

Open book and notes.

Max points = 50

Time = 50 min

Do all questions.

1. (Compression; 11 points)
 - (a) (4 points) Create a Huffman tree for symbols with the following frequencies: {8, 4, 1, 2, 6, 9, 10}.
 - (b) (7 points) Show that the successive values computed during execution of the Huffman algorithm (by adding the two smallest values) are nondecreasing.
2. (Error Correction; 21 points)
 - (a) (4 points) Let W' be a set obtained from a dependant set W by either removing an element or adding an element. Given W' describe how to determine W .
 - (b) (5 points) How many errors can be detected and how many corrected given the following set of codewords: {00000, 01110, 11001, 10111}?
 - (c) (4 points) Given the set of codewords as above, what can the receiver say if she receives the string 01010? What if she receives 10100?
 - (d) (8 points) Show that in any Hadamard matrix, the top row has all 1s, and every other row (if any) has equal number of 0s and 1s.
3. (Cryptography; 18 points)
 - (a) (7 points) There are 5 candidates in an election; number them 0 through 4. Each voter votes for exactly one candidate and sends its vote to a trusted party. Show that if a voter encrypts his vote directly using public key cryptography, it is vulnerable. Suggest a technique for secure transmission, again using public key cryptography. Show how the trusted party can decrypt and count the votes it receives.
 - (b) (4 points) A sender transmits a sequence of blocks using the following scheme. Encrypt the first block by doing exclusive-or of the block with a secret key, and subsequent blocks by doing exclusive-or with the previous encrypted block. Show that this scheme is insecure.
 - (c) (7 points) A sender transmits a sequence of blocks using the following scheme. Encrypt the first block by doing exclusive-or of the block with a secret key, and subsequent blocks by doing exclusive-or of the block with the plaintext of the previous block. Show that this scheme is secure if the eavesdropper can only apply exclusive-or over the blocks.

Hint: Show that no matter how many blocks are exclusive-ored, the result is never a single block of plaintext.