

Open book and notes.

**Problem** Variables  $x$  and  $y$  are unknowns in the following two equations.

$$x \oplus y = u,$$

$$x \oplus u = v,$$

$$\text{where } u = 0\ 1\ 0\ 1 \text{ and } v = 1\ 0\ 1\ 1$$

What are  $x$  and  $y$  ?

**Solution**

$$\begin{aligned} & x \oplus y = u \text{ and } x \oplus u = v \\ \Rightarrow & \{\text{taking } \oplus \text{ of left and right sides}\} \\ & x \oplus y \oplus x \oplus u = u \oplus v \\ \Rightarrow & \{\text{simplify lhs}\} \\ & y \oplus u = u \oplus v \\ \Rightarrow & \{\text{cancel } u \text{ from both sides}\} \\ & y = v \\ \Rightarrow & \{v = 1\ 0\ 1\ 1\} \\ & y = 1\ 0\ 1\ 1 \end{aligned}$$

Next,

$$\begin{aligned} & x \oplus y = u \\ \Rightarrow & \{\text{substitute } v \text{ for } y\} \\ & x \oplus v = u \\ \Rightarrow & \{\text{apply } \oplus v \text{ to both sides}\} \\ & x = u \oplus v \\ \Rightarrow & \{u = 0\ 1\ 0\ 1 \text{ and } v = 1\ 0\ 1\ 1\} \\ & x = 1\ 1\ 1\ 0 \end{aligned}$$