Object Allocation Over a Network of Objects: Mobile Agents with Strict Preferences

Extended Abstract

Fu Li
University of Texas at Austin
Austin, TX, USA
fuli.theory.research@gmail.com

C. Gregory Plaxton
University of Texas at Austin
Austin, TX, USA
plaxton@cs.utexas.edu

Vaibhav B. Sinha
University of Texas at Austin
Austin, TX, USA
vaibhavsinha@utexas.edu

ABSTRACT

In recent work, Gourvès, Lesca, and Wilczynski propose a variant of the classic housing markets model where the matching between agents and objects evolves through Pareto-improving swaps between pairs of adjacent agents in a social network. To explore the swap dynamics of their model, they pose several basic questions concerning the set of reachable matchings. In their work and other follow-up works, these questions have been studied for various classes of graphs: stars, paths, generalized stars (i.e., trees where at most one vertex has degree greater than two), trees, and cliques. For generalized stars and trees, it remains open whether a Pareto-efficient reachable matching can be found in polynomial time.

In this paper, we pursue the same set of questions under a natural variant of their model. In our model, the social network is replaced by a network of objects, and a swap is allowed to take place between two agents if it is Pareto-improving and the associated objects are adjacent in the network. In those cases where the question of polynomial-time solvability versus NP-hardness has been resolved for the social network model, we are able to show that the same result holds for the network-of-objects model. In addition, for our model, we present a polynomial-time algorithm for computing a Pareto-efficient reachable matching in generalized star networks. Moreover, the object reachability algorithm that we present for path networks is significantly faster than the known polynomial-time algorithms for the same question in the social network model.

KEYWORDS

Housing markets; Distributed process; Algorithms; Complexity

ACM Reference Format:

Fu Li, C. Gregory Plaxton, and Vaibhav B. Sinha. 2021. Object Allocation Over a Network of Objects: Mobile Agents with Strict Preferences: Extended Abstract. In *Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021), Online, May 3–7, 2021*, IFAAMAS, 3 pages.

1 OVERVIEW

Problems related to resource allocation under preferences are widely studied in both computer science and economics. Research in this area seeks to gain mathematical insight into the structure of resource allocation problems, and to exploit this structure to design fast algorithms. In one important class of resource allocation problems, sometimes referred to as one-sided matching problems [8], we seek to allocate indivisible objects to a set of agents, where each agent has preferences over the objects and wants to receive at most one object (unit demand). The allocation should enjoy one or more strong game-theoretic properties, such as Pareto-efficiency.

Of particular relevance to the present paper is the line of research initiated by Gourvès et al. [4] on decentralized allocation in housing markets. They propose a model in which agents have strict preferences and are embedded in an underlying social network. A pair of agents are allowed to swap objects with each other only if (1) they will be better off after the swap, and (2) they are directly connected (socially tied) via the network. The underlying social network is modeled as an undirected graph, and five different graph classes are considered: paths, stars, generalized stars, trees, and general graphs. The swap dynamics of the model are investigated by considering three computational questions. The first question, Reachable Object, asks whether there is a sequence of swaps that results in a given agent being matched to a given target object. The second question, Reachable Matching, asks whether there is a sequence of swaps that results in a given target matching. The third question, Pareto Efficiency, asks how to find a sequence of swaps that results in a Pareto-efficient matching with respect to the set of reachable matchings.

Gourvès et al. [4] studied each of the three questions in the context of the aforementioned graph classes, with the goal of either exhibiting a polynomial-time algorithm or establishing NP-hardness. The work of Gourvès et al. [4] left three of these problems open: Reachable Object on paths and Pareto Efficiency on generalized stars and trees. Subsequently, two sets of authors independently presented polynomial-time algorithms for Reachable Object on paths [1, 5]. Both groups obtained an $O(n^4)$-time algorithm by carefully studying the structure of swap dynamics on paths and then reducing the problem to 2-SAT. The complexity of Pareto Efficiency remains open for generalized stars and for trees.

Bentert et al. [1] established that Reachable Object on cliques is NP-complete. Müller and Bentert [9] further established that Reachable Matching on cliques is NP-complete. It is easy to extend the latter result to show that Pareto Efficiency on cliques is NP-hard. These three hardness results for cliques subsume the corresponding results obtained previously for general graphs by Gourvès et al.

