Problem Set #4

This problem set is due at the start of class on Tuesday, April 17th.

In class we presented a mechanism (based on the Hungarian algorithm) for computing a buyer-optimal stable outcome for any given instance of the assignment game. In the following questions, let $\mathcal{M}$ denote this mechanism.

1. Let $I = (P, Q, \alpha)$ be an instance of the assignment game. Assume that on instance I, mechanism $\mathcal{M}$ assigns item j to buyer i. Prove that the price $\mathcal{M}$ charges buyer i for item j is equal to the seller-optimal price of item j in assignment game instance $I' = (P - i, Q, \alpha)$.

2. Let $I = (P, Q, \alpha)$ and $I' = (P, Q, \alpha')$ be two instances of the assignment game, let buyer i belong to P, and assume that for any buyer i' not equal to i, and any item j in Q, we have $\alpha_{i',j} = \alpha'_{i',j}$. Let (u, v, x) (resp., (u', v', x')) denote a possible outcome of mechanism $\mathcal{M}$ on instance I (resp., I'). Prove or disprove: If $x_{i,j} = x'_{i,j} = 1$, then $v = v'$.

3. In class we argued that mechanism $\mathcal{M}$ computes the VCG outcome in a setting where there is an agent for each buyer and the mechanism plays the role of the sellers. Now consider applying the VCG mechanism in a setting the mechanism does not represent any of the buyers or sellers; in other words, there is a separate agent to model each buyer and each seller (each item has a distinct seller). In the latter setting, the VCG allocation still corresponds to a maximum-weight matching. Derive formulas (similar to those presented in class for the setting where the mechanism represents all of the sellers) for the VCG payments of the buyer and seller agents. CLARIFICATIONS ADDED 4/6/12: (1) please assume that the sellers do not assign any value to the items that they are selling; (2) as in some of the examples in Section 9.3.5 of the AGT text that we discussed in class, you will need to use the modified Clarke pivot rule discussed in the last paragraph of Section 9.3.4, since we want the mechanism to pay the sellers for any items that they sell (the original Clarke pivot rule is designed to enforce the "no positive transfers" property, which does not allow the mechanism to make positive payments).