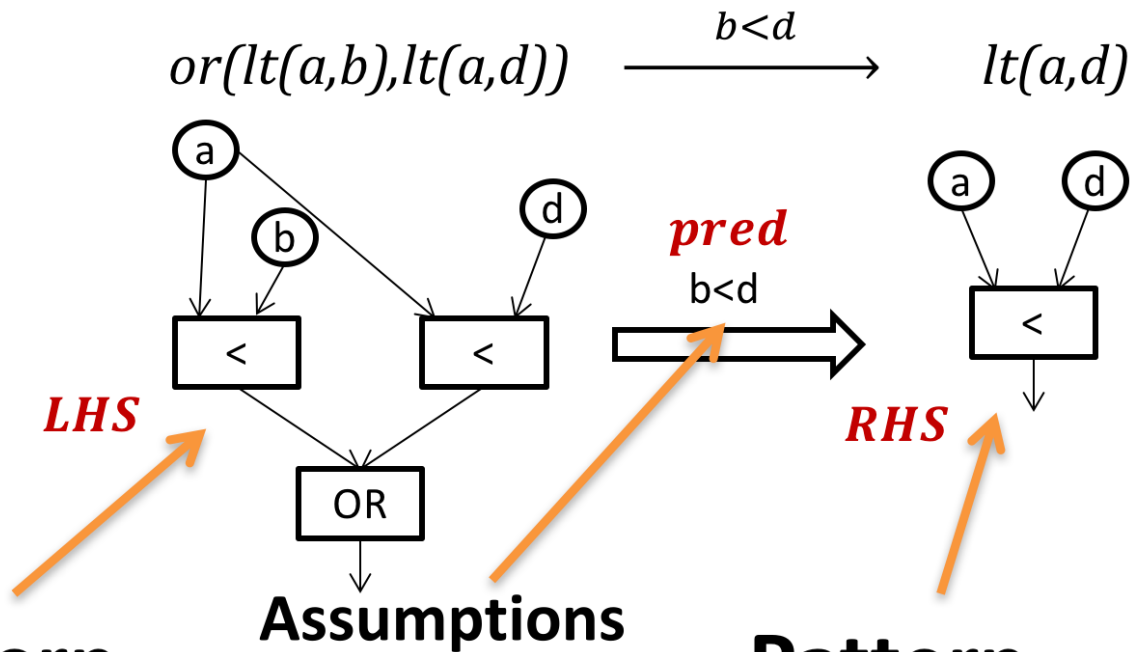
Conditional Rewrite Rules



● **Pattern** $\xrightarrow{\text{Assumptions}}$ **Pattern**

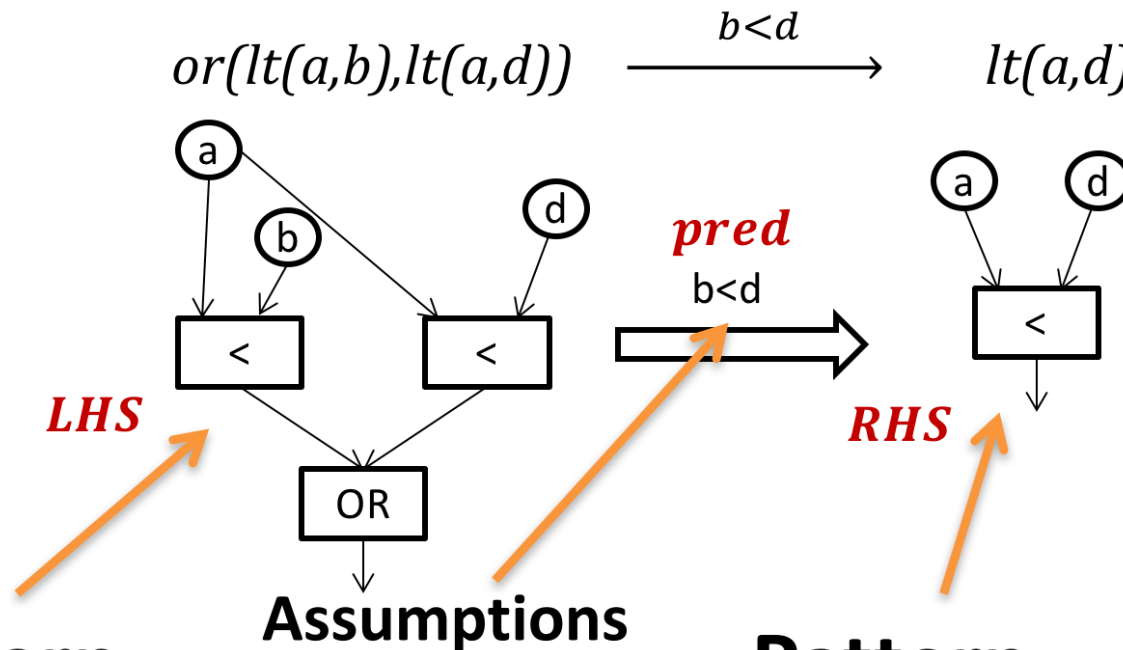
● **Inputs :** (a) (b) (d)

Conditional Rewrite Rules



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Conditional Rewrite Rules



- **Pattern** $\xrightarrow{\text{Assumptions}}$ **Pattern**
- **Inputs :** $(a) (b) (d)$
- $\forall x \text{ pred}(x) \Rightarrow (LHS(x) == RHS(x))$
- Sketch Simplifier: apply in order at each node

Rule application code

Code for implementing Rewrite Rules

$$\begin{aligned} & \text{and}(\text{lt}(\text{plus}(a,e),x), \\ & \quad \text{lt}(\text{plus}(e,b),x)) \\ & \xrightarrow{b < a} \text{lt}(\text{plus}(a,e),x) \end{aligned}$$

Rule application code

Code for implementing Rewrite Rules

```
if(nfather->type == LT && nmother->type == LT){
  // (a+e<x) & (b+e<x) ---> a+e<x when b<a
  if(nfather->mother->type == PLUS && nmother->mother-
>type == PLUS){
    bool_node* nfm = nfather->mother;
    bool_node* nmm = nmother->mother;

    bool_node* nmmConst = nmm->mother;
    bool_node* nmmExp = nmm->father;
    if(isConst(nmmExp)){
      bool_node* tmp = nmmExp;
      nmmExp = nmmConst;
      nmmConst = tmp;
    }
    bool_node* nfmConst = nfm->mother;
    bool_node* nfmExp = nfm->father;
    if(isConst(nfmExp)){
      bool_node* tmp = nfmExp;
      nfmExp = nfmConst;
      nfmConst = tmp;
    }
  }
  if(isConst(nfmConst) && isConst(nmmConst) && nfmExp== nmmExp){
    if(val(nfmConst) < val(nmmConst)){
      return nmother;
    }else{
      return nfather;
    }
  }
}
```

$$\begin{array}{l} \text{and}(\text{lt}(\text{plus}(a,e),x), \\ \quad \text{lt}(\text{plus}(e,b),x)) \\ b < a \\ \longrightarrow \text{lt}(\text{plus}(a,e),x) \end{array}$$

Problem Statement

Given a corpus of benchmark problems
(formulas) from a domain:

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Solution: **SWAPPER** framework

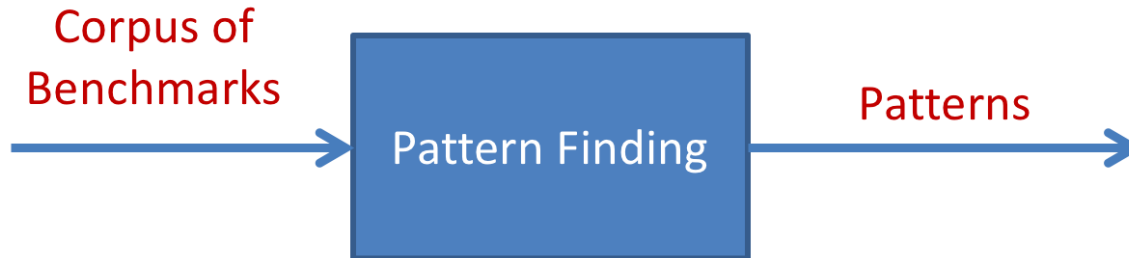
SWAPPER framework

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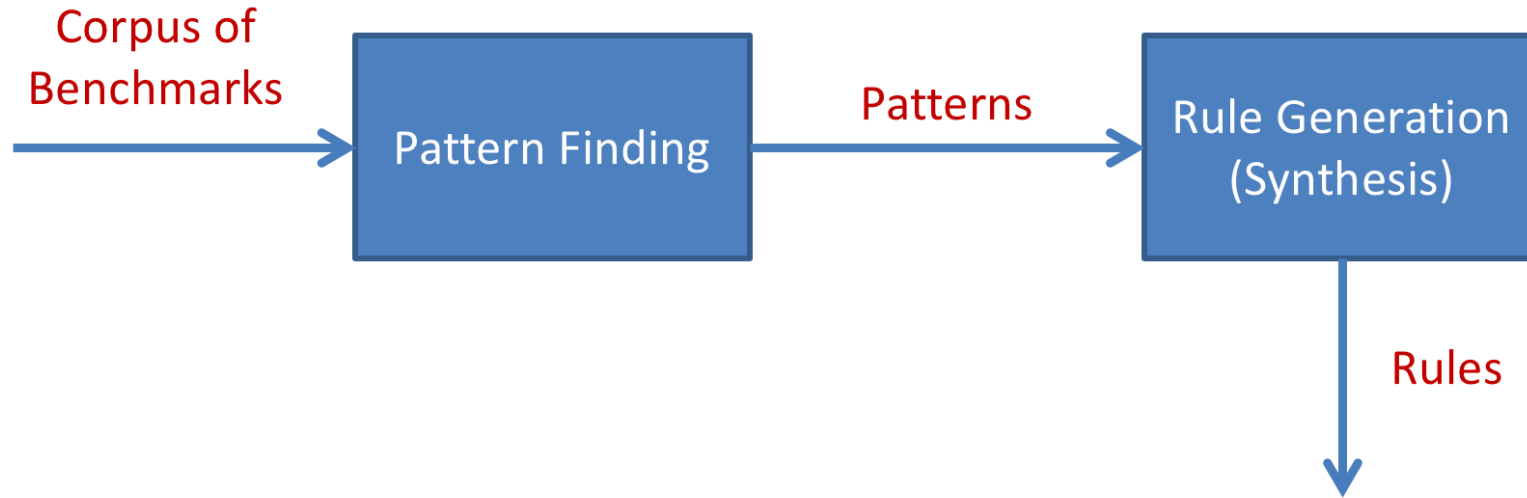
Corpus of
Benchmarks