We study a natural variant of the decentralized housing markets model of Gourvès et al. [4]. Instead of enforcing locality constraints on trade via a network where the locations of the agents are fixed (since they correspond to the vertices of the network) and the objects move around (due to swaps), we consider a network where the locations of the objects are fixed and the agents move around. We refer to these two models as the object-moving model and the

Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021), U. Endriss, A. Nowé, F. Dignum, A. Lomuscio (eds.), May 3–7, 2021, Online.

Table 1: This table presents known complexity results for various questions related to the object-moving model of Gourvès et al. [4]. The results in parentheses follow directly from other table entries. For the agent-moving model, we obtain the same results, except that we also give a polynomial-time algorithm for Pareto Efficiency on generalized stars.

	Reachable Object	Reachable Matching	Pareto Efficiency
Star	poly-time	(poly-time)	poly-time
Path	poly-time	(poly-time)	poly-time
Generalized Star	NP-complete	(poly-time)	open
Tree	(NP-complete)	poly-time	open
Clique	NP-complete	NP-complete	NP-hard

agent-moving model. Table 1 summarizes the current state of the art for the object-moving model.

Related work. For the object-moving model, Huang and Xiao [5] study Reachable Object with weak preferences, i.e., where an agent can be indifferent between different objects. Bentert et al. [1] establish NP-hardness for Reachable Object on cliques, and consider the case where the preference lists have bounded length. Saffidine and Wilczynski [10] propose a variant of Reachable Object where we ask whether a given agent is guaranteed to achieve a specified level of satisfaction after any maximal sequence of rational exchanges. Müller and Bentert [9] study Reachable Matching on cliques and cycles. Aspects related to social connectivity are also addressed in recent work on envy-free allocations [2, 3] and on trade-offs between efficiency and fairness [6].

2 OUR RESULTS

We initiate the study of the agent-moving model by revisiting each of the questions associated with Table 1 in the context of the agent-moving model. We emphasize that the sole difference between the agent-moving model and the object-moving model is that the locality constraint prevents an agent a currently matched to an object b from trading with an agent a' currently matched to an object b' unless objects b and b' (two vertices in a given network of objects) are adjacent, rather than requiring agents a and a' (two vertices in a given network of agents) to be adjacent. Both models also require swaps to be Pareto-improving. The two models have strong similarities. In fact, for all of the questions in Table 1 for which a polynomial-time algorithm or hardness result has been established in the object-moving model, we establish a corresponding result in the agent-moving model. Moreover, for Pareto Efficiency on generalized stars, which is open in the object-moving model, we provide a polynomial-time algorithm in the agent-moving model. In some cases, it is relatively straightforward to adapt known results for the object-moving model to the agent-moving model. Below we highlight our four main technical contributions, which address more challenging cases.

Our first main technical result is an $O(n^2)$ time algorithm for Reachable Object on paths in the agent-moving model, which is much faster than the known $O(n^4)$-time algorithms for Reachable Object on paths in the object-moving model. (Here n denotes the number of agents/objects; the size of the input is quadratic in n since the preference list of each agent is of length n.) The speedup is due to a simpler local characterization of the reachable matchings on a path in the agent-moving model.

In our second main technical result, we obtain the same $O(n^2)$ time bound for Pareto Efficiency on paths. Our algorithms for Reachable Object and Pareto Efficiency are based on an efficient subroutine for solving a certain constrained reachability problem. Roughly speaking, this subroutine determines all of the possible matches for a given agent when certain agent-object pairs are required to be matched to one another. Our implementation involves a trivial $O(n^2)$-time preprocessing phase followed by an $O(n)$-time greedy phase. The preferences of the agents are only examined during the preprocessing phase. The proof of correctness of the greedy phase is somewhat nontrivial. We solve Reachable Object on paths using a single application of the subroutine, yielding an $O(n^2)$ bound. Our polynomial-time algorithm for Pareto Efficiency on paths uses n applications of our algorithm for Reachable Object on paths. Since the preprocessing phase only needs to be performed once, the overall running time remains $O(n^2)$.

