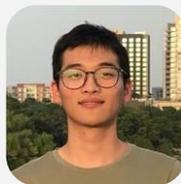


Causal Dynamics Learning for Task-Independent State Abstraction

Zizhao Wang, Xuesu Xiao, Zifan Xu, Yuke Zhu, and Peter Stone



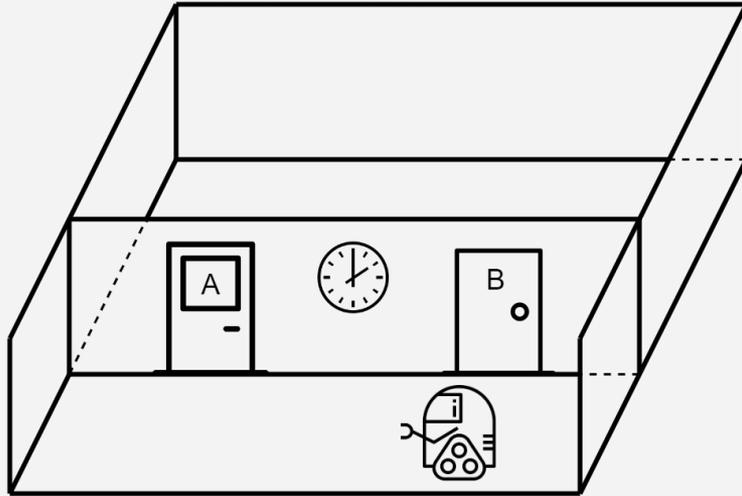
Sony AI

Motivation

Real-world dynamics are usually *sparse*.

- The transition of each state variable only depends on a few state variables.

For example, for an environment with a robot, two doors and a clock on the wall:

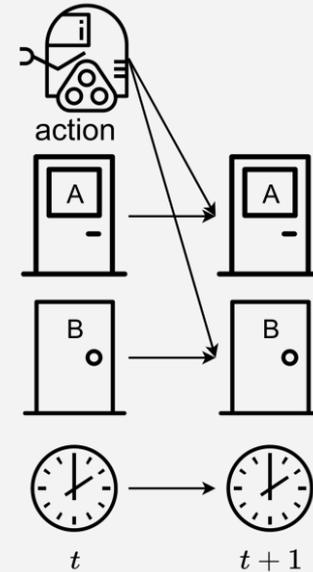
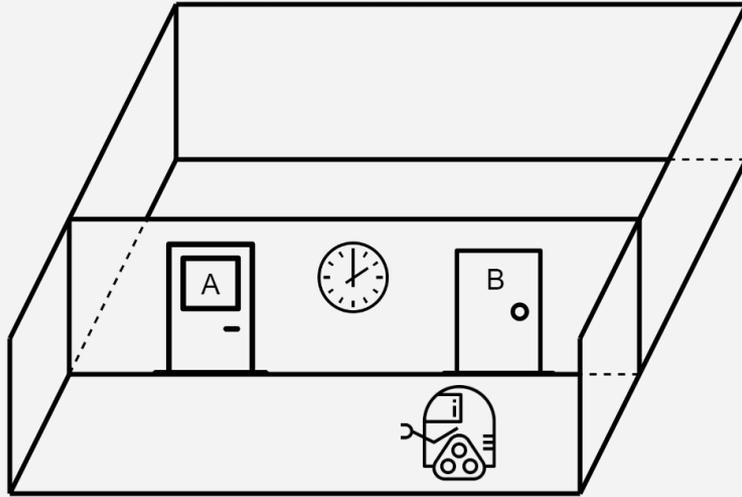


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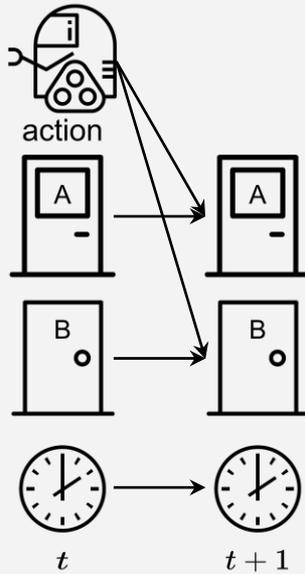
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sparse real-world dynamics

