CS313K: Logic, Sets, and Functions

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Lecture 15 – Chap 4 (4.9, 4.10)

Derived Rule: Hyp (for IF)

If you are trying to simplify an `if` while maintaining propositional equivalence and you see the test of the `if` in a propositional position in the true branch, you may replace it by `t`.

The symmetric replacement (by `nil`) is permitted in the false branch.

$$
\begin{array}{l}
\quad \texttt{(if } \alpha \texttt{ (... } \underset{\uparrow_\pi}{\alpha} \texttt{ ...) } \beta \texttt{)} \\
\leftrightarrow \\
\quad \texttt{(if } \alpha \texttt{ (... t ...) } \beta \texttt{)}
\end{array}
$$

where π admits propositional replacement.

$$\texttt{(if } \alpha \texttt{ (... } \underset{\uparrow_\pi}{\alpha} \texttt{ ...) } \beta\texttt{)} \leftrightarrow \texttt{(if } \alpha \texttt{ (... t ...) } \beta\texttt{)}$$

Proof

Case 1: $\alpha \neq \texttt{nil}$

$lhs \leftrightarrow$ {Hyp}

$\texttt{(if t (... t ...) } \beta\texttt{)}$

$rhs \leftrightarrow$ {Hyp}

$\texttt{(if t (... t ...) } \beta\texttt{)}$

`Case 2:` $\alpha =$ `nil`

$lhs \leftrightarrow$ $\{\text{Hyp}\}$

`(if nil (`$\ldots$ α $\ldots$`)` β`)`

$\leftrightarrow$ $\{$`if-ax2`$\}$

β

$rhs \leftrightarrow$ $\{\text{Hyp}\}$

`(if nil (`$\ldots$ `t` $\ldots$`)` β`)`

$\leftrightarrow$ $\{$`if-ax2`$\}$

β $\square$