Boolean Synthesis via Decomposition

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Boolean Synthesis

Boolean Synthesis [1]

Given: Boolean formula $F(\vec{x}, \vec{y})$ representing a relation over input variables $\vec{x} = \{x_1, \dots, x_m\}$ and output variables $\vec{y} = \{y_1, \dots, y_n\}$

Obtain: Boolean function $g : \{0,1\}^m \to \{0,1\}^n$ such that, for all $\vec{x}$,

$$F(\vec{x}, g(\vec{x})) \Leftrightarrow \exists \vec{y}.F(\vec{x}, \vec{y})$$

- F is called the *specification*.
- g is called the *implementation*.

Example: The two's complement of a two-bit integer x_1x_0 is a two-bit integer y_1y_0 such that $x_1x_0 + y_1y_0 = 0$. We can synthesize a function that computes the two's complement as follows:

$$F(x_0, x_1, y_0, y_1) = \neg(x_0 \oplus y_0) \wedge \neg(x_1 \oplus y_1 \oplus (x_0 \wedge y_0))$$

$$\Downarrow$$

$$g(x_0, x_1) = \begin{cases} y_0 := x_0 \\ y_1 := x_1 \oplus x_0 \end{cases}$$

Despite extensive research on the subject, Boolean synthesis remains a challenging NP-hard problem.

A standard strategy for handling hard problems is decomposing them into smaller problems. Our goal is to apply this concept to Boolean synthesis.

Decomposition using Factored Formulas

One way to decompose Boolean synthesis is to use factored formulas [2, 3]:

$$F(\vec{x}, y_1, y_2, y_3, y_4) = F_1(\vec{x}, y_2, y_4) \wedge F_2(\vec{x}, y_1, y_2, y_3) \wedge F_3(\vec{x}, y_3)$$

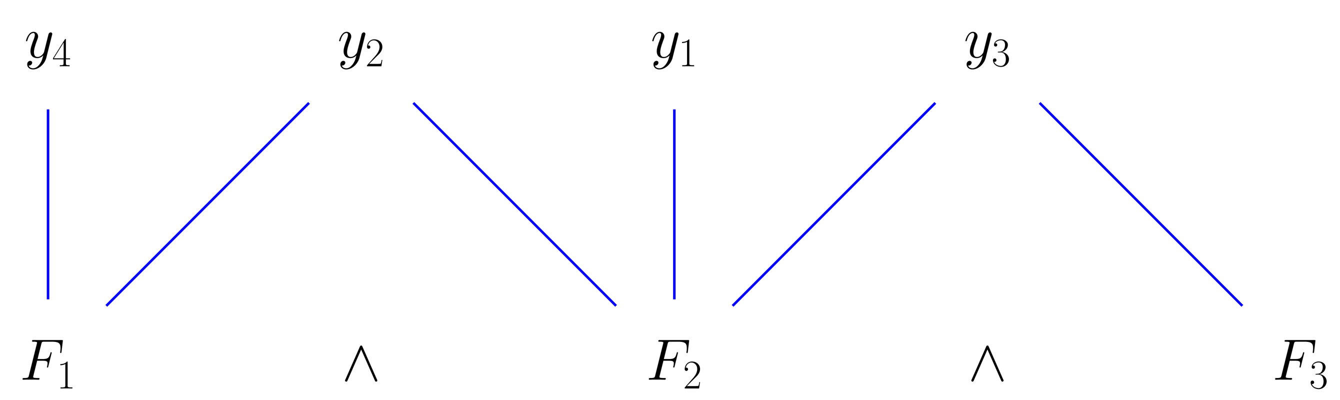
Pros:

- Easy to perform decomposition.
- Specifications are often already given as a conjunction of constraints.
- Each factor uses only a subset of the variables.