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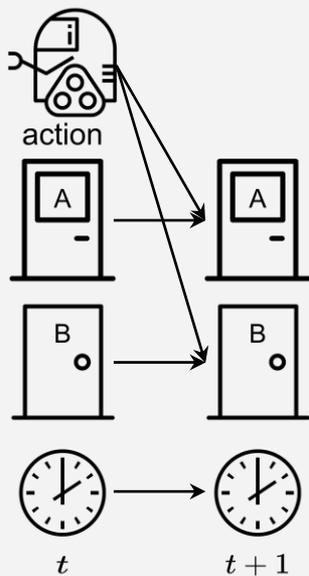
But most model-based RL work uses dense dynamics models (fully-connected networks).



sparse real-world dynamics

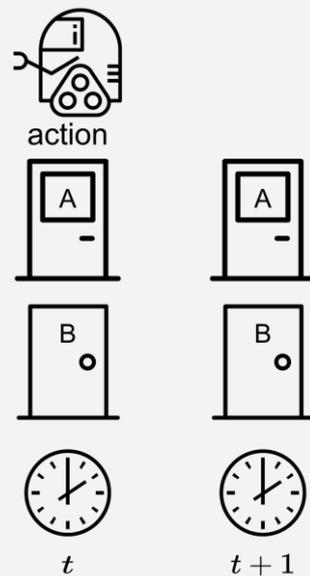
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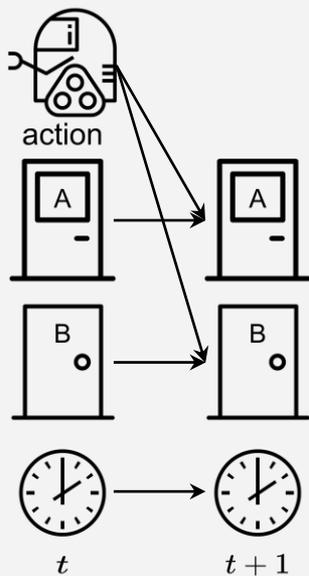
VS



dense dynamics model

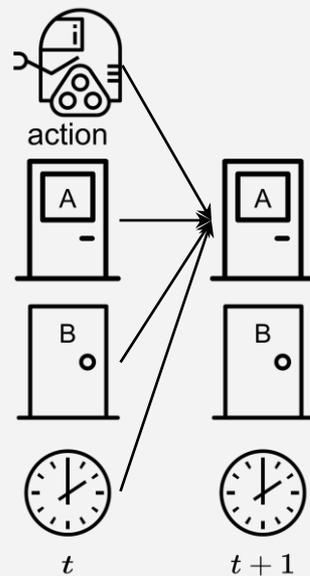
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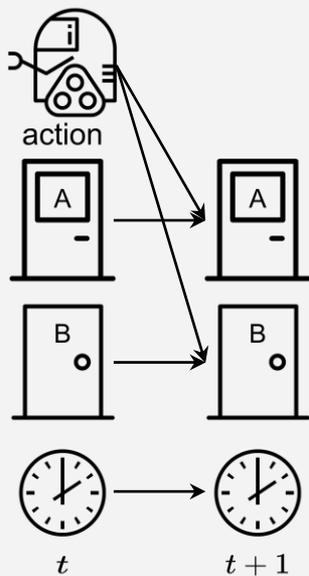
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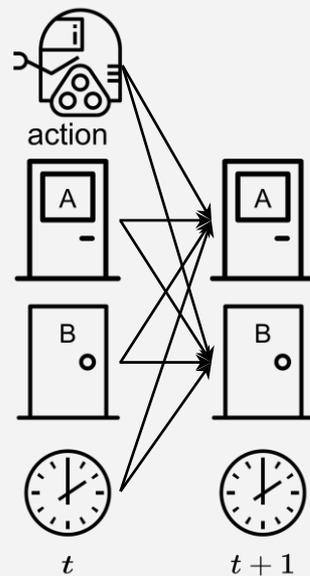
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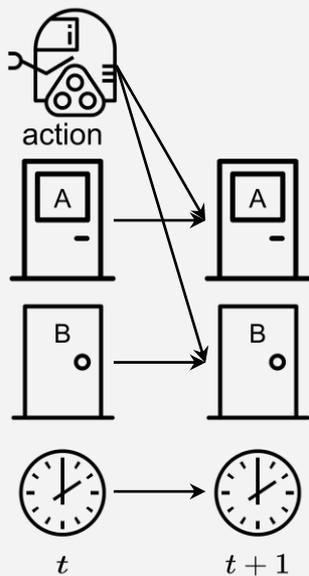
VS