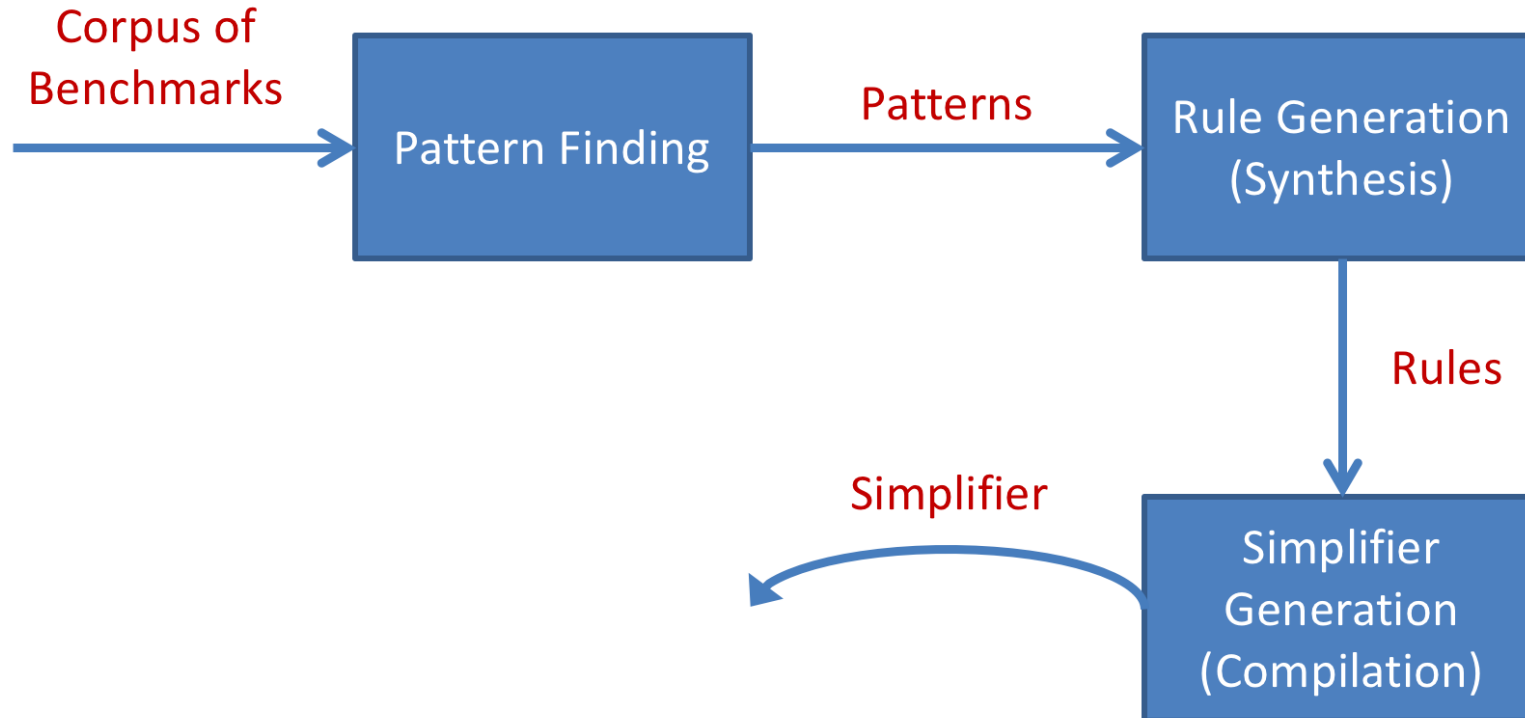
SWAPPER framework



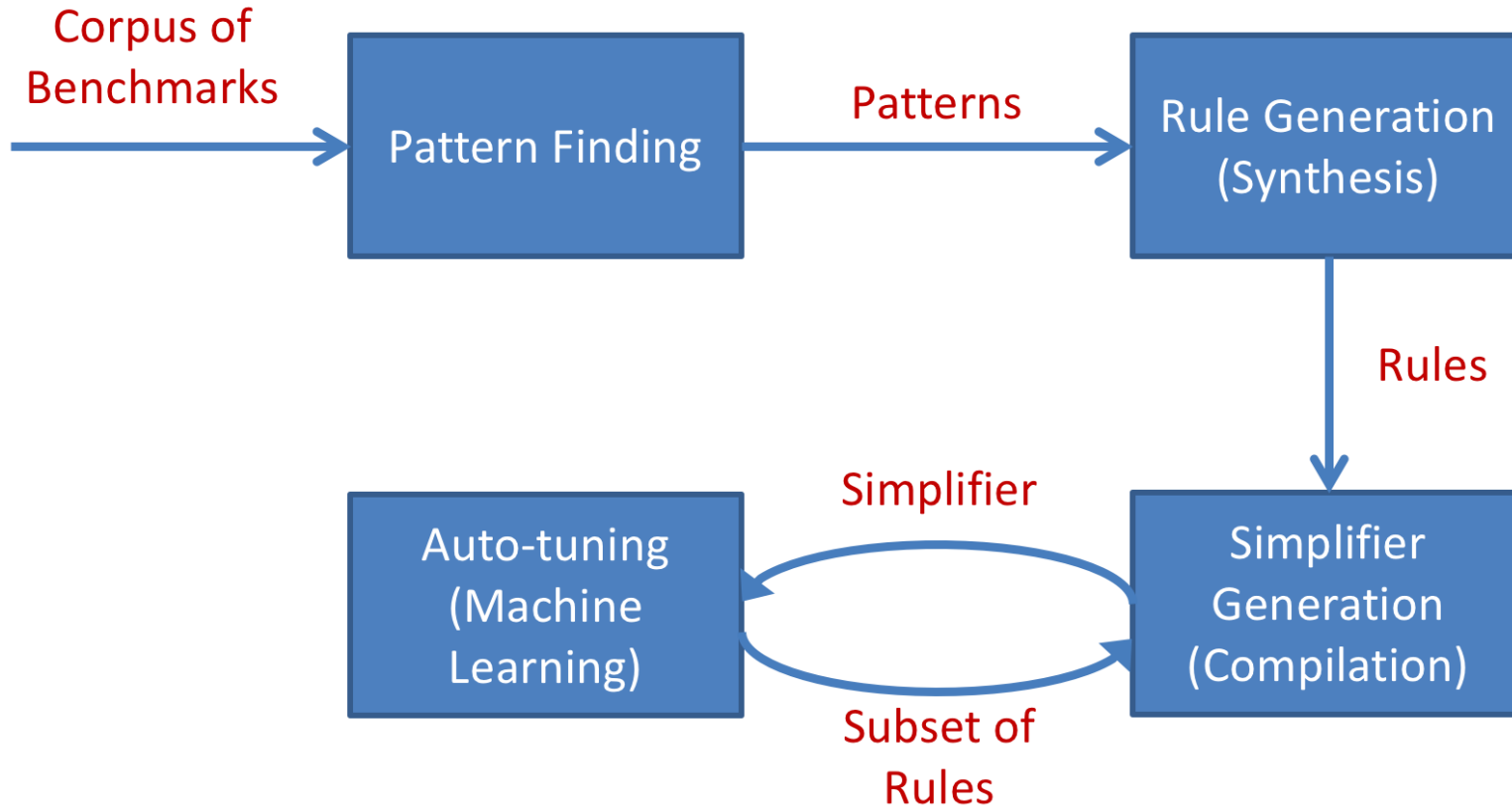
SWAPPER framework



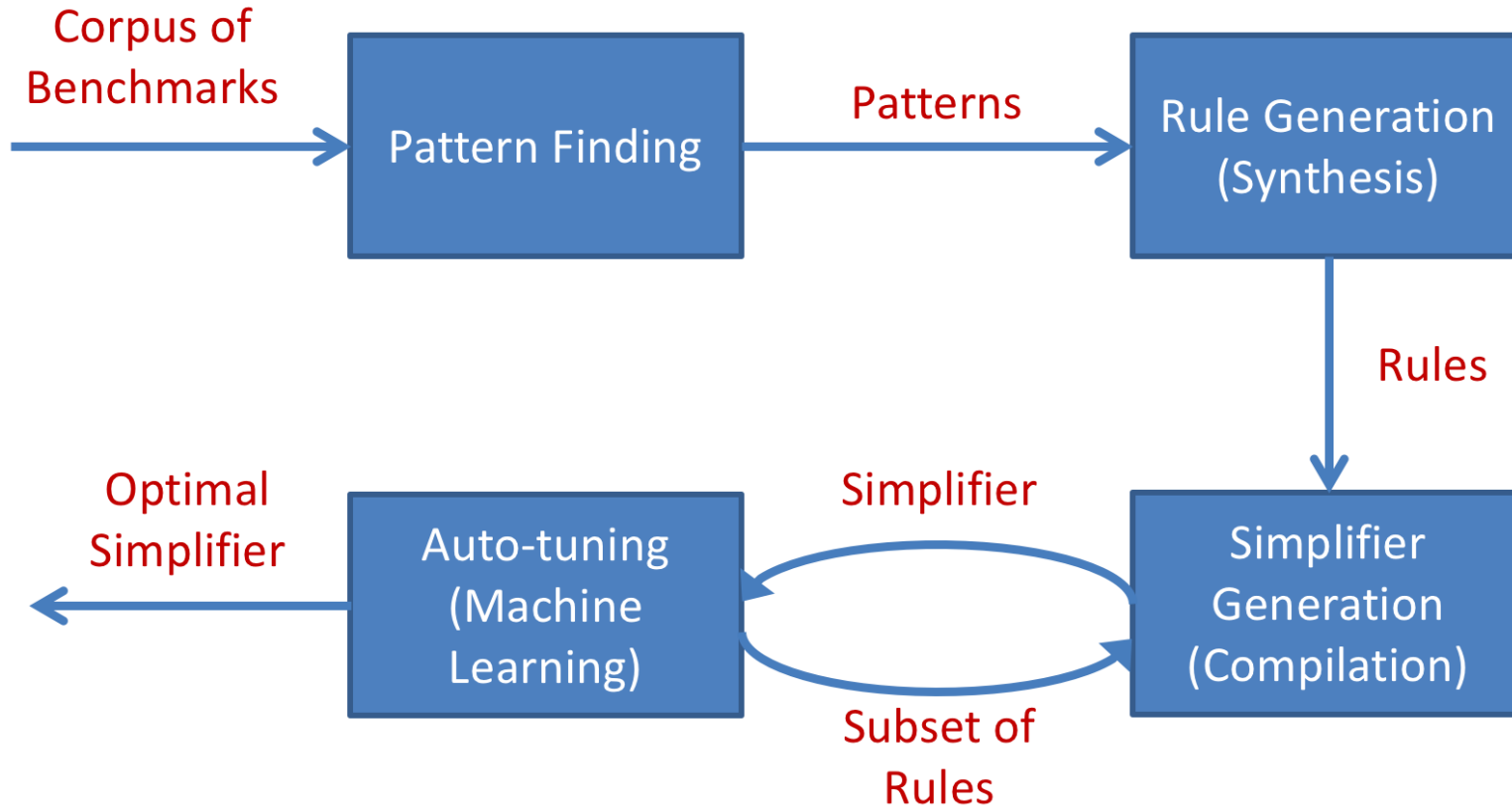
SWAPPER framework



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SWAPPER framework



Related Work

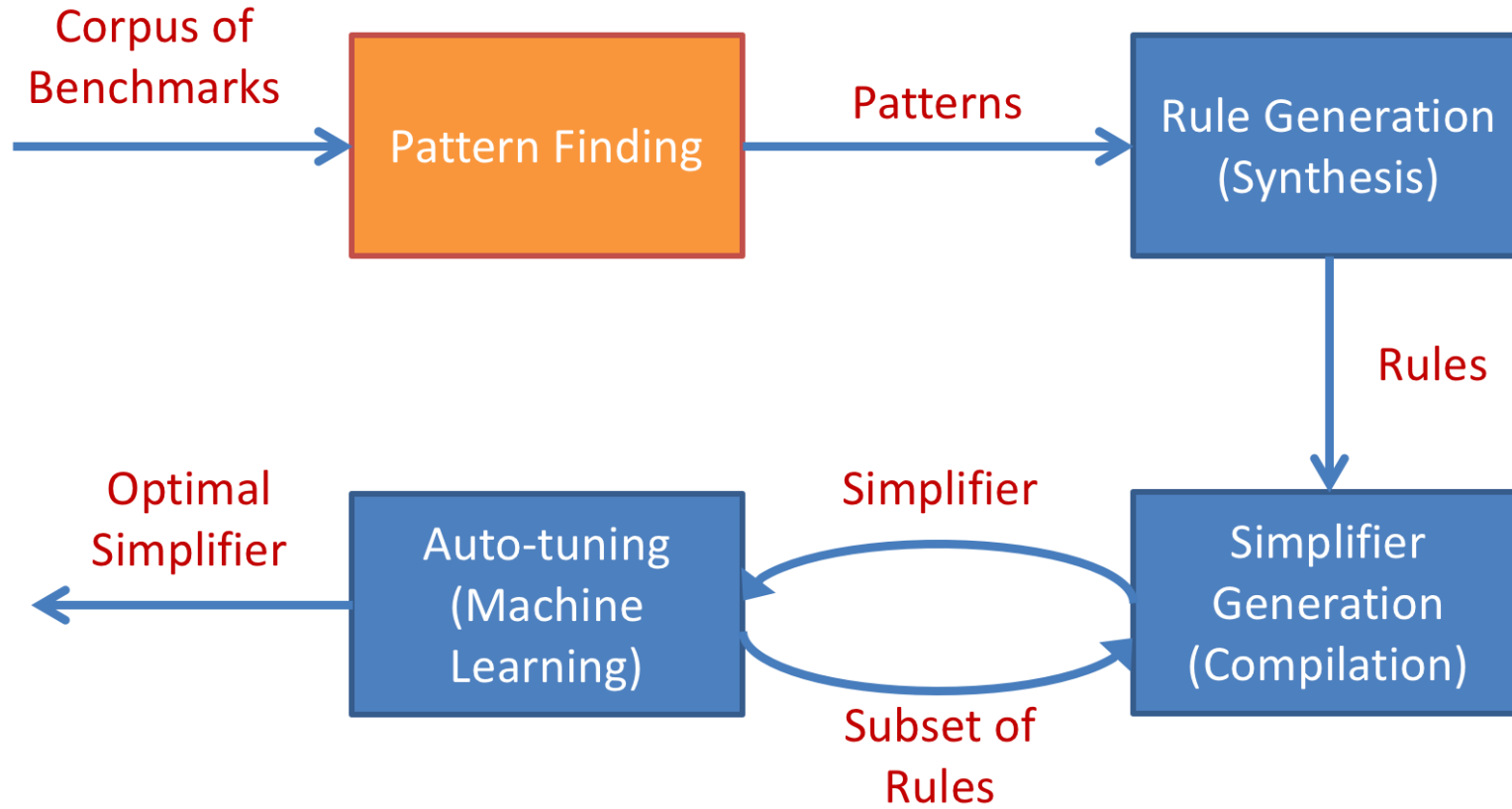
Related Work

- Peephole optimizations
 - Alive DSL [PLDI 15] : no synthesis of rules
 - Automatic generation by enumeration [ASPLOS 06]: no semantic guards

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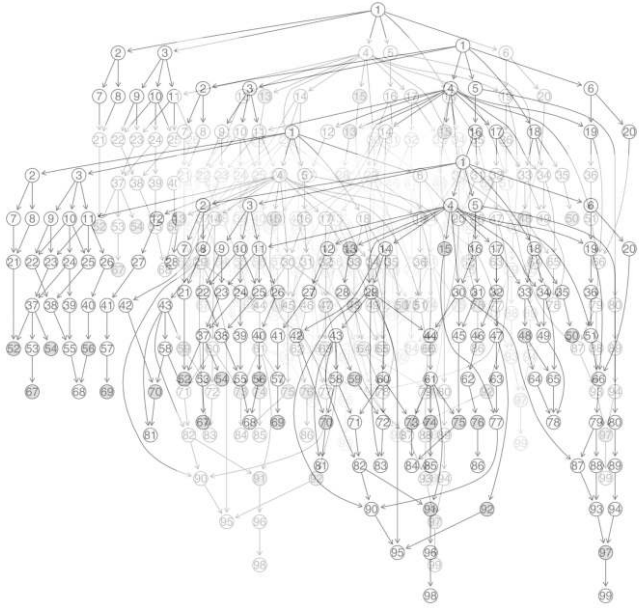
- Peephole optimizations
 - Alive DSL [PLDI 15] : no synthesis of rules
 - Automatic generation by enumeration [ASPLOS 06]: no semantic guards
- Term/Graph Rewriting:
 - Stratego/XT [ASF+SDF 97], GrGen

SWAPPER framework



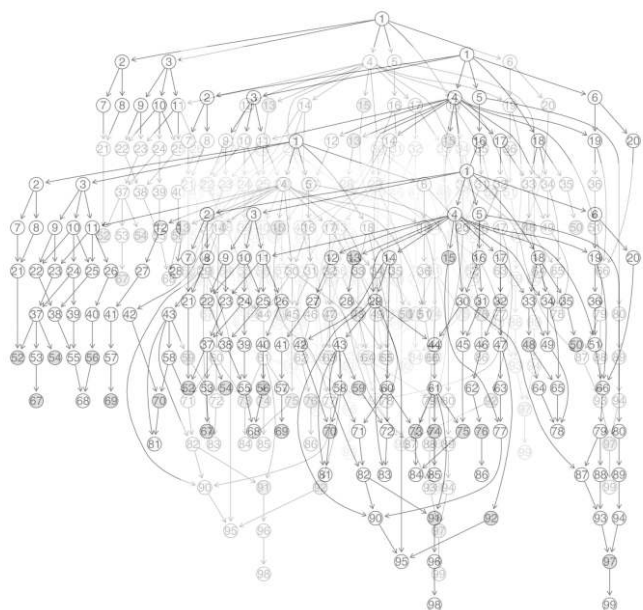
Pattern Finding

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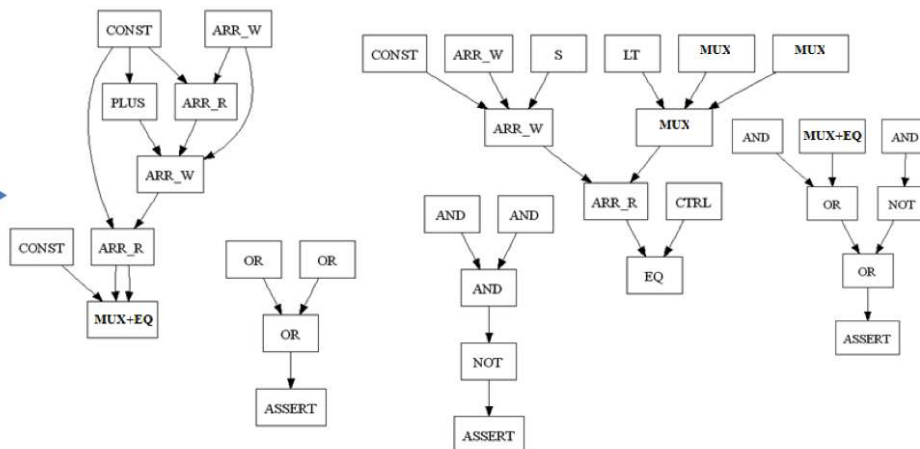


