

CS 313E (Mitra)

Climbing Stairs Problem

Given a staircase with N steps in total to reach the top where you can climb either 1, 2, or 3 steps at a time, your goal is to find out the number of distinct ways you can reach the top. For example, in a staircase with 3 stairs, you can reach the top the following ways:

1. 1 step + 1 step + 1 step
2. 1 step + 2 steps
3. 2 steps + 1 step
4. 3 steps

This gives you a total of 4 ways to reach the top.

If we consider a random step K , this can be reached from the $(K-1)^{\text{th}}$, $(K-2)^{\text{th}}$, and $(K-3)^{\text{th}}$ steps by climbing 1, 2, or 3 steps respectively. Because of this property, the number of ways to reach the K^{th} step is simply the sum of the number of ways to reach the 3 steps before it. Therefore, this problem can be solved using dynamic programming if we let $DP[i]$ be the number of ways to reach the i^{th} step by using the following recurrence:

$$DP[i] = DP[i-1] + DP[i-2] + DP[i-3]$$

if $i-1$, $i-2$, or $i-3$ are negative simply let $DP[\text{negative}] = 0$

Using this formula, complete the table below to find the number of ways to reach the top of a staircase with 10 steps.

Stair, i	$DP[i]$
0	1
1	1
2	2
3	4
4	
5	
6	
7	
8	
9	
10	