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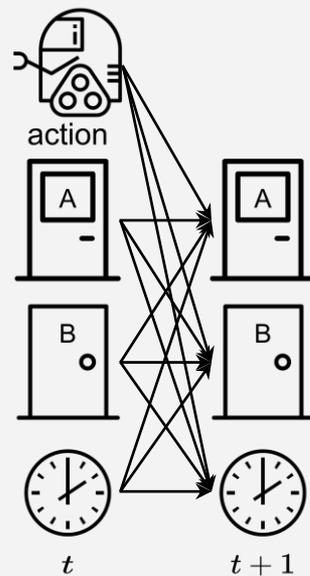
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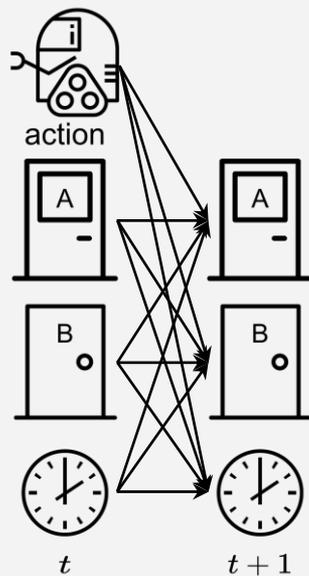
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dense dynamics model

Motivation

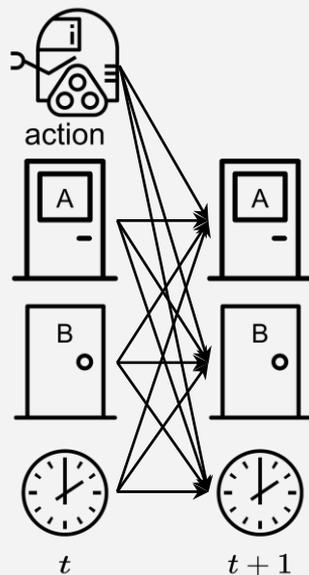
dense dynamics model



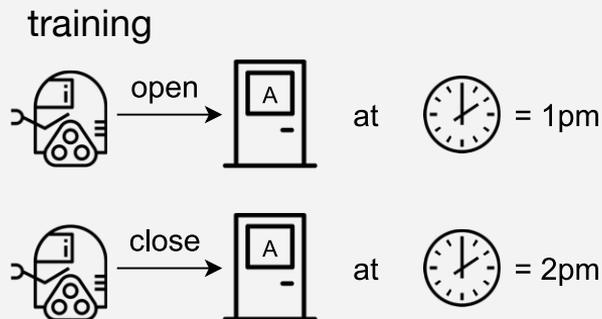
generalizes badly
due to spurious correlation