**Corpus of Formulas
(DAGs)**

Pattern Finding



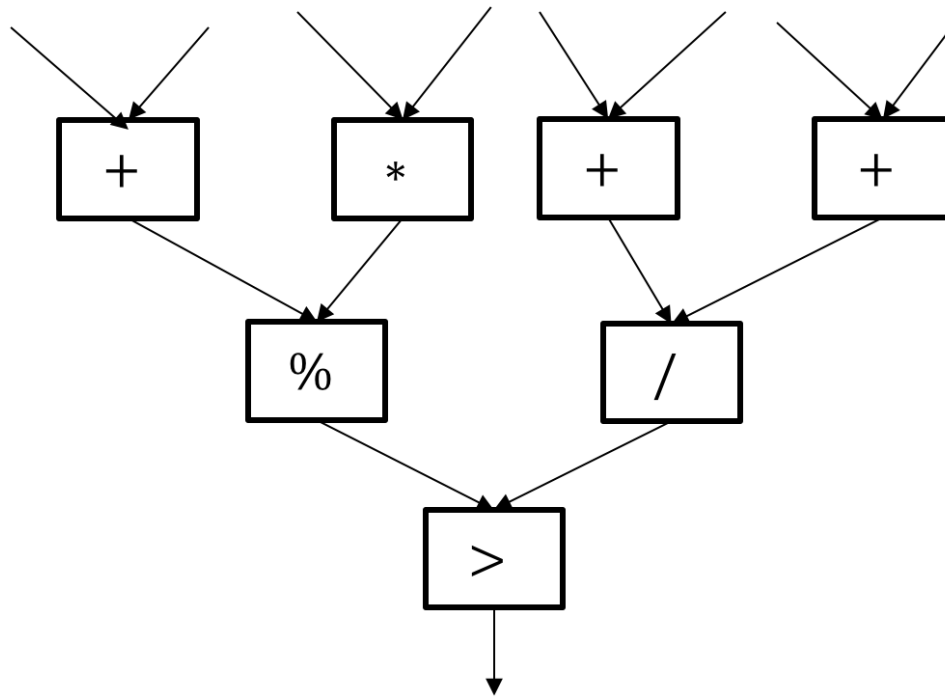
**Corpus of Formulas
(DAGs)**



**Patterns
(Sub-formulas)**

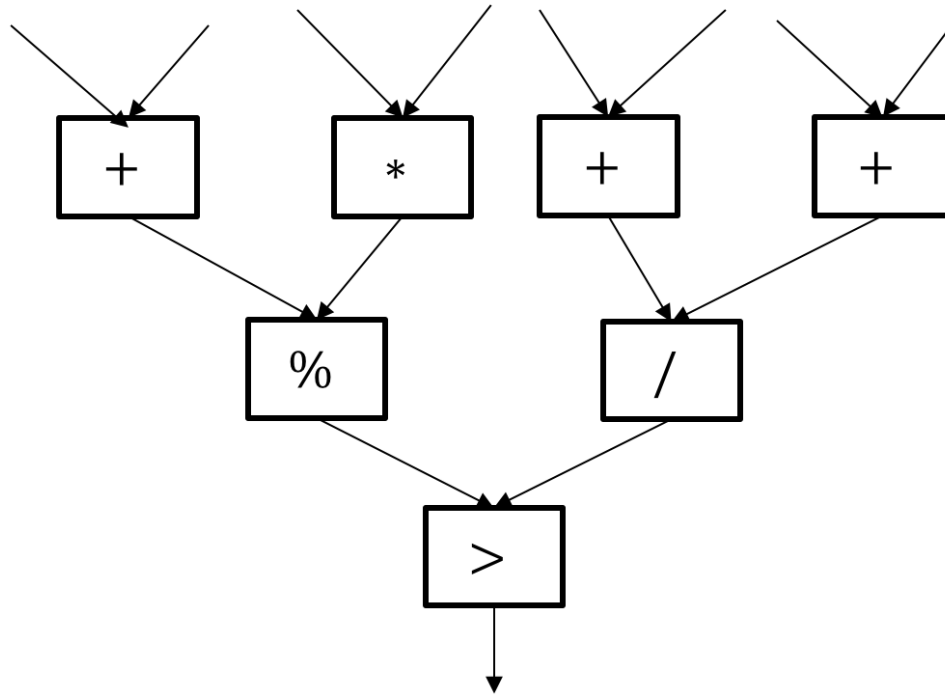
Commonly Occurring Patterns \Rightarrow More applicable rules
Different from Motif Discovery

Representative Sampling



Formula Trees

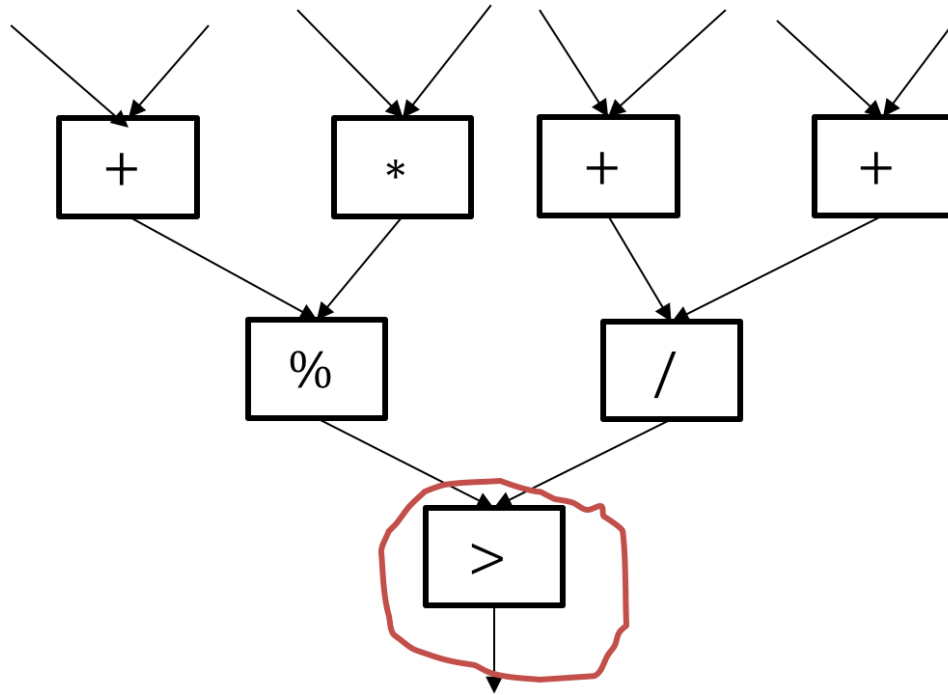
Representative Sampling



$$\frac{1}{|nodes|}$$

Formula Trees

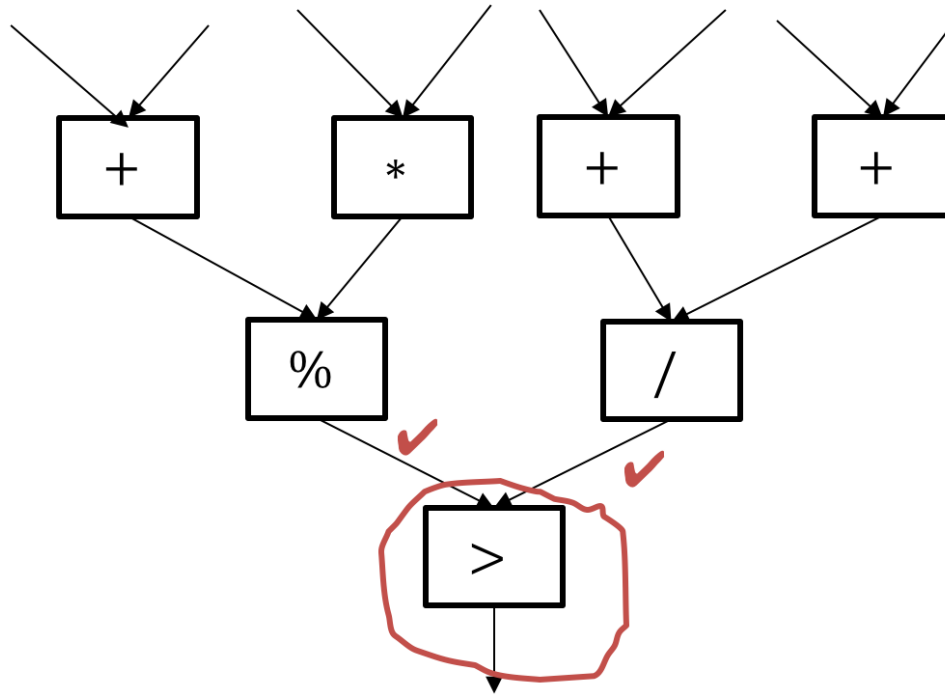
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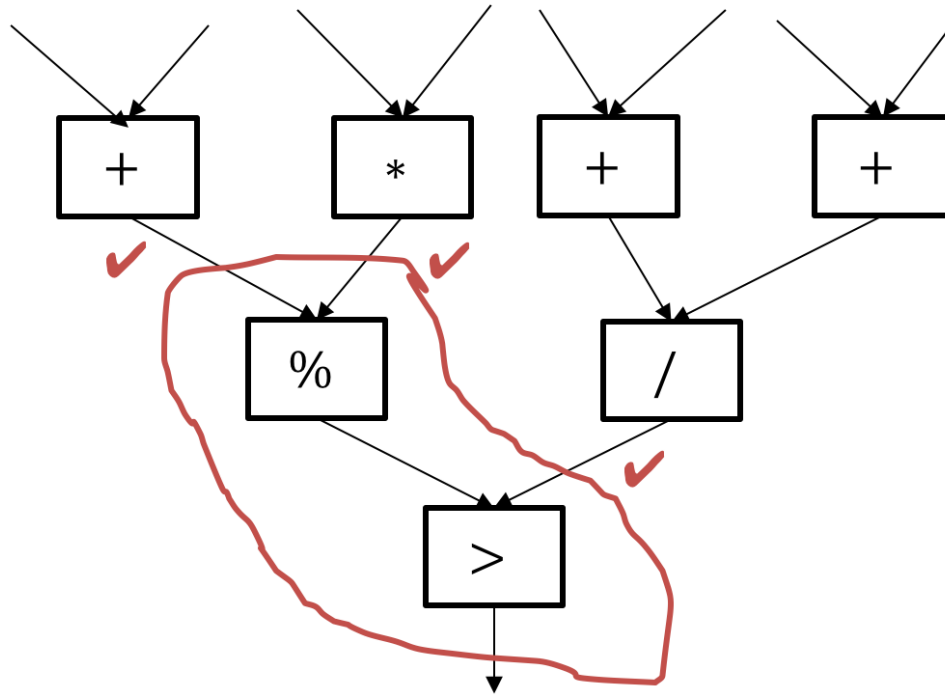
Representative Sampling



$$\frac{1}{|\mathit{nodes}|} \times \frac{1}{2}$$

Formula Trees

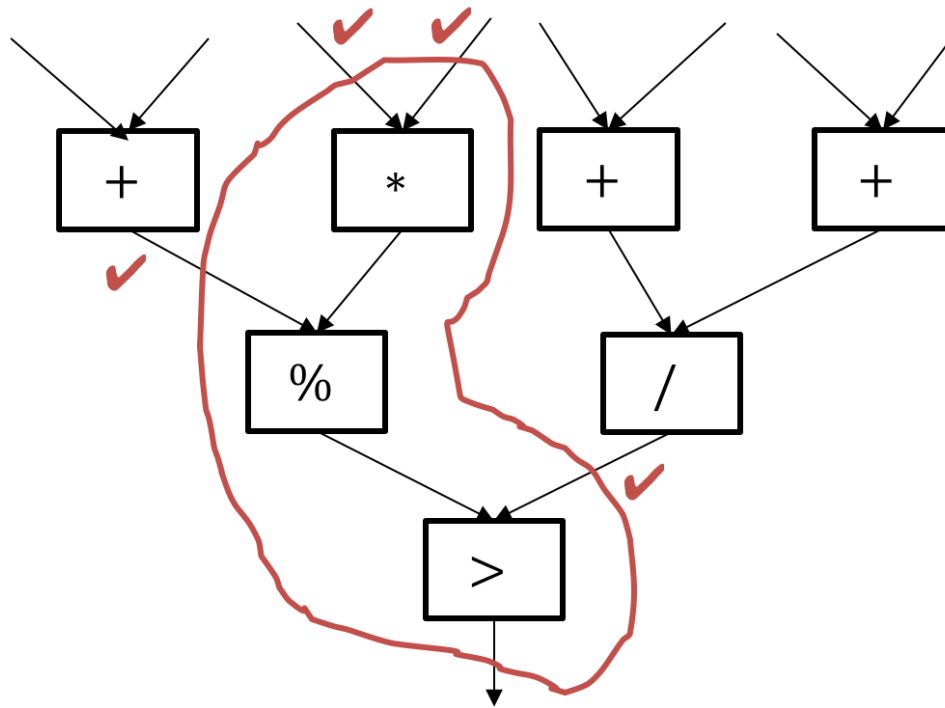
Representative Sampling



$$\frac{1}{|\text{nodes}|} \times \frac{1}{2} \times \frac{1}{3}$$

Formula Trees

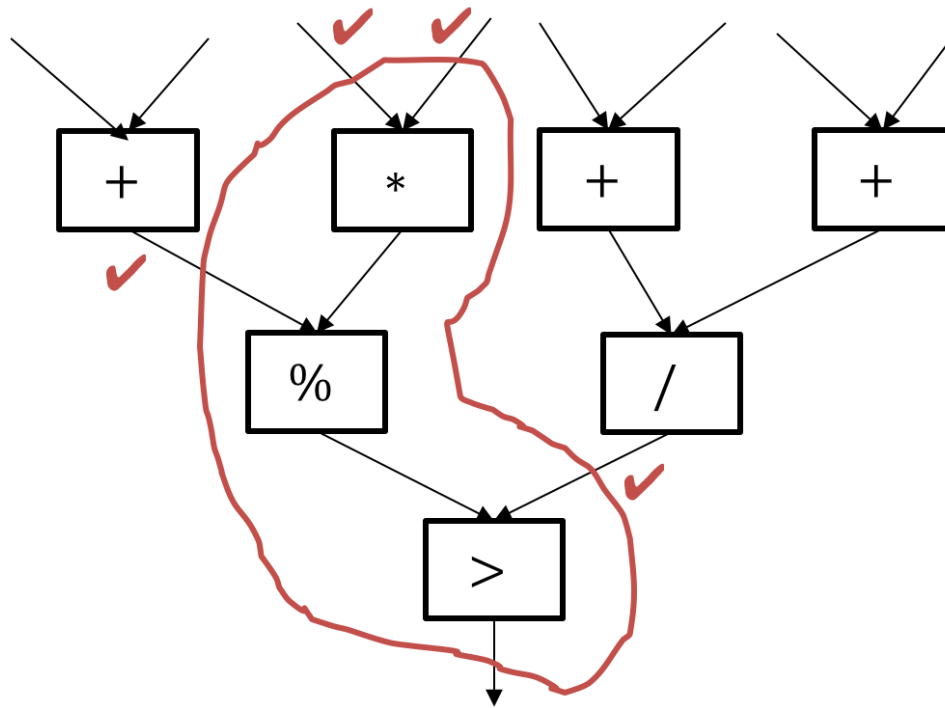
Representative Sampling



$$\frac{1}{|\mathit{nodes}|} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \dots$$