In our third main technical result, we present a polynomial-time algorithm for Pareto Efficiency on generalized stars, which remains open in the object-moving model. To tackle this problem, we use the serial dictatorship algorithm with the novel idea of dynamically choosing the dictator sequence. We also leverage our techniques for solving Pareto Efficiency on paths.

The faster time bounds discussed above for the case of paths suggest that the agent-moving model is simpler than the object-moving model, at least from an upper bound perspective. Accordingly, we can expect it to be a bit more challenging to establish the NP-completeness results stated in Table 1 for the agent-moving model than for the object-moving model. In our fourth main technical result, we adapt an NP-completeness proof developed by Bentert et al. [1] in the context of the object-moving model to the more challenging setting of the agent-moving model. Specifically, we modify their reduction from 2P1N-SAT to establish that Reachable Object on cliques remains NP-complete in the agent-moving model.

Proofs of all of our results are provided in our full paper [7].

3 FUTURE WORK

In this paper, we have presented a polynomial-time algorithm for Pareto Efficiency on generalized stars in the agent-moving model, a problem that remains open in the object-moving model. It is natural to ask whether our techniques can be extended to address this open problem. Our algorithm relies on the polynomial-time solvability of the Reachable Object problem for the center agent, which allows us to compute an object that is matched to the center agent in some Pareto-efficient matching. In the object-moving model, no polynomial-time algorithm is known to compute an agent that is matched to the center object in some Pareto-efficient matching. (We do know how to compute the agents that can be reached by the center object in polynomial time, but it isn't clear how to use this information to compute a Pareto-efficient matching in polynomial time.) An interesting direction for future research in the agent-moving model is to determine whether our techniques for solving Pareto Efficiency on generalized stars can be extended to trees. It would also be interesting to study strategic aspects of the agent-moving model.

REFERENCES

[1] Matthias Bentert, Jiehua Chen, Vincent Froese, and Gerhard J. Woeginger. 2019. Good things come to those who swap objects on paths. *arXiv:1905.04219* (May 2019).

[2] Aurélie Beynier, Yann Chevaleyre, Laurent Gourvès, Ararat Harutyunyan, Julien Lesca, Nicolas Maudet, and Anaëlle Wilczynski. 2019. Local envy-freeness in house allocation problems. *Autonomous Agents and Multi-Agent Systems* 33, 5 (2019), 591–627.

[3] Vittorio Bilò, Ioannis Caragiannis, Michele Flammini, Ayumi Igarashi, Gianpiero Monaco, Dominik Peters, Cosimo Vinci, and William S. Zwicker. 2019. Almost envy-free allocations with connected bundles. In *Proceedings of the 10th Innovations in Theoretical Computer Science Conference*. 14:1–14:21.

[4] Laurent Gourvès, Julien Lesca, and Anaëlle Wilczynski. 2017. Object allocation via swaps along a social network. In *Proceedings of the 26th International Joint Conference on Artificial Intelligence*. 213–219.

[5] Sen Huang and Mingyu Xiao. 2019. Object reachability via swaps along a line. In *Proceedings of the 33rd AAAI Conference on Artificial Intelligence*. 2037–2044.

[6] Ayumi Igarashi and Dominik Peters. 2019. Pareto-optimal allocation of indivisible goods with connectivity constraints. In *Proceedings of the 33rd AAAI Conference on Artificial Intelligence*. 2045–2052.

[7] Fu Li, C. Gregory Plaxton, and Vaibhav B. Sinha. 2021. *Object Allocation Over a Network of Objects: Mobile Agents with Strict Preferences*. Technical Report TR–21–01. Department of Computer Science, University of Texas at Austin.

[8] David F. Manlove. 2013. *Algorithmics of Matching Under Preferences*. World Scientific.

[9] Luis Müller and Matthias Bentert. 2020. On Reachable Assignments in Cycles and Cliques. *arXiv:2005.02218* (May 2020).

[10] Abdallah Saffidine and Anaëlle Wilczynski. 2018. Constrained swap dynamics over a social network in distributed resource reallocation. In *Proceedings of the 11th International Symposium on Algorithmic Game Theory*. 213–225.