Motivation

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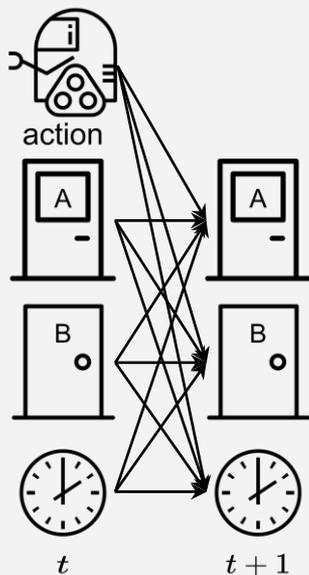
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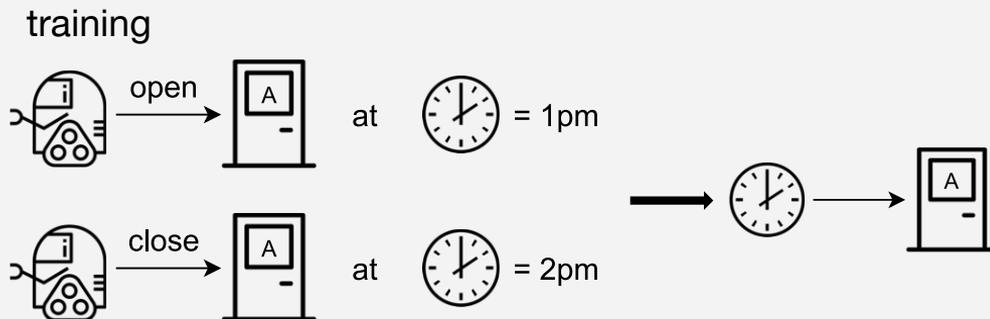
overfit to data noise, etc

Motivation

dense dynamics model



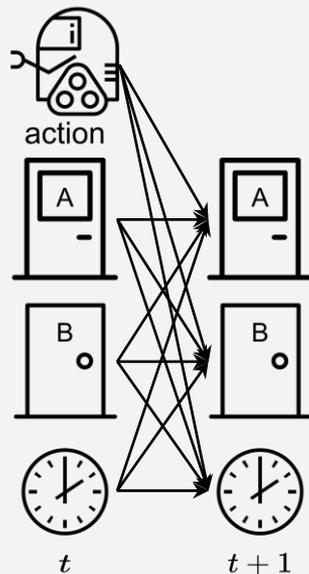
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overfit to data noise, etc