Formula Trees

Representative Sampling



$$\frac{1}{|\mathit{nodes}|} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \dots$$

Formula Trees

Probability independent of structure

Representative Sampling

- Naïve algorithm
 - Sample a node at random
 - Maintain a set of “boundary” edges
 - Sample from the boundary and repeat

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- Naïve algorithm
 - Sample a node at random
 - Maintain a set of “boundary” edges
 - Sample from the boundary and repeat
- Works for K-ary trees

Pattern Finding: Sampling

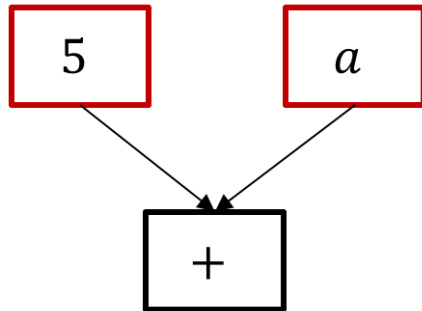
- Issues:

Pattern Finding: Sampling

- Issues:
 - Dealing with missing edges (e.g. reaching top-most nodes or variable arity)

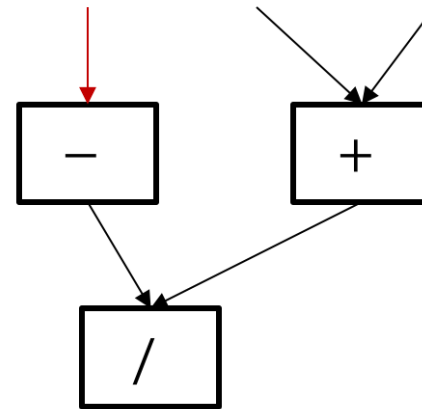
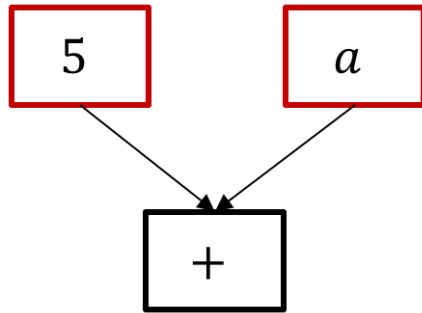
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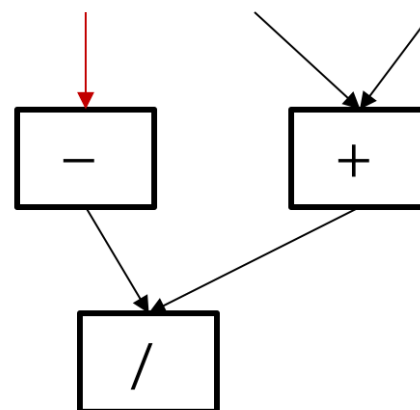
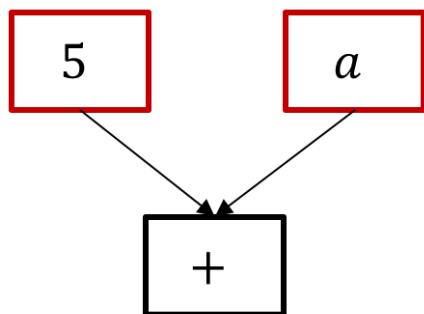
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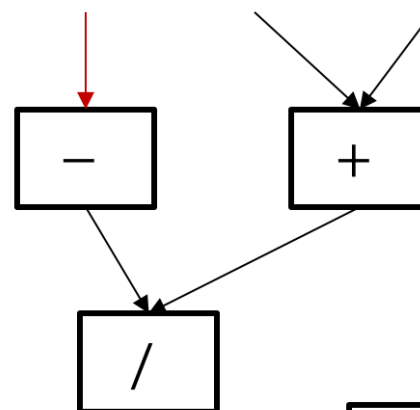
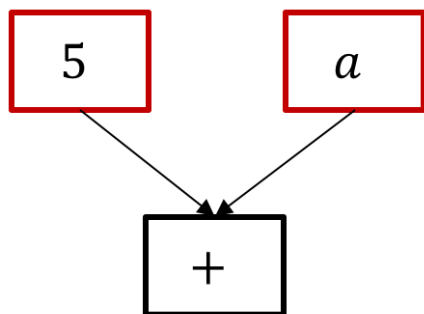
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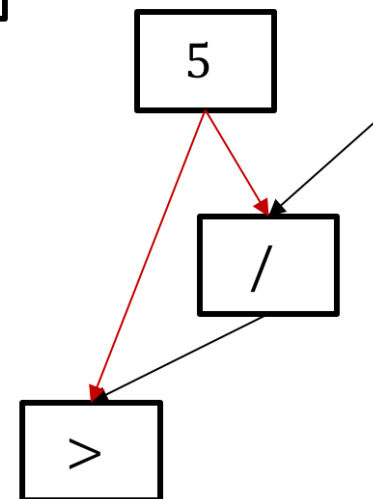
- For DAGs, Finding same pattern in multiple ways

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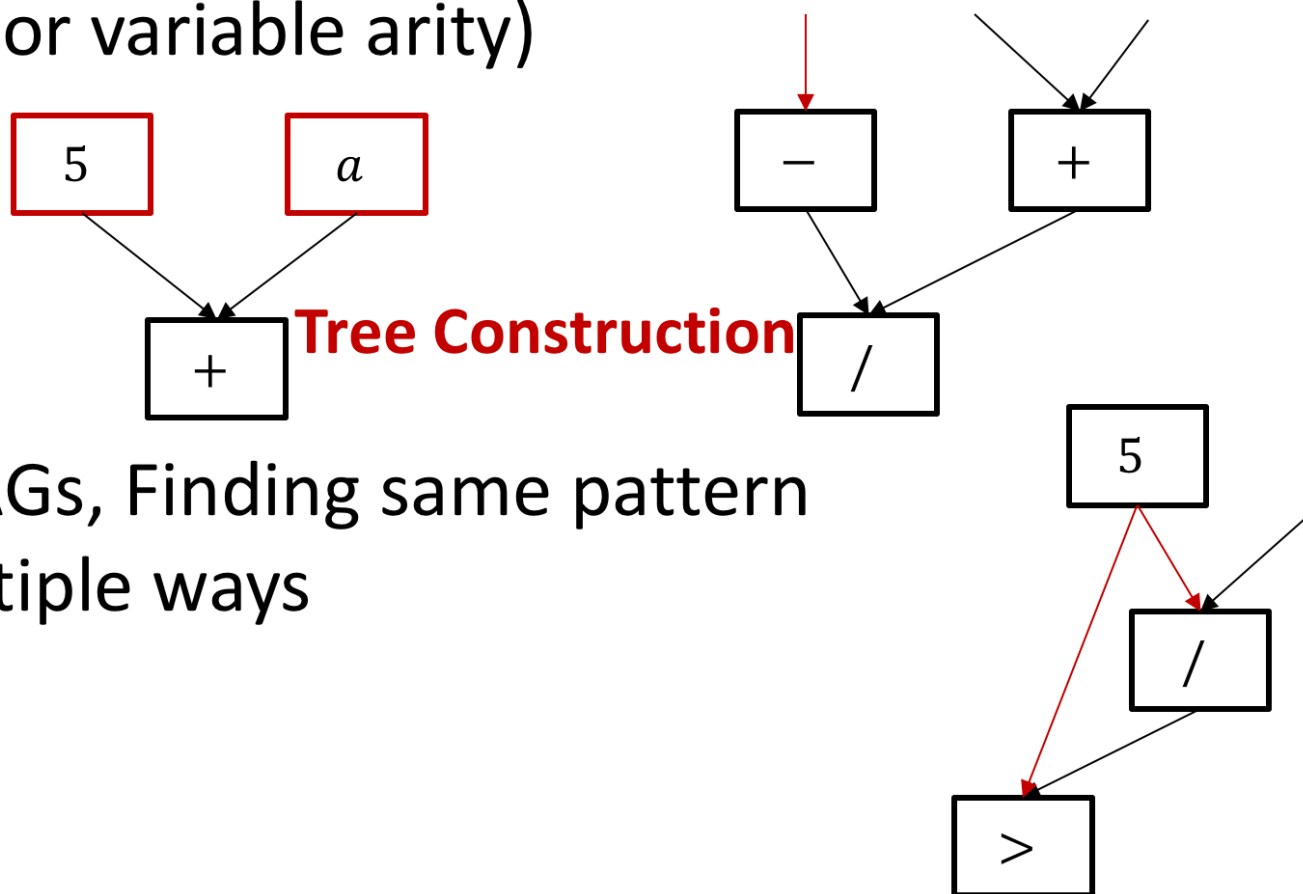


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Pattern Finding: Sampling

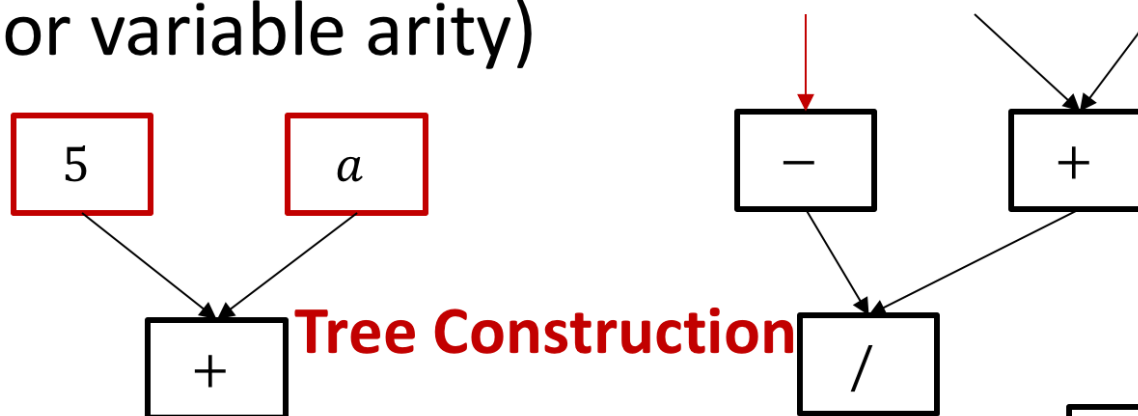
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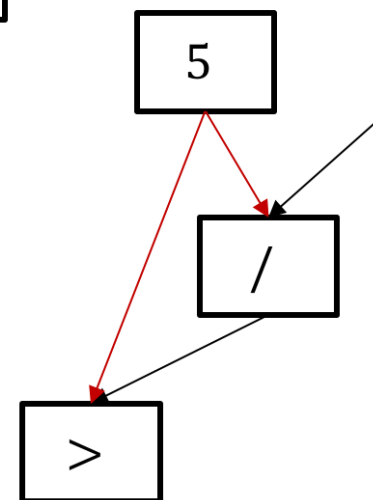
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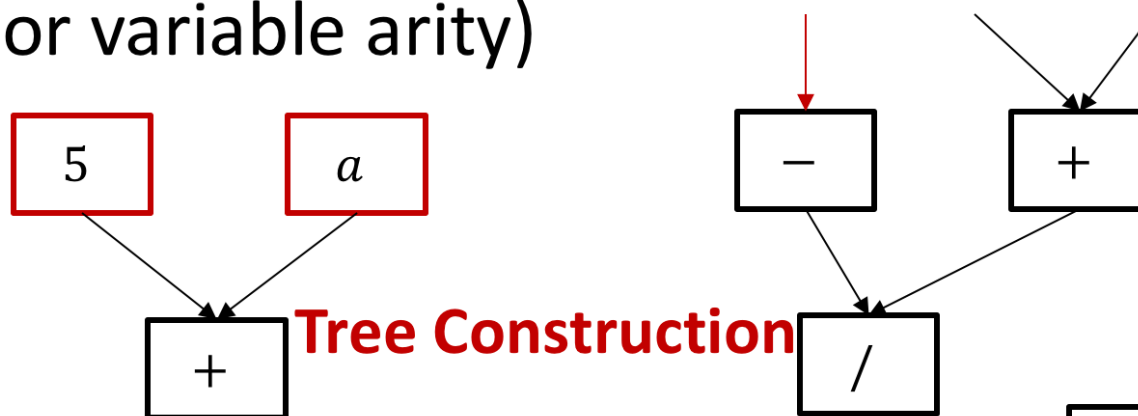
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BFS Ordering



