

Problem A sender uses the following encoding scheme: for each two bit block, he replicates the block 3 times; so, 00 is transmitted as 000000 and 10 as 101010. How many errors can be detected and how many corrected with this scheme?

Solution Let x and y be distinct blocks of two bits each. If they differ in a single bit, their codewords differ in 3 bits, one for each replication of the block. And if they differ in 2 bits their codewords differ in 6 bits. Therefore, the minimum Hamming distance between any two codewords is 3. The receiver can detect up to 2 errors ($3-1$) and correct up to 1 error (less than $3/2$).