CS313K: Logic, Sets, and Functions

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(Lecture 1)

Syllabus

`http://www.cs.utexas.edu/users/moore/classes/cs313k`

The Last Real Lecture

This will be the last real lecture you get from me!

The typical class will be

- you ask me questions
- I ask you questions (via iClicker quizzes)
- I talk about whatever your answers bring to mind

Read the Book

"Lecture! Then ask questions!" – *Anonymous*

"You can't ask questions about things you haven't talked about." – *Anonymous*

Read the Book!

You are supposed *to have read the assigned material before class*.

This is not high school.

Read the material before class.

I'm here to answer your questions, not read to you.

Why Do We Study

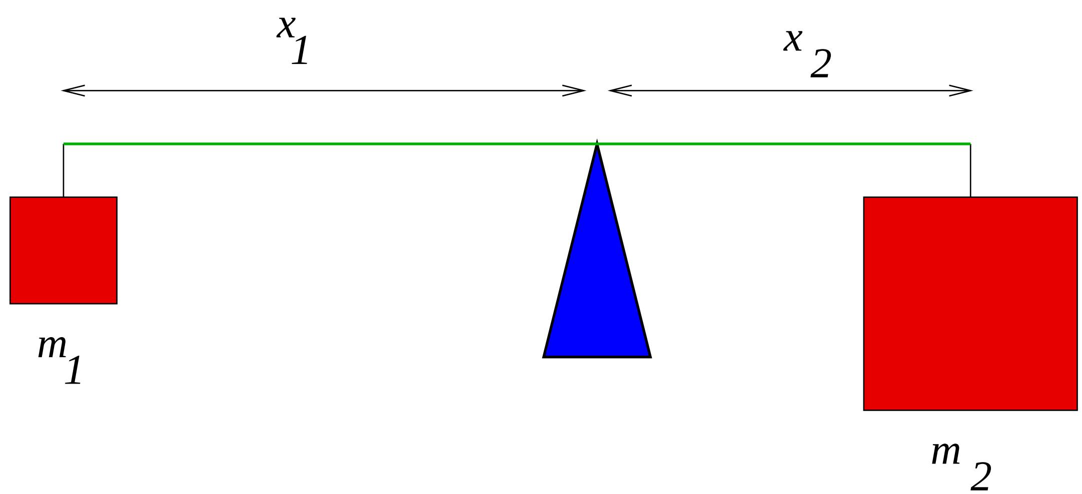
- logic?
- sets?
- functions?

How do you know your programs are right?

What does “right” even mean?

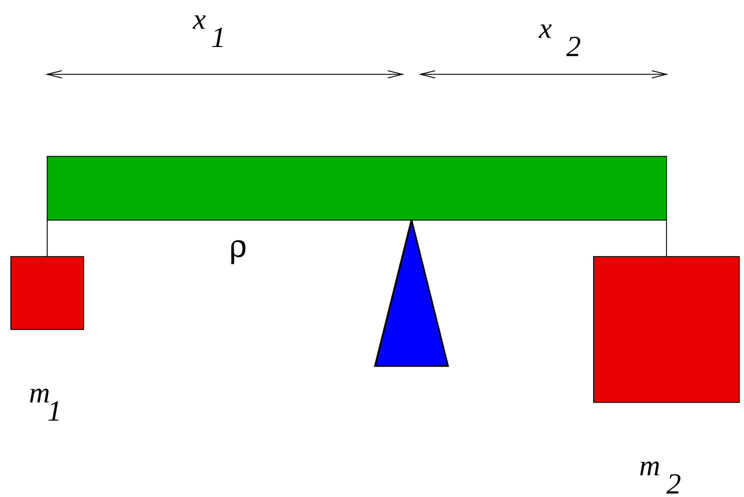
If you’re building a bridge, putting a satellite into orbit, or balancing a beam you can use algebra and calculus to calculate whether your plan will work.

Proposition 6: *Commensurable magnitudes are in equilibrium at distances reciprocally proportional to the weights.* — Archimedes



$$m_1 \times x_1 = m_2 \times x_2.$$

The mathematics necessary to describe even simple physical systems may be complicated.



$$m_1 x_1 + \int_0^{x_1} \rho x dx = m_2 x_2 + \int_0^{x_2} \rho x dx$$

But what mathematics is used to model and predict the behavior of computer programs?

But what mathematics is used to model and predict the behavior of computer programs?

- logic!
- sets!
- functions!

```
public static String majority(String[] a){
    int k = 0;
    String c="";
    for (int i=0; i<a.length; i++)
{if (k==0){
    c=a[i];k=1;
   } else {
     if (c.equals(a[i])) {k++;
     } else {k--;};
 }
}
    return c;
}
```

What Does This Code Do?