Motivation

dense dynamics model



generalizes badly
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training



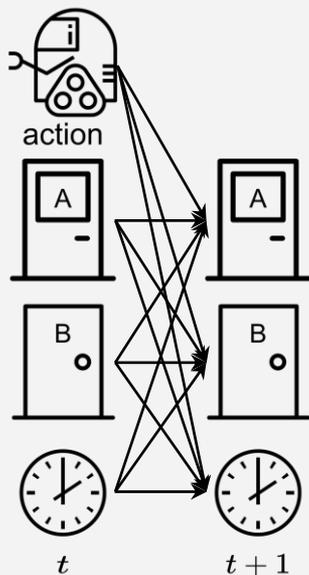
testing



overfit to data noise, etc

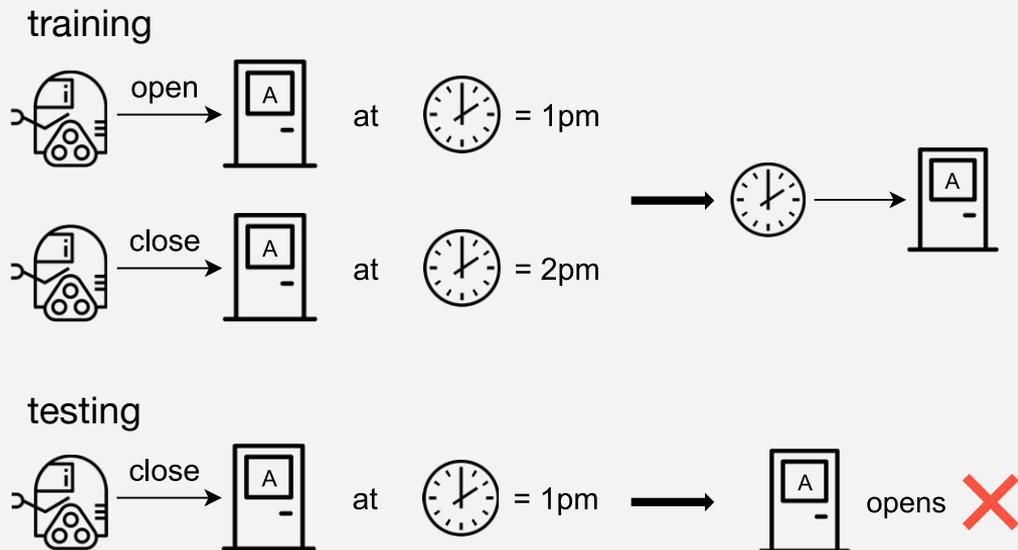


Motivation



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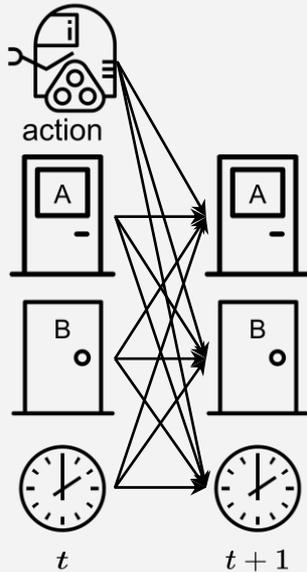
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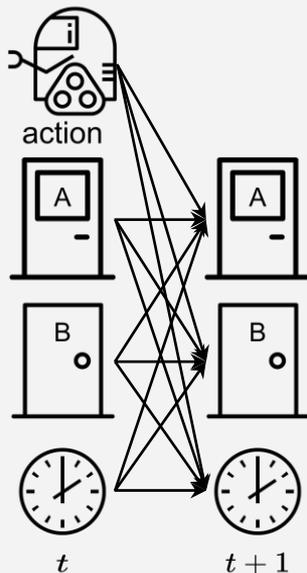
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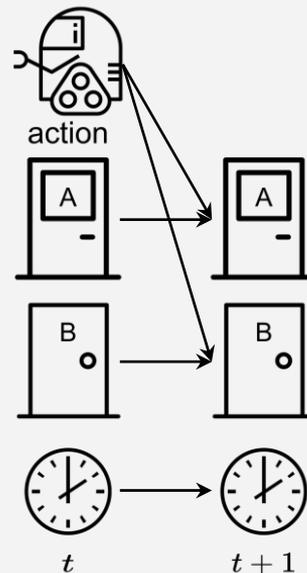
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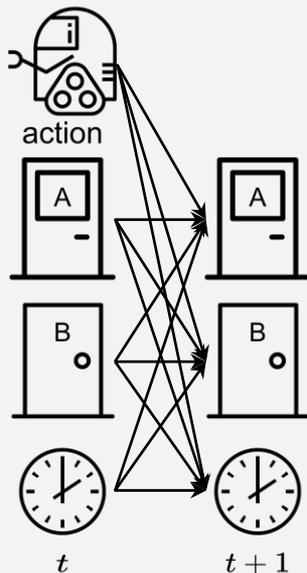
causal dynamics learning (CDL)



only keep causal edges, robust to outliers,

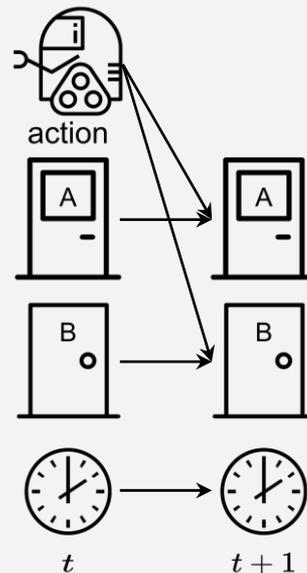
Motivation

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causal dynamics learning (CDL)



only keep causal edges, robust to outliers,
e.g., clock outliers won't affect door A & B prediction

