

1. This is an old puzzle attributed to Alcuin of York (735 – 804) – a farmer needs to carry a wolf, a goat, and a cabbage across a river. The farmer only has a small boat, that can carry the farmer and only one object (an animal or vegetable). He can cross the river repeatedly. However, in the absence of the farmer the wolf will eat the goat, and the goat will eat the cabbage. Construct a graph to solve this problem. Each vertex of the graph is an allowable state. For example, we can use the pair of symbols (FG, WC) to denote that the farmer and goat are on the first shore and the wolf and cabbage are on the other shore. The starting state is (FWGC, 0) where 0 denotes nothing is on the other shore. The ultimate solution will be (0, FWGC). There are two different solutions to the problem each using seven crossings. Suppose that the farmer must pay a toll of one dollar whenever he crosses the river with an animal (but not a vegetable). What is the cost of the two solutions?

2. Use the above graph model to solve the jealous husbands problem. Two married couples, each a husband and a wife, want to cross a river. They can only use a boat that can carry one or two people from one shore to the other shore. Each husband is extremely jealous and is not willing to leave his wife with the other husband, either in the boat or on shore. How can these four people reach the opposite shore? Use symbols H, W, h, w to denote the two couples. What if there are three couples, what would the graph look like? Is there a solution if the number of couples is four or greater?

3. Suppose that you have a three gallon jug and a five gallon jug. You may fill either with water, you may empty either jug, and you may transfer water from either jug into the other jug. Use a path in a directed graph to show that you can end up with a jug containing exactly one gallon. Use an ordered pair (a, b) to indicate how much water is in each jug. Represent these ordered pairs by vertices. Add an edge for each allowable operation with jugs.