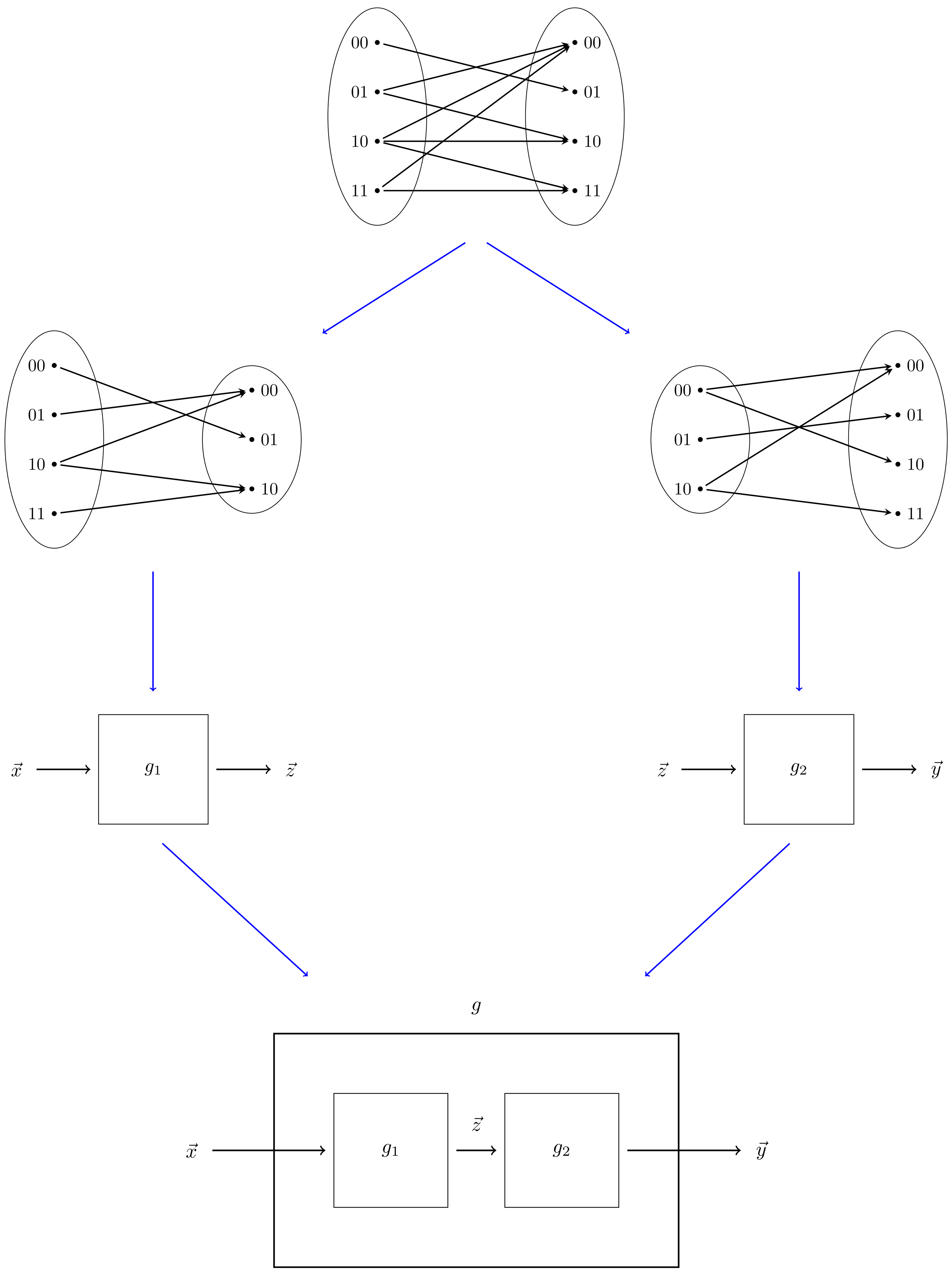
Cons:

- Dependences between factors.
- Highly non-trivial to combine implementations of $F_1, \dots, F_k$ into an implementation of F [2].

This form of decomposition has been shown to significantly improve synthesis algorithms [2, 3]. However, dealing with the dependences between factors prevents us from taking full advantage of the decomposition [3]:



Towards Sequential Decomposition



Sequential Decomposition

Given: Boolean formula $F(\vec{x}, \vec{y})$ representing a relation over input variables $\vec{x} = \{x_1, \dots, x_m\}$ and output variables $\vec{y} = \{y_1, \dots, y_n\}$

Obtain: Formulas $F_1(\vec{x}, \vec{z})$ and $F_2(\vec{z}, \vec{y})$ for intermediate variables $\vec{z} = \{z_1, \dots, z_k\}$ such that, if g_1 implements F_1 and g_2 implements F_2, then $g_2 \circ g_1$ implements F.

References

[1] Dror Fried, Lucas M. Tabajara, and Moshe Y. Vardi.
BDD-Based Boolean Functional Synthesis.
In *Computer Aided Verification - 28th International Conference, CAV 2016, Toronto, ON, Canada, July 17-23, 2016, Proceedings, Part II.* Springer, 2016.

[2] Ajith K. John, Shetal Shah, Supratik Chakraborty, Ashutosh Trivedi, and S. Akshay.
Skolem Functions for Factored Formulas.
In *Formal Methods in Computer-Aided Design, FMCAD 2015, Austin, Texas, USA, September 27-30, 2015.*, pages 73–80, 2015.

[3] Lucas M. Tabajara and Moshe Y. Vardi.
Factored Boolean Functional Synthesis.
In *Formal Methods in Computer-Aided Design, FMCAD 2017, Vienna, Austria, October 2-6, 2017.*, 2017.