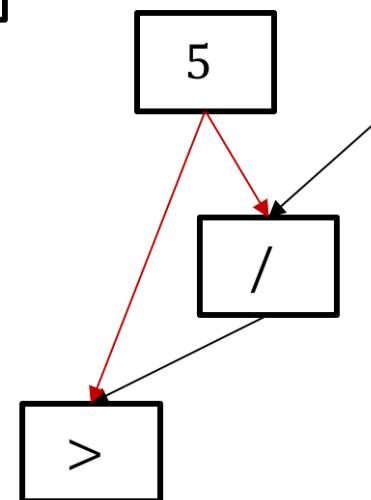
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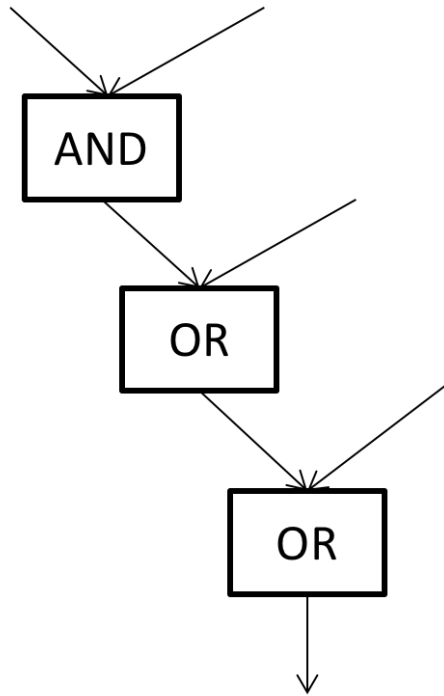


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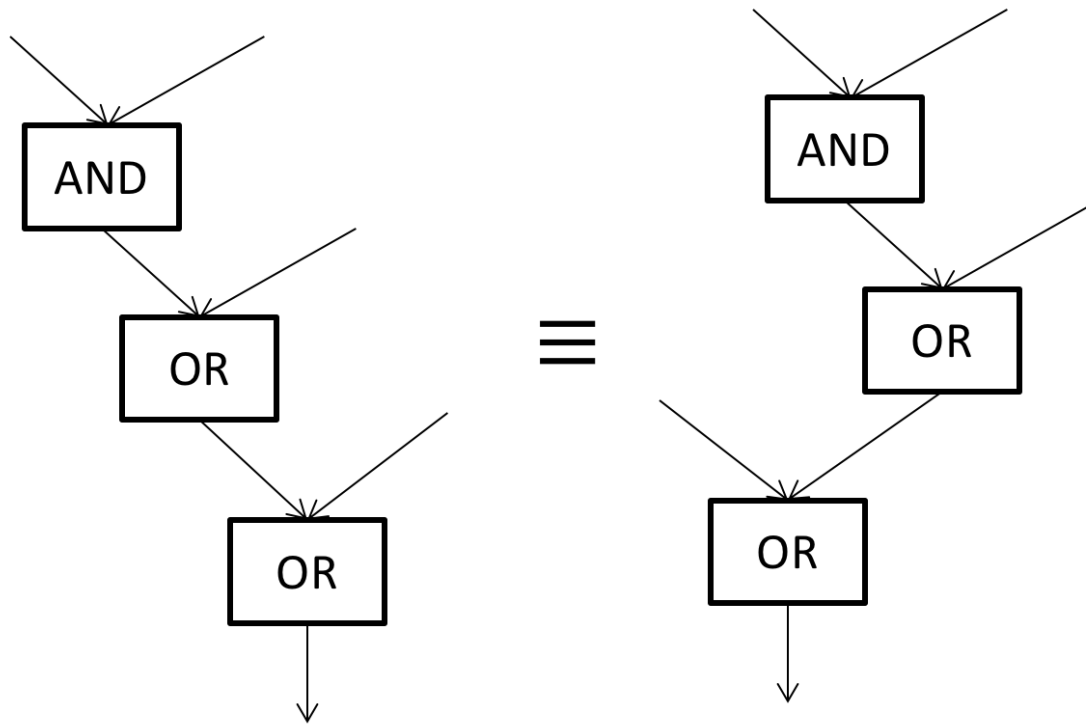
BFS Ordering



Pattern Finding: Book-keeping

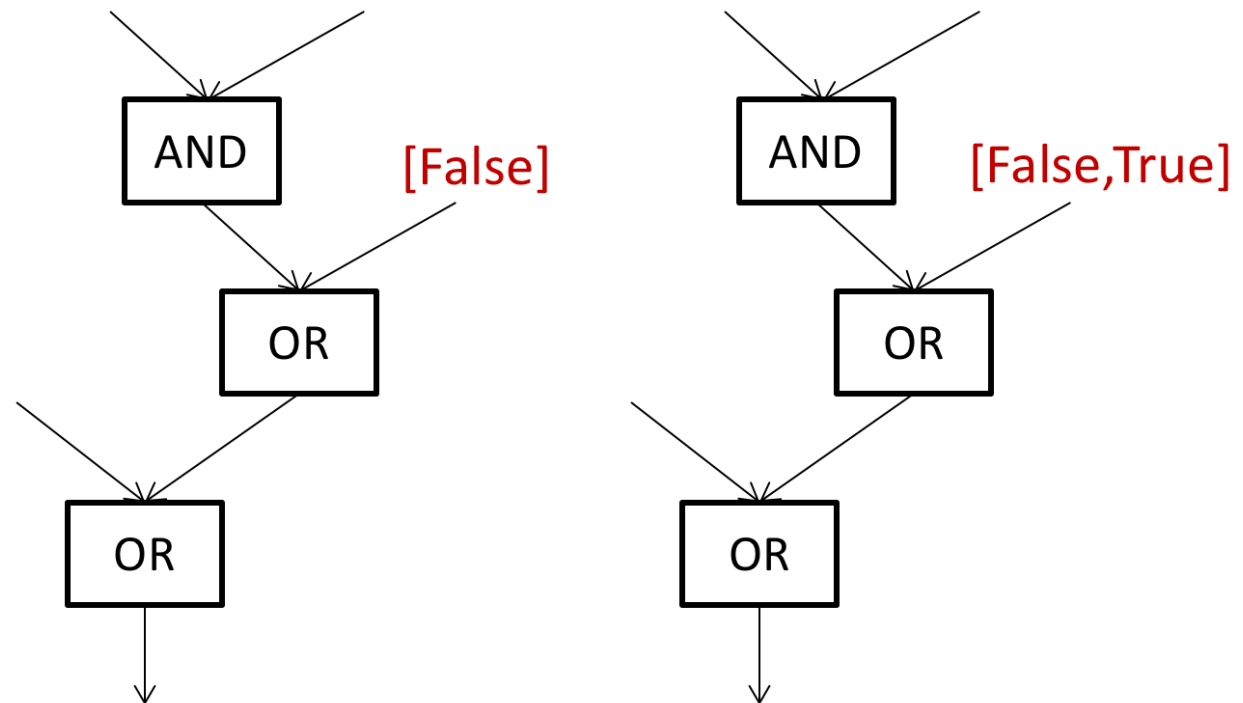


Pattern Finding: Book-keeping



- Aggregation modulo symmetries

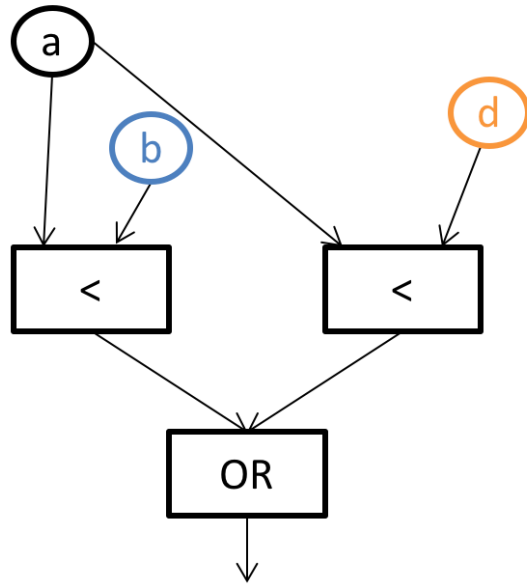
Pattern Finding: Book-keeping



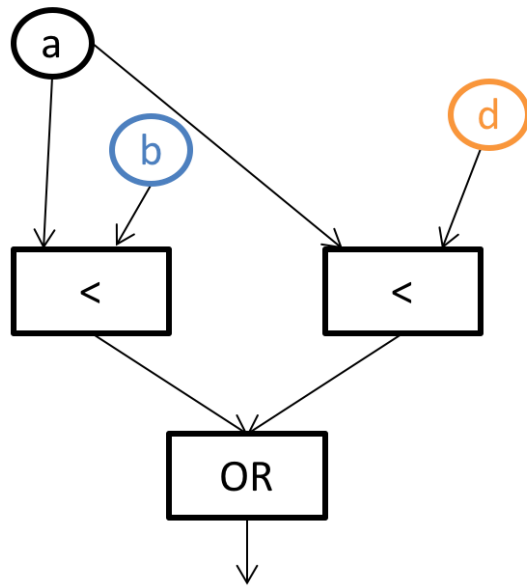
- Aggregation modulo symmetries
- Handling **contextual information** around formulas

Contextual Information: *static()*

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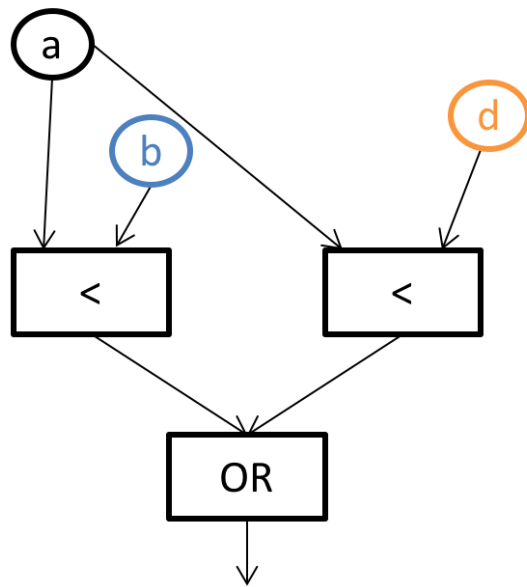
Contextual Information: *static()*



$$\mathit{static}(b) = (-\infty, \mathbf{0}]$$

$$\mathit{static}(d) = (\mathbf{0}, \infty)$$

Contextual Information: *static()*



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Can infer strong assumptions like:

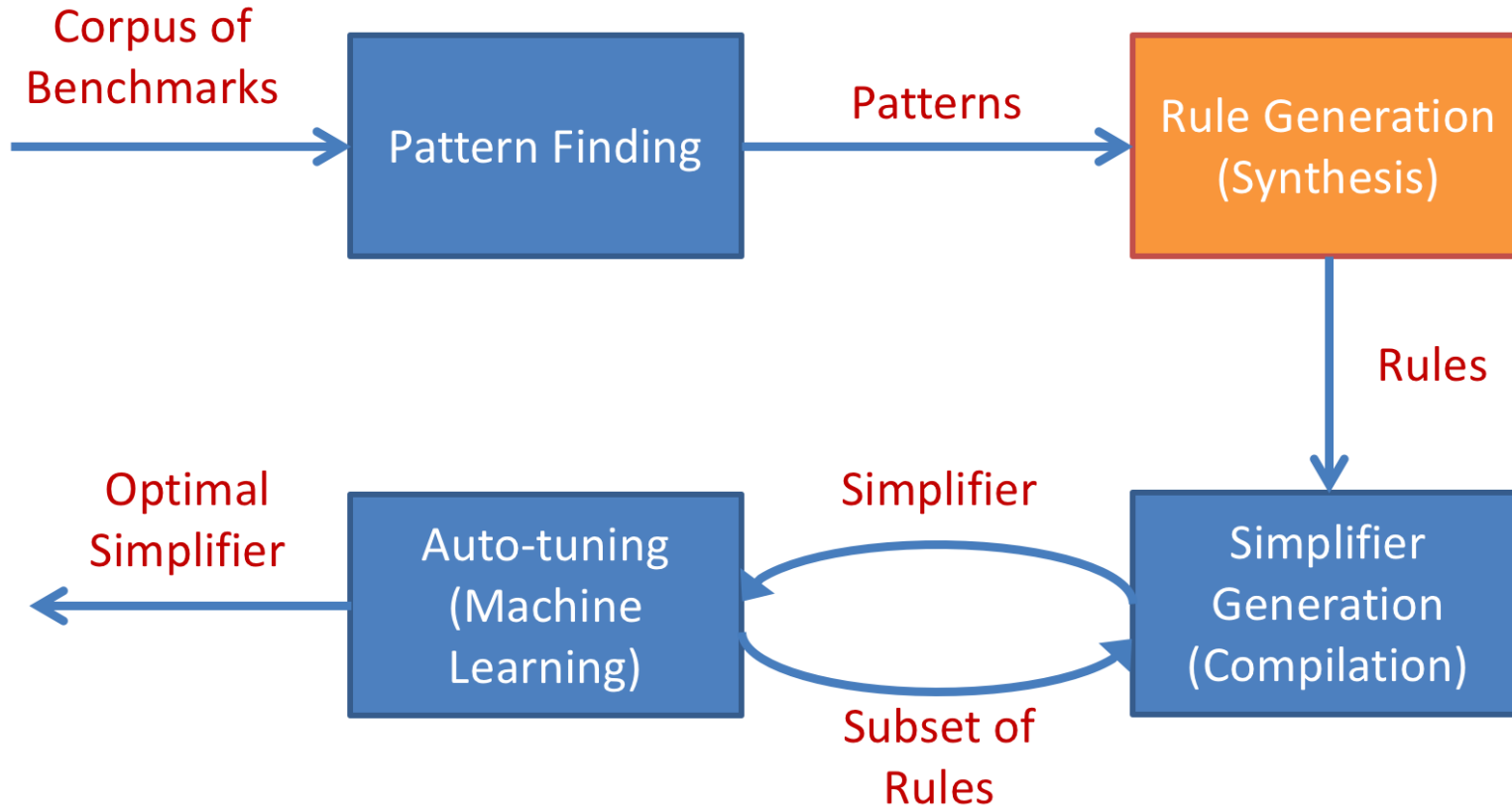
$$b < d$$

$$b \neq d$$

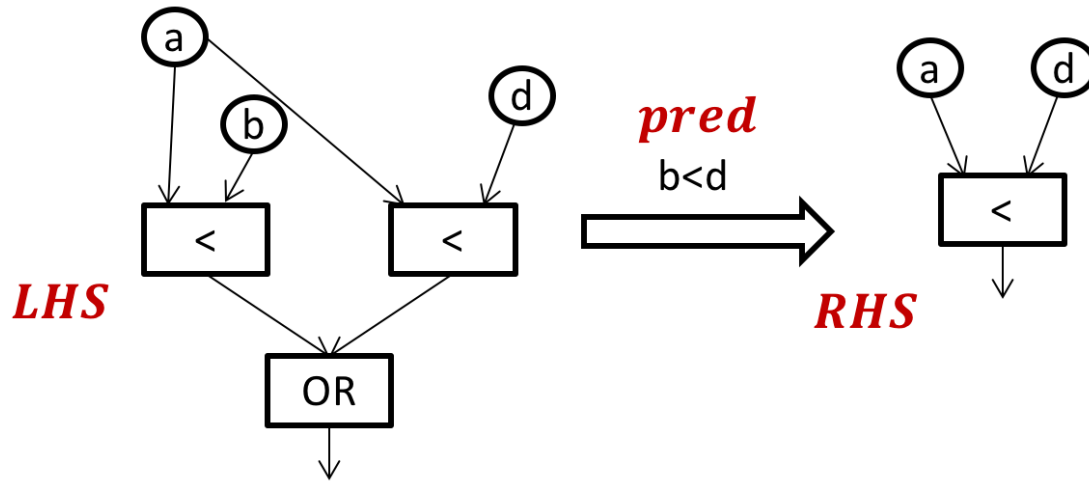
$$b \leq \mathbf{0}$$

$$\mathbf{0} < d$$

SWAPPER framework



Conditional Rewrite Rules



- **Inputs (x) :** a b d
- $\forall x \text{ pred}(x) \Rightarrow (LHS(x) == RHS(x))$