Problem Setup

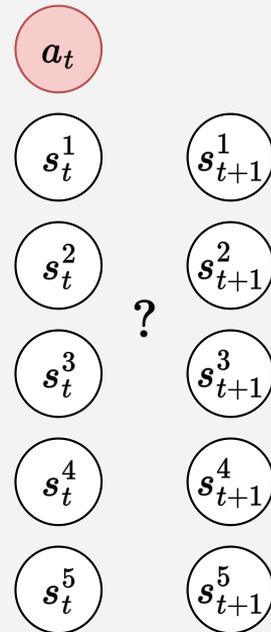
$$\langle \mathcal{S}, \mathcal{A}, \mathcal{P} \rangle$$

S: state space (known, *high-level* variables are given)

We leave handling low-level, partially-observable state space (e.g., images) as future work.

A: action space (known)

P: transition probability (not known)



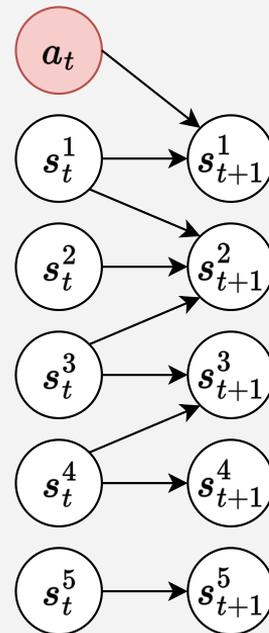
Problem Setup

Goals

1. Learn a causal dynamics model from transition data

$$\mathcal{P}(s_{t+1}|s_t, a_t) = \prod_{i=1}^{d_S} \mathcal{P}(s_{t+1}^i | \mathbf{PA}_{s^i})$$

\mathbf{PA}_{s^i} are parents of s^i during the data generation process.



Problem Setup

Goals

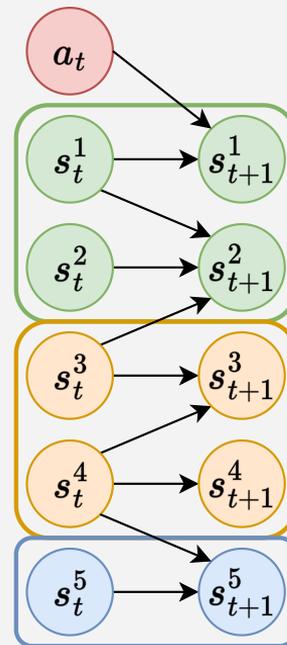
1. Learn a causal dynamics model from transition data
2. Split state variables into three categories