`A A A C C B B C C C B C C`

i

$c : k$

What Does This Code Do?

```
 A A A C C B B C C C B C C
⇑
"":0
```

What Does This Code Do?

```
A A A C C B B C C C B C C
 ⇑
 A:1
```

What Does This Code Do?

```
A A A C C B B C C C B C C
    ⇑
    A:2
```

What Does This Code Do?

```
A  A  A  C  C  B  B  C  C  C  B  C  C
        ⇑
        A:3
```

What Does This Code Do?

```
A A A C C B B C C C B C C
      ⇑
      A:2
```

What Does This Code Do?

```
A  A  A  C  C  B  B  C  C  C  B  C  C
```

$$\Uparrow$$

```
A:1
```

What Does This Code Do?

```
A  A  A  C  C  B  B  C  C  C  B  C  C
                  ⇑
                  A:0
```

What Does This Code Do?

`A A A C C B B C C C B C C`

$\Uparrow$

`B:1`

What Does This Code Do?

```
A A A C C B B C C C B C C
                ⇑
                B:0
```

What Does This Code Do?

```
A A A C C B B C C C B C C
                  ⇑
                  C:1
```

What Does This Code Do?

```
A A A C C B B C C C B C C
                  ⇑
                  C:2
```

What Does This Code Do?

```
A A A C C B B C C C B C C
                       ⇑
                       C:1
```

What Does This Code Do?

```
A A A C C B B C C C B C C
                       ⇑
                       C:2
```

What Does This Code Do?

`A A A C C B B C C C B C C`

$\Uparrow$

`C:3`

Why Is It Correct?

"The majority will win a fair fight."

```
public static String majority(String[] a){
    int k = 0;
    String c="";
    for (int i=0; i<a.length; i++)
{if (k==0){
    c=a[i];k=1;
   } else {
     if (c.equals(a[i])) {k++;
     } else {k--;};
 }
}
    return c;
}
```

Note

This course is not about Java! You're not going to be writing any Java!

This course could be called "Mathematics for Computer Science."

You'll be writing mathematics in here.

But what mathematics?

Showing an example of an algorithm does not answer the question "what does this code do?"

An argument like "The majority will win a fair fight." isn't convincing that *this* code is correct.

By writing down formulas to describe what we expect of our algorithms we can then use math to analyze whether our algorithms work, when we can use them, how to extend them, etc.

But what formulas?

What Does This Code Do?

$$(\exists \texttt{e} \in \texttt{a} : \texttt{(hm e a)} > \frac{|\texttt{a}|}{2})$$

$$\longrightarrow$$

$$\texttt{(hm (majority a) a)} > \frac{|\texttt{a}|}{2}$$

Translation: "If there is a majority element in `a` (i.e., an element `e` that occurs more times than half the length of `a`), then the mathematical function `majority` returns a majority element."

We'll define the mathematical function `majority` to be a *mathematical model* of the Java method of that name.

How many times does e occur in a?

$$hm(e,a) = \begin{cases} 0 & \text{if } endp(a) \\ 1 + hm(e, rest(a)) & \text{if } e = first(a) \\ hm(e, rest(a)) & \text{otherwise} \end{cases}$$

But we'll define it in ACL2:

```
(defun hm (e a)
  (if (endp a)
      0
      (if (equal e (first a))
          (+ 1 (hm e (rest a)))
          (hm e (rest a)))))
```

```
public static String majority(String[] a){
    int k = 0;
    String c="";
    for (int i=0; i<a.length; i++)
{if (k==0){
    c=a[i];k=1;
   } else {
     if (c.equals(a[i])) {k++;
     } else {k--;};
 }
}
    return c;
}
```

Math Translation

```
(defun maj (c i a)
  (if (endp a)
      c
    (if (equal i 0)
        (maj (first a) 1 (rest a))
      (maj c
           (+ i (if (equal c (first a)) +1 -1))
           (rest a)))))

(defun majority (a)
  (maj nil 0 a))
```

Theorem

$$(\exists \texttt{e} \in \texttt{a} : (\texttt{hm e a}) > \frac{|\texttt{a}|}{2})$$

$$\longrightarrow$$

$$(\texttt{hm (majority a) a}) > \frac{|\texttt{a}|}{2}$$

Lemma

$$\texttt{i} \in \mathbf{N}$$

$$\wedge$$

$$\frac{\texttt{i} + \texttt{(len a)}}{2} < \texttt{(hm e a)} + \texttt{(if e=c i 0)}$$

$$\longrightarrow$$

$$\texttt{(maj c i a)} = \texttt{e}$$

Proof: Easy, by the induction suggested by `(maj c i a)`.

□

Don't Forget!

Buy the notes **before Thursday's class**

Buy an iClicker **before Thursday's class**

Register the iClicker **before Thursday's class**

Bring them to class

Check the reading and homework assignments

Read the assigned material **before Thursday's class**

Start working on the homework due next Tuesday

Be ready for an iClicker quiz on Thursday