A THEORY OF SATISFIABILITY-PRESERVING PROOFS IN SAT SOLVING

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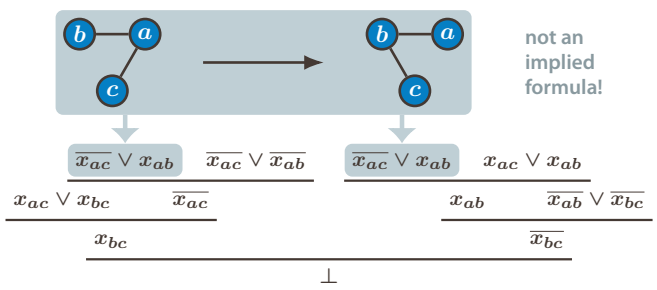
logics LOGICAL METHODS IN COMPUTER SCIENCE

Reasoning without loss of generality

Given three people a, b and c, either one of them is a friend of everybody else, or one of them is a friend of nobody else.

$$\vDash (x_{ab} \wedge x_{ac}) \vee (x_{ab} \wedge x_{bc}) \vee (x_{ac} \wedge x_{bc}) \vee (\overline{x_{ab}} \wedge \overline{x_{ac}}) \vee (\overline{x_{ab}} \wedge \overline{x_{bc}}) \vee (\overline{x_{ac}} \wedge \overline{x_{bc}})$$

Without loss of generality, we can assume that, if c is a friend of a, then c is also a friend of b; if this is not the case, simply swap a and b. Assume the claim is false. Then, if b is a friend of c, it follows that a cannot be a friend of b nor c; and if b is not a friend of c, then by our assumption without loss of generality, c is not a friend of a.



Interpretation overwrite

$$B = \{\overline{x_1}, x_3, \overline{x_6}\}$$

	I	B	$I + B$
x_1	1	0	0
x_2	0		0
x_3	0	1	1
x_4	1		1
x_5	1		1
x_6	0	0	0

Conditional overwrite

$$I + (B :\!- \psi) = \begin{cases} I + B & \text{if } I \vDash \psi \\ I & \text{if } I \nvDash \psi \end{cases}$$

Pizza connective

$I \vDash \nabla(B :\!- \psi).\, \varphi$ if and only if $I + (B :\!- \psi) \vDash \varphi$

The semantics of DRAT / DPR

C is a RAT clause in F upon l

If F holds, then without loss of generality C holds as well: if this is not the case for an interpretation I, overwriting the literal l in I yields the interpretation $I + l$, which satisfies both F and C.

$$F \vDash \nabla(l :\!- \overline{C}).\, F \wedge C$$

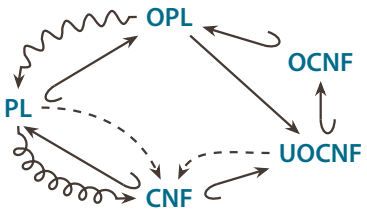
C is a PR clause in F upon B

$$F \vDash \nabla(B :\!- \overline{C}).\, F \wedge C$$

DRAT / DPR operates as a truth-preserving proof system in overwrite logics.

Overwrite logics

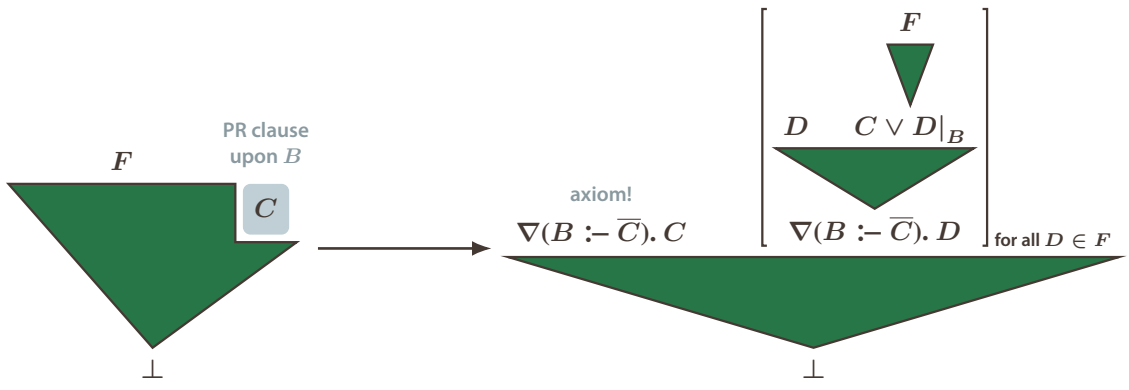
PL	standard propositional logic
CNF	standard clausal normal form
OPL	overwrite propositional logic
OCNF	overwrite clausal normal form
UOCNF	uniformly overwrite clausal normal form



polynomial simulation
embedding
exponential simulation
exponential separation
polynomial reduction for satisfiability

UOCNF is to CNF what DRAT / DPR is to DRUP

Overwrite proof systems



Proofs become truth-preserving, with nodes represented by overwrite clauses.

Inferences that cannot be done in DRAT / DPR due to interference become available.

Deletion becomes irrelevant, allowing tree-shaped proofs.

References

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