$$\mathcal{S} = \mathcal{S}^c \times \mathcal{S}^r \times \mathcal{S}^i$$

\mathcal{S}^c : space of **controllable** state variables

\mathcal{S}^r : space of **action-relevant** state variables

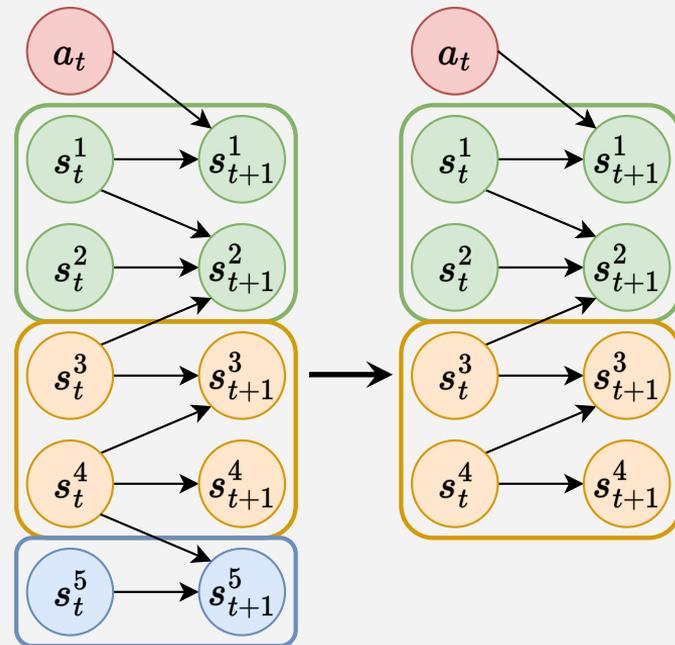
\mathcal{S}^i : space of **action-irrelevant** state variables



Problem Setup

Goals

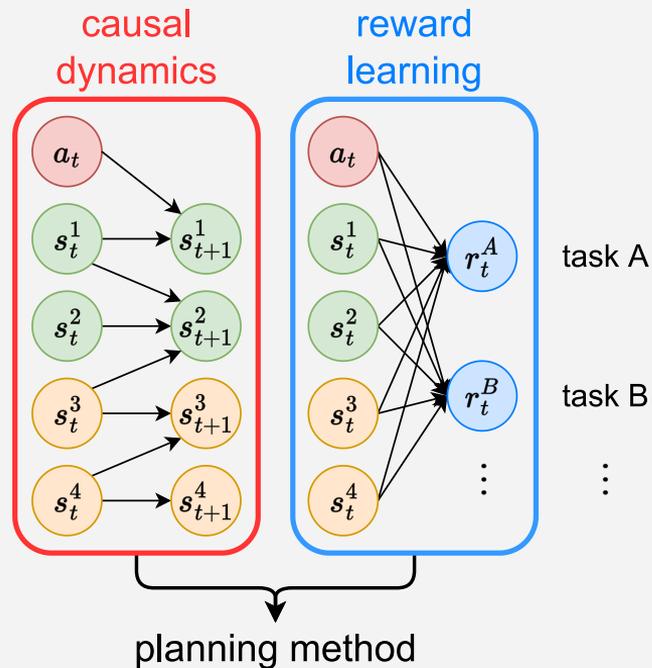
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3. Derive a state abstraction by omitting **action-irrelevant** state variables



Problem Setup

Goals

1. Learn a causal dynamics model from transition data
2. Split state variables into three categories
3. Derive a state abstraction by omitting **action-irrelevant** state variables
4. Use the abstracted causal dynamics to learn (many) downstream tasks



Related Work

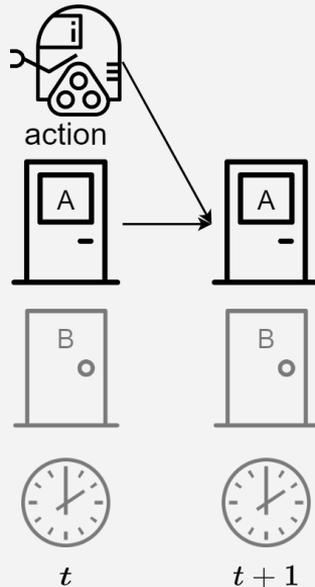
Bisimulation^[1] ϕ : bisimulation considers two states the same $\phi(x) = \phi(x')$ if

$$\begin{aligned} R(x, a) &= R(x', a), \\ \sum_{x'' \in \phi^{-1}(s)} P(x''|x, a) &= \sum_{x'' \in \phi^{-1}(s)} P(x''|x', a) \end{aligned}$$

Related Work

Compared to CDL,

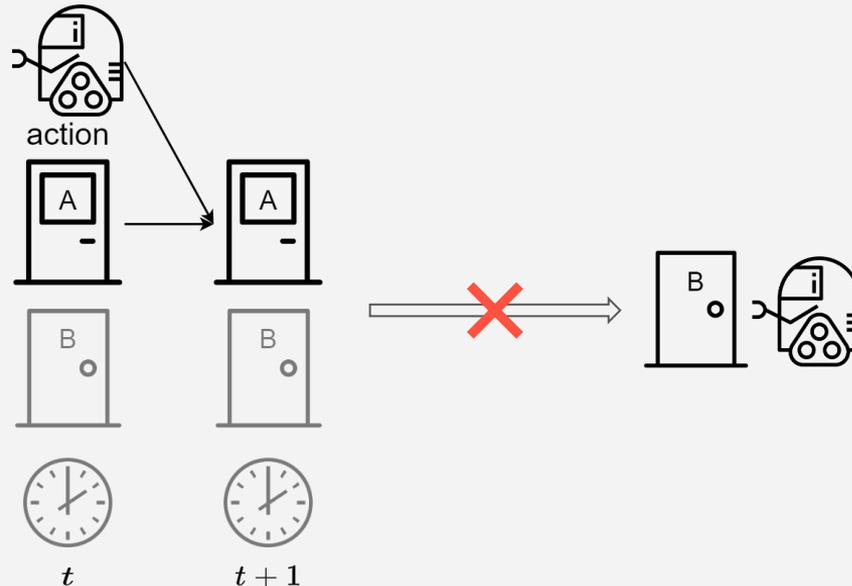
- Bisimulation is reward-specific (applicable to limited tasks).
e.g., the bisimulation abstraction learned from “opening door A” can’t be used for “opening door B.



Related Work

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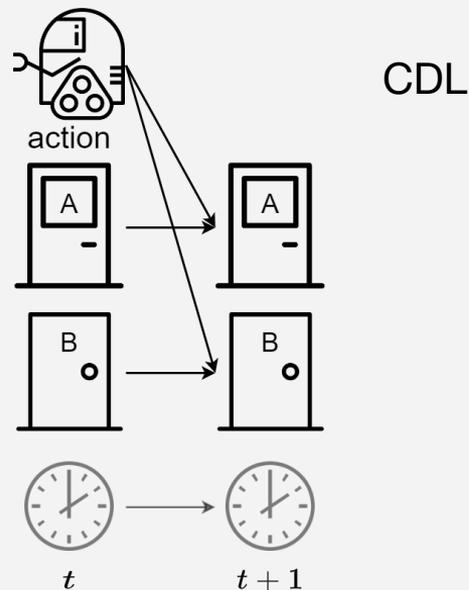
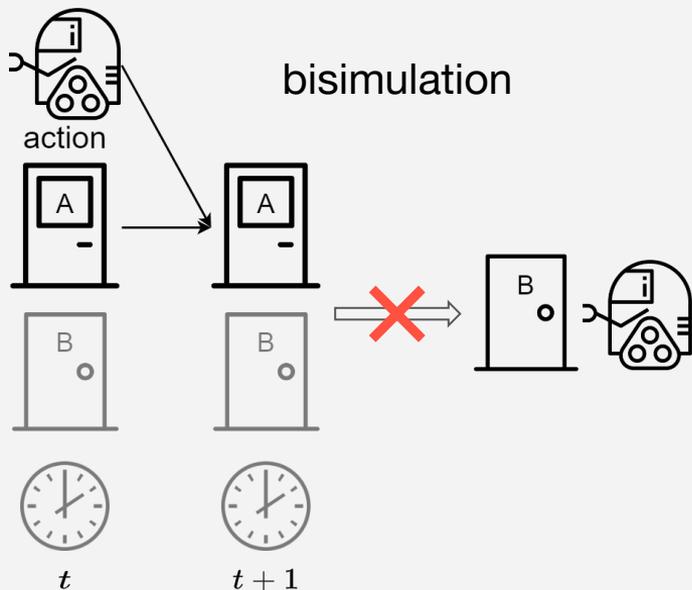
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