Rule Generation: The Problem

- Given a **pattern** $LHS(x)$, **assumptions** $static(x)$ and **grammars** for $pred$ and RHS , find $pred(x)$, $RHS(x)$ such that:

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 - $\forall x : static(x) \Rightarrow pred(x)$
 - $\forall x : pred(x) \Rightarrow (LHS(x) == RHS(x))$
 - $size(RHS) < size(LHS)$
 - $pred(x)$ is one of the weakest (most permissive) candidates

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$pred(x) \equiv x_i \mathbf{binop} x_j$ for integer x_i, x_j

| x_i | $\neg x_j$ for boolean x_i, x_j

| **True**

$x = (x_1, x_2, \dots, x_n)$ and $1 \leq i \neq j \leq n$

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binop $\equiv < | > | \leq | \geq | == | \neq$

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| x_i | $\neg x_j$ for boolean x_i, x_j

| **True**

$x = (x_1, x_2, \dots, x_n)$ and $1 \leq i \neq j \leq n$

binop $\equiv < | > | \leq | \geq | == | \neq$

- **Grammar for $RHS(x)$:** complete DAGs

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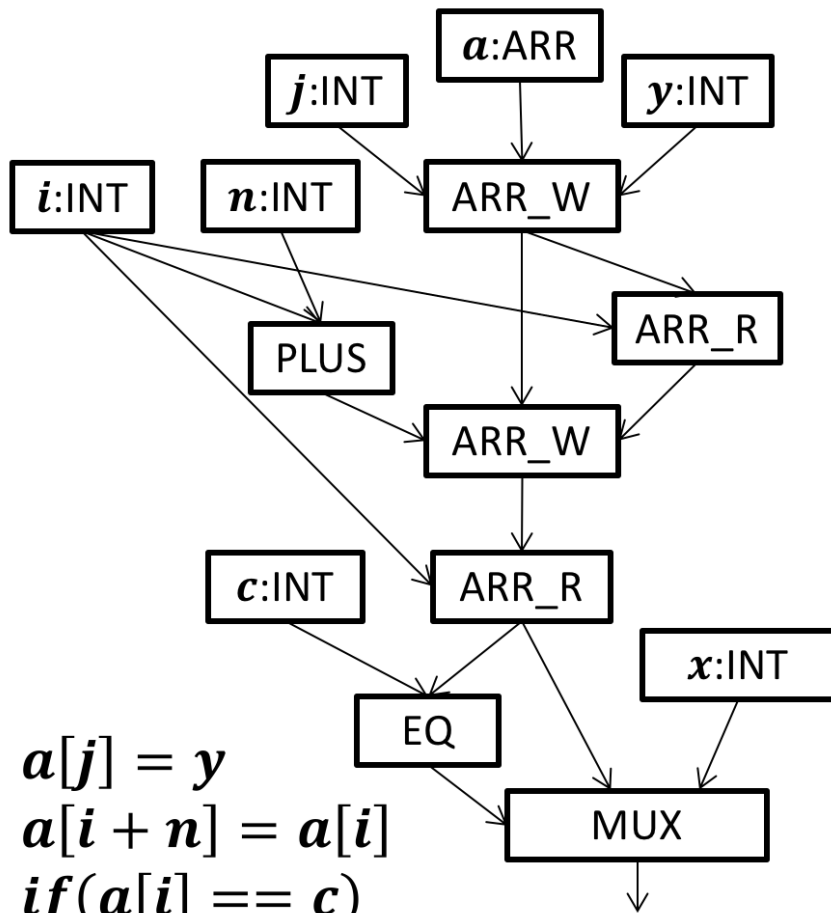
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+ Enumerative predicate refinement

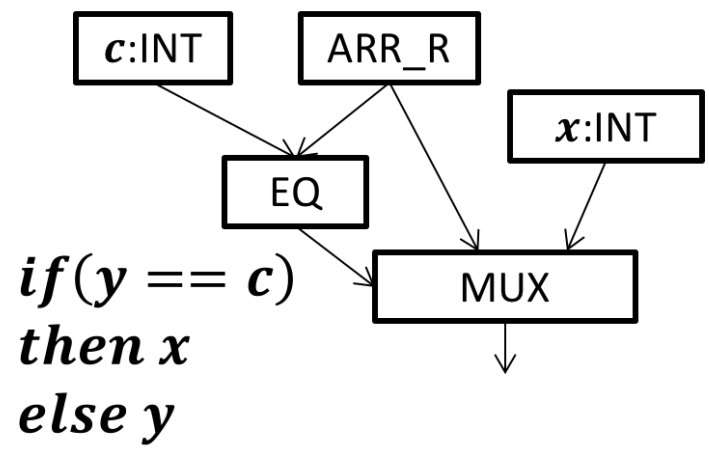
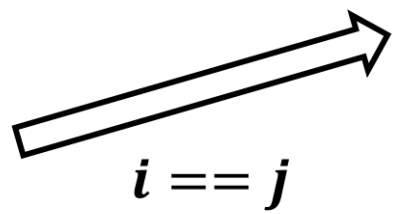
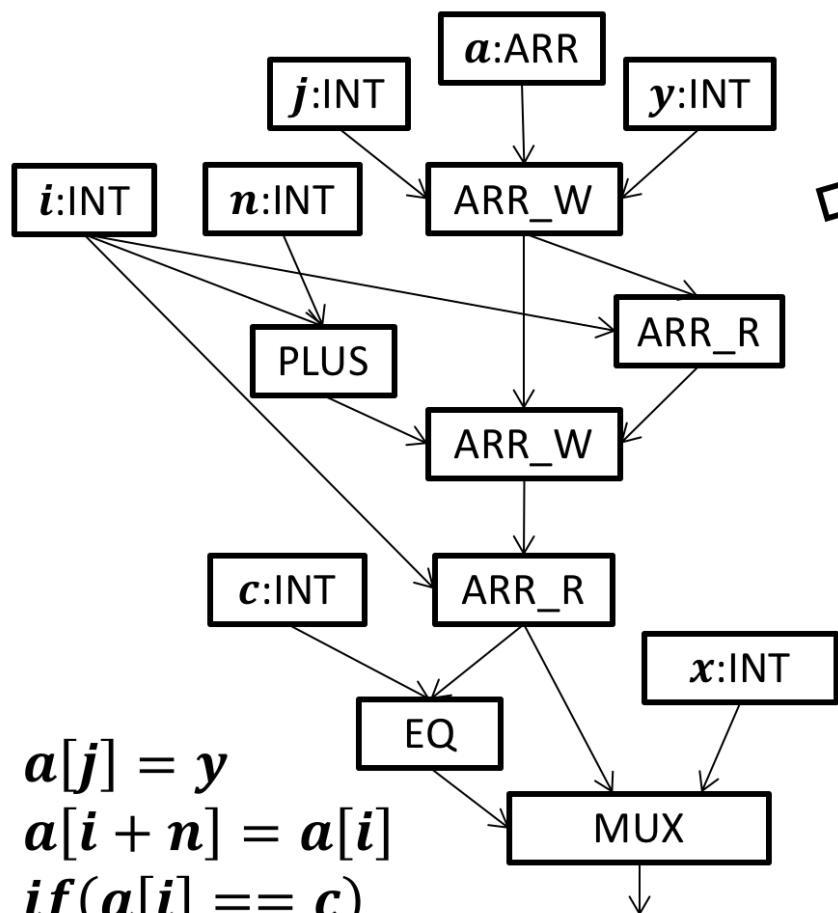
Rule Generation: Example

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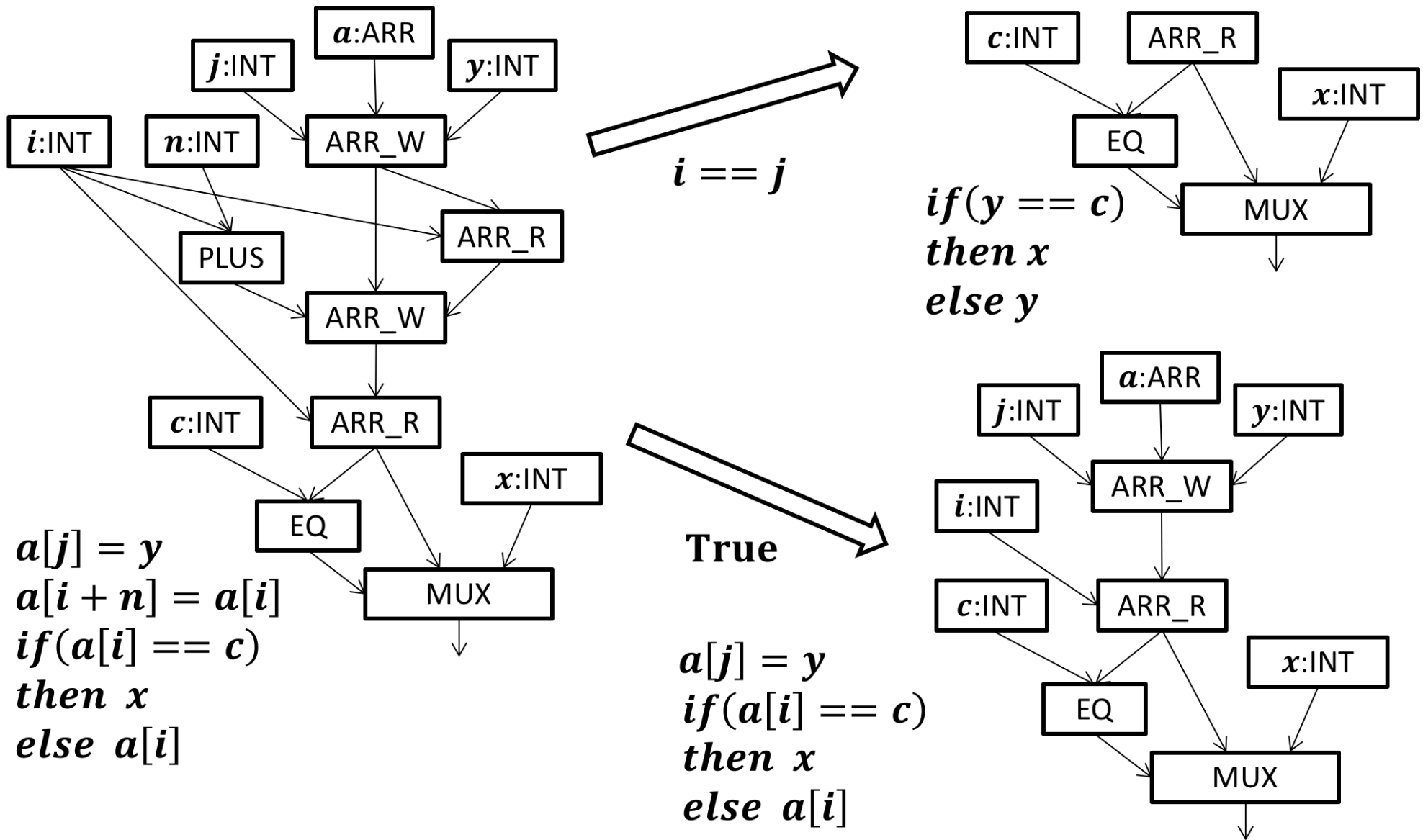
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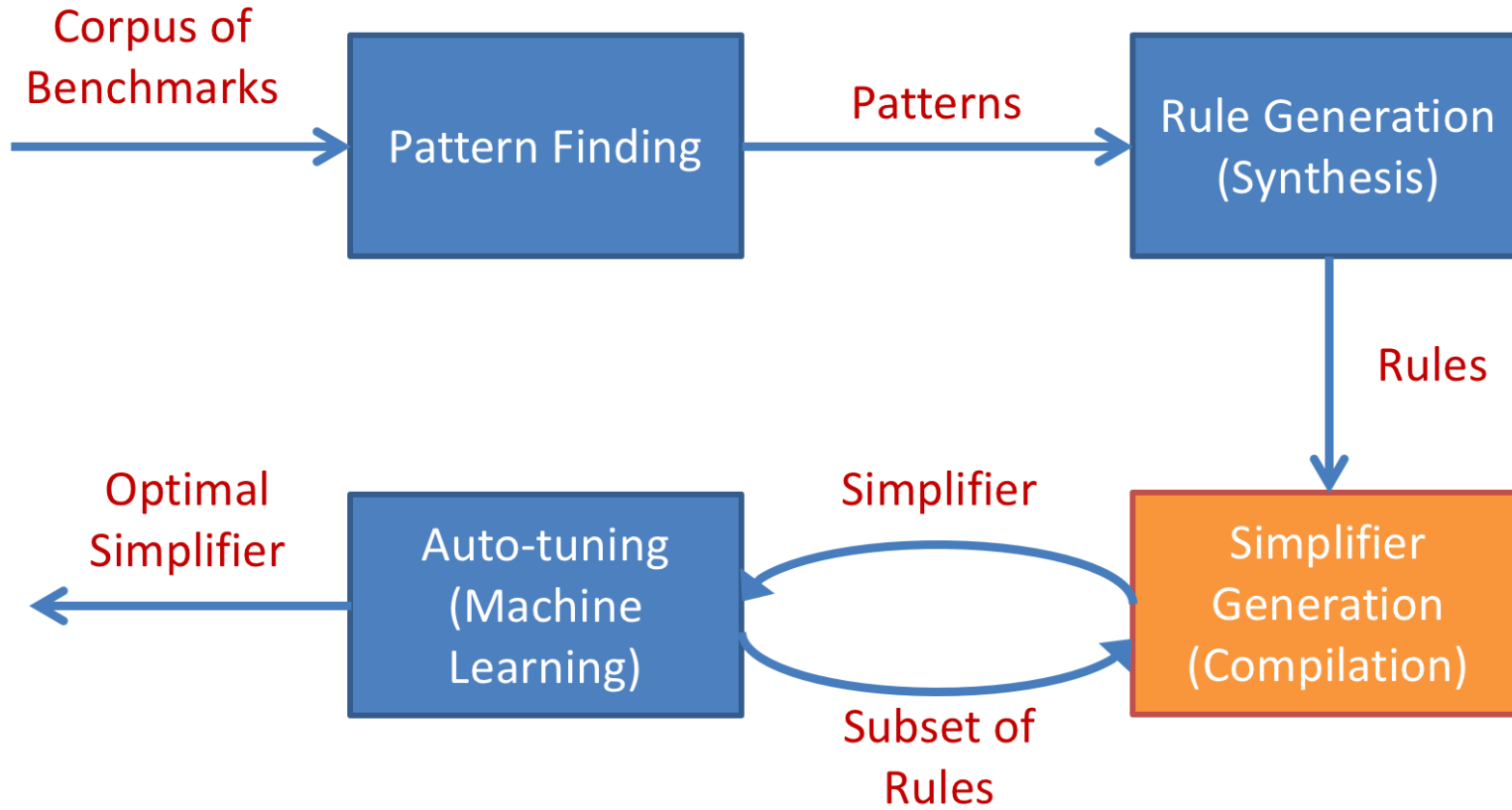


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SWAPPER framework



Simplifier Generation

- Default node traversal and rule application strategy
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 - Fully verifying the rule at each stage

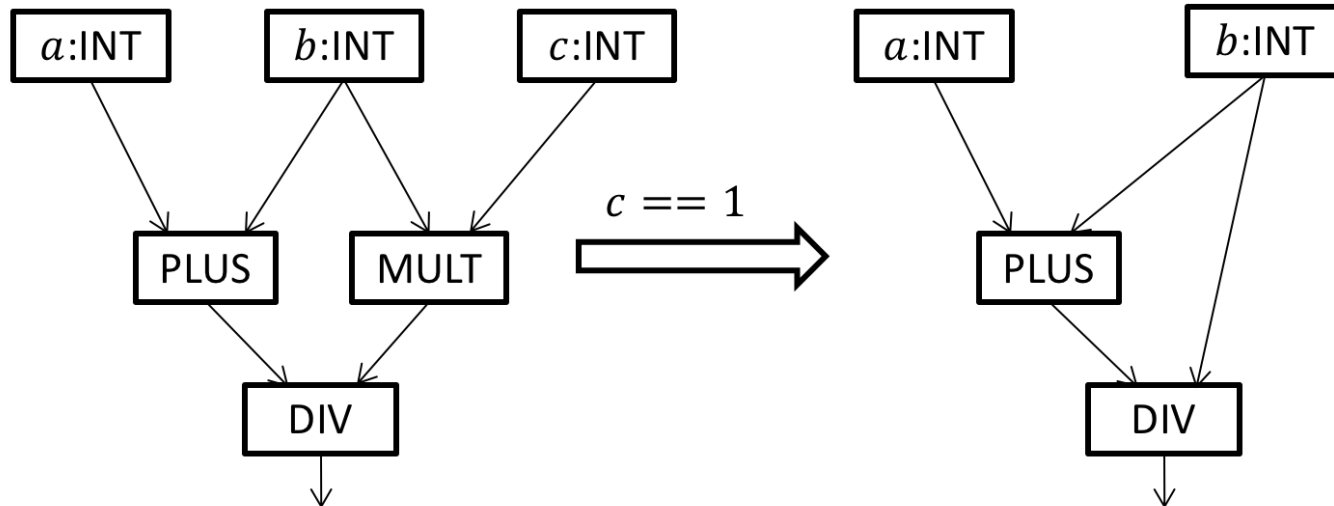
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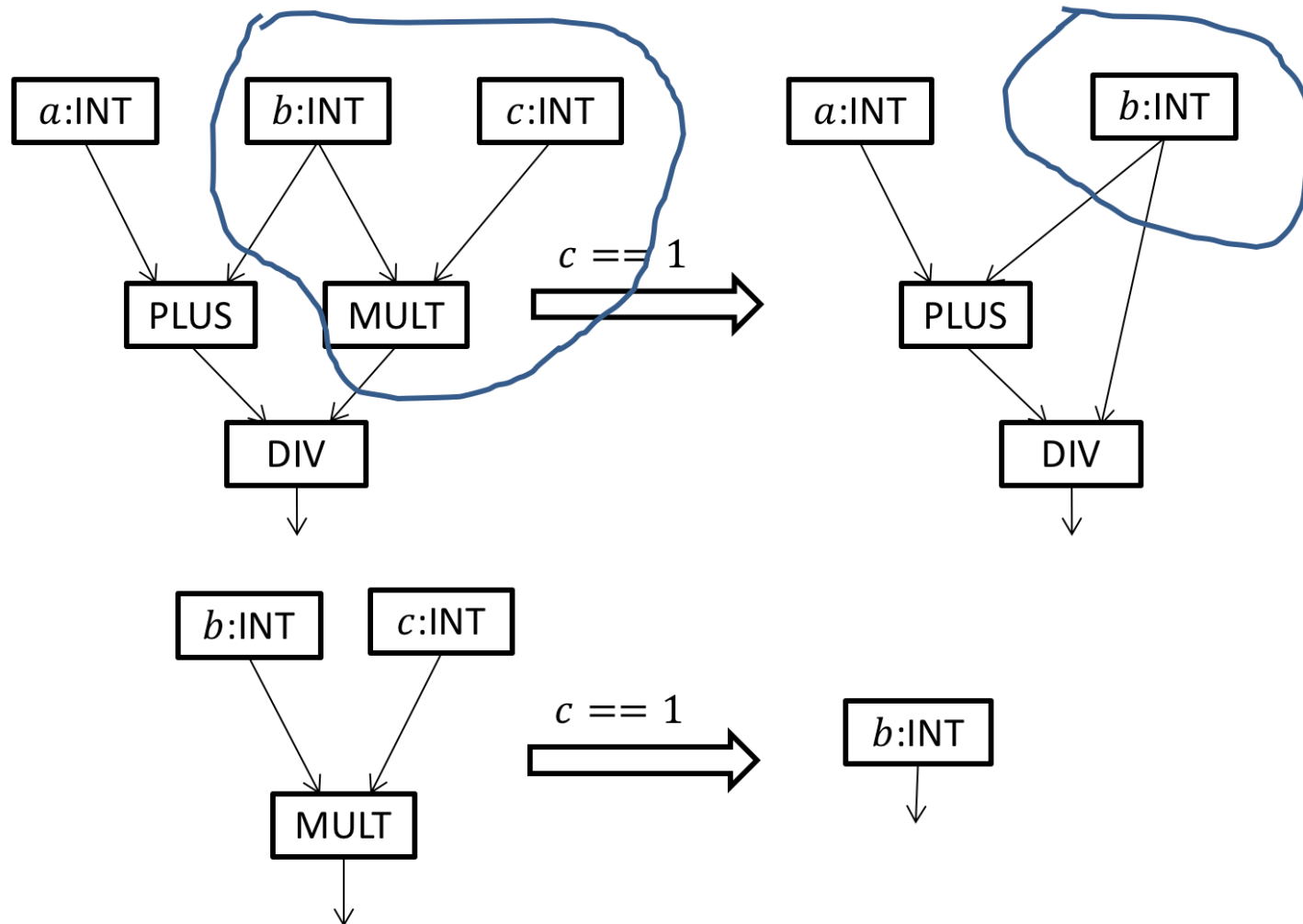
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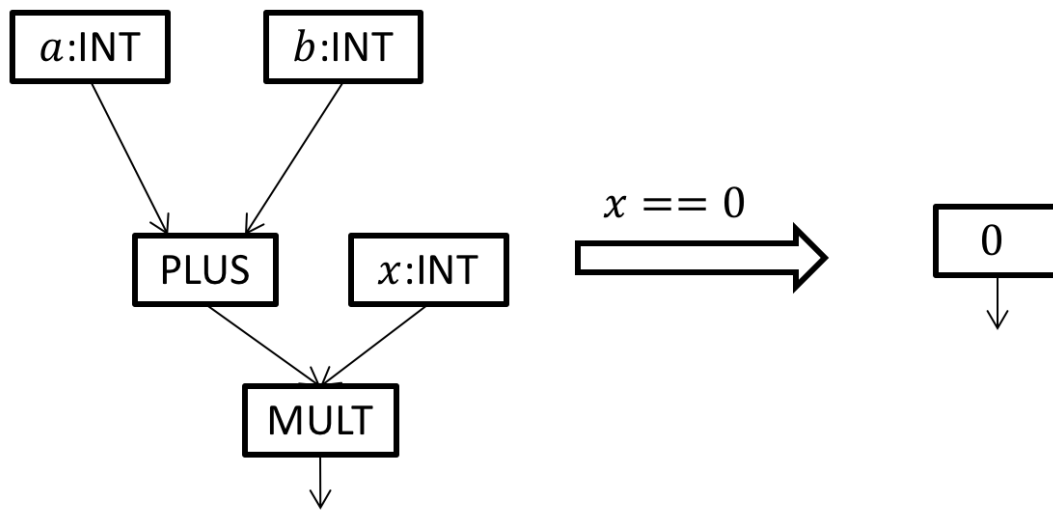


Simplifier Generation: Rule Generalization

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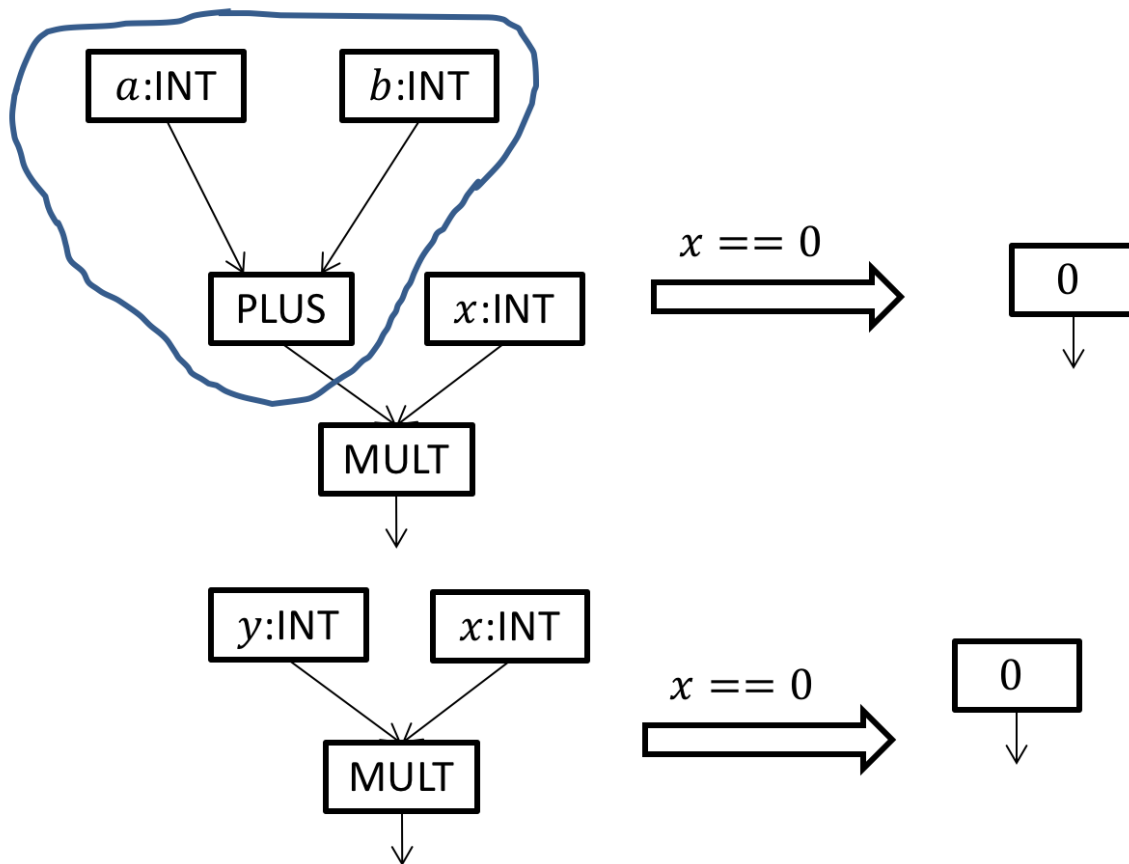
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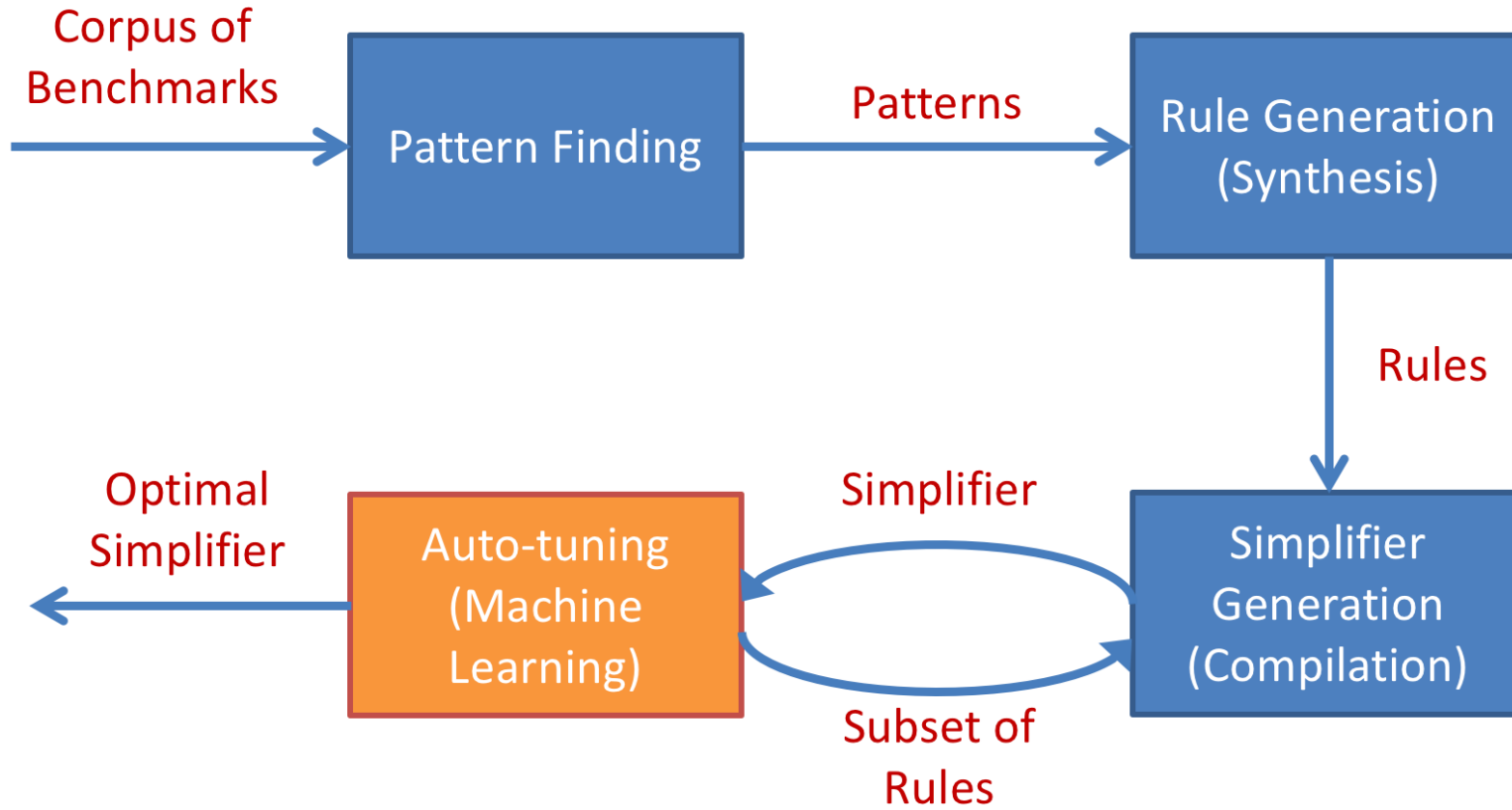


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SWAPPER framework



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- Identifies the *best* subset of rules



Ansel et al, PACT 2014

<http://opentuner.org>

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Auto tuning

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- Problem Setup:
 - Search space parameters:
 - Permutation of rules
 - Number of rules to be used
 - Optimization Function: Weighted Solution time



Ansel et al, PACT 2014

<http://opentuner.org>

Experiments

Domains & Benchmarks

Domain	Benchmark DAGs Used	Avg. Number of Terms
AutoGrader	45	23289
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 - **Auto-generated**: Baseline + generated rules

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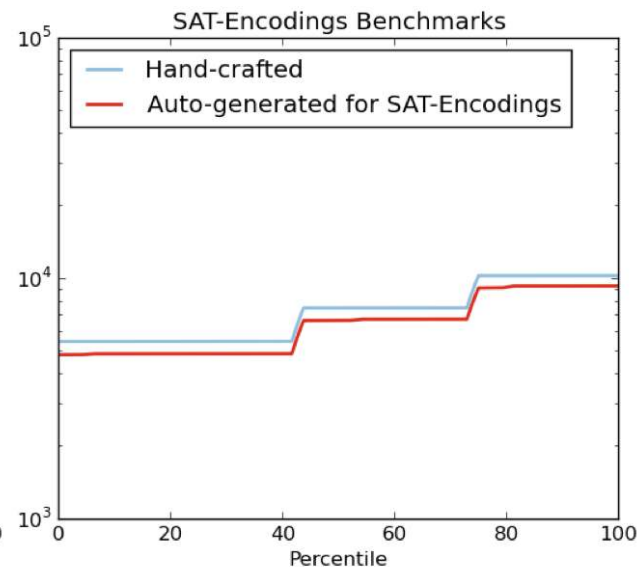
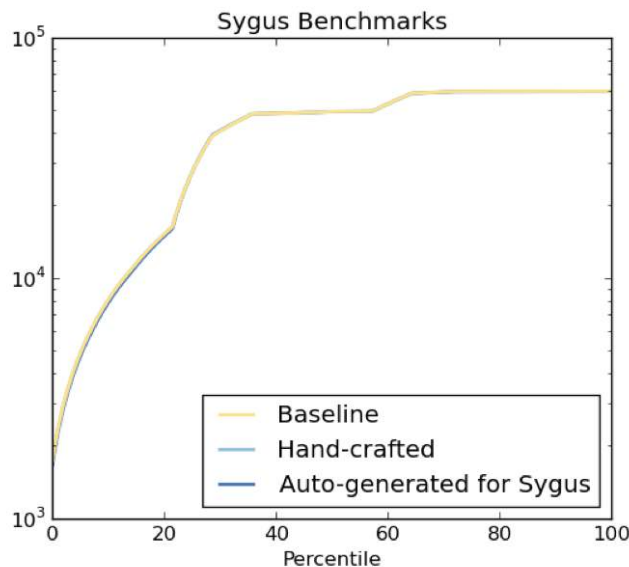
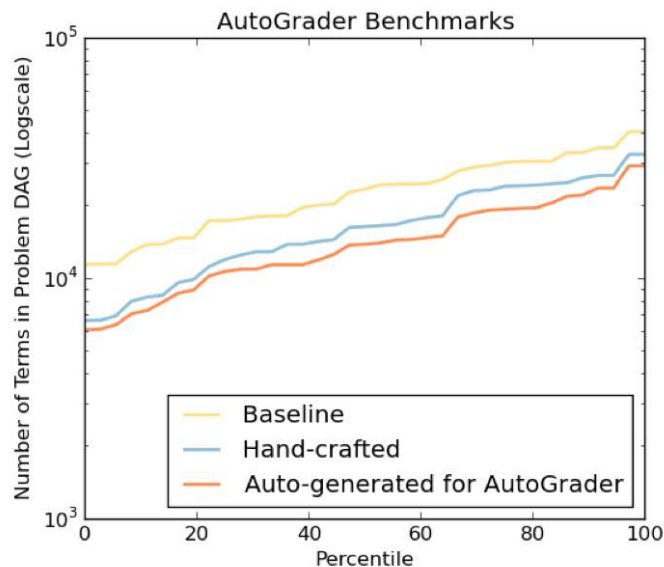
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 - For **AutoGrader** and **SyguS** domains

SWAPPER : Generated Rules

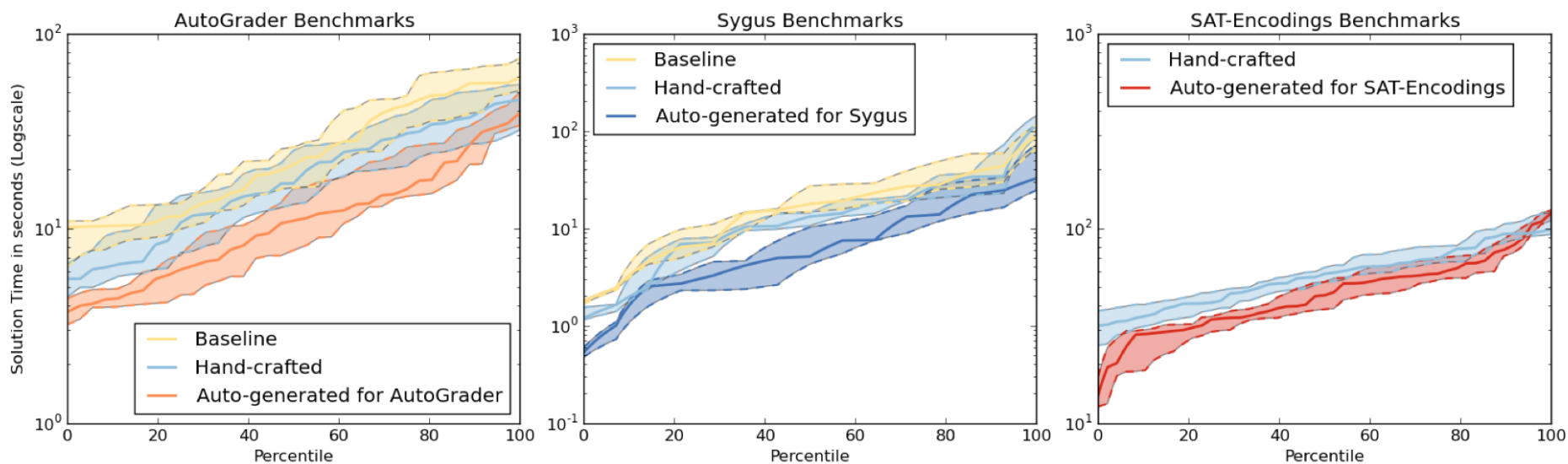
	AutoGrader	SyguS	SAT Encodings
Obtained	201	163	117
Optimal	135	65	68

Impact on Sizes



- **AutoGrader: 13.8% reduction**
- **Sygus: 1.1% reduction**
- **SAT Encodings: 11% reduction**

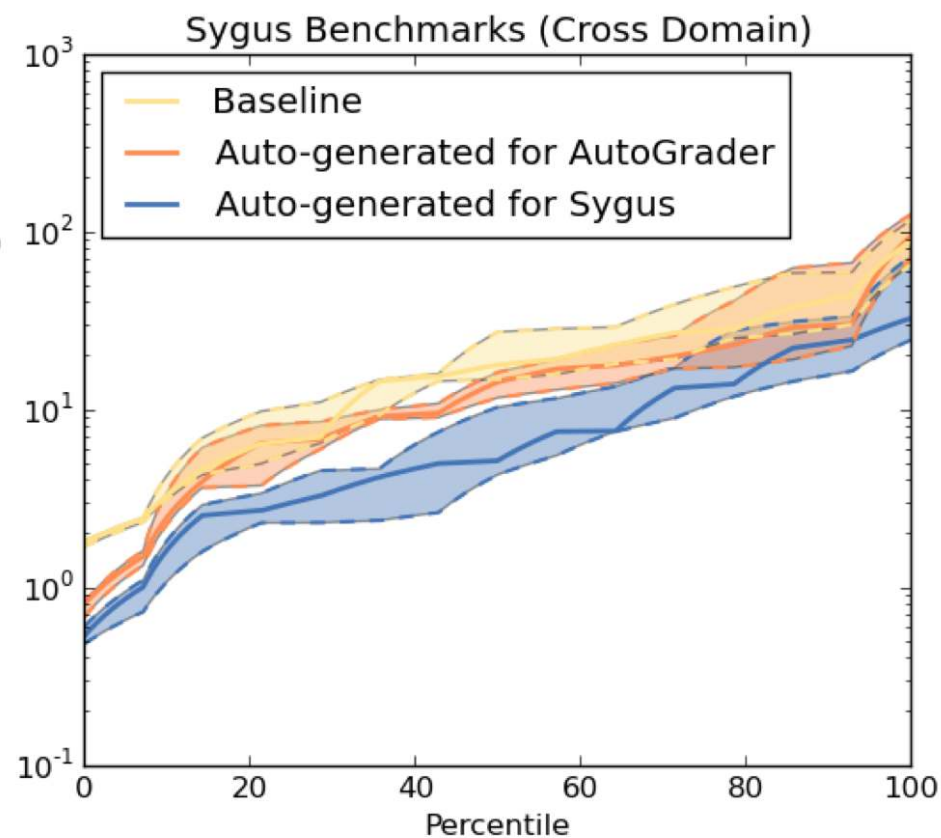
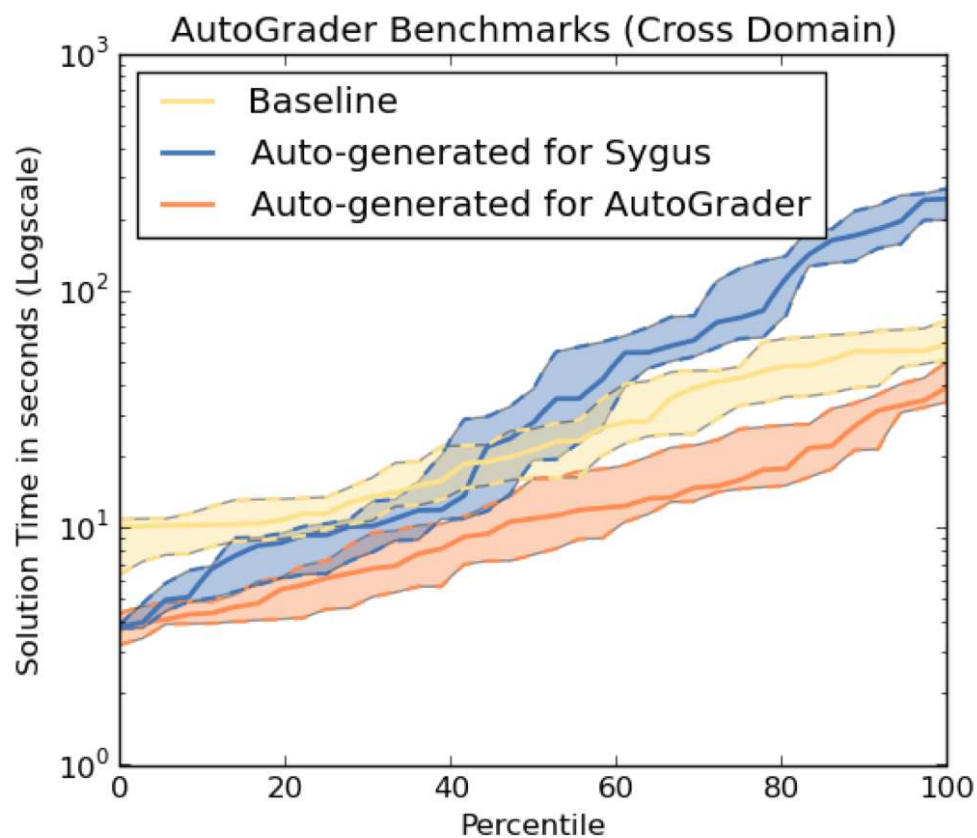
Impact on Running times



- Medians with quartile confidence intervals
- **AutoGrader:** 21s → 13s average times
- **Sygus:** 20s → 8s average times
- **SAT Encodings:** 59s → 51s average times

Domain Specificity

Impact on times across domains



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Time and Cost Estimation (on AWS, parallelism of 40 threads)

Domain	Pattern Finding	Rule Generation	Auto-Tuning	Total Time (in hours)	Cost
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Can reduce time by increasing parallelism or smarter evaluations with timeouts

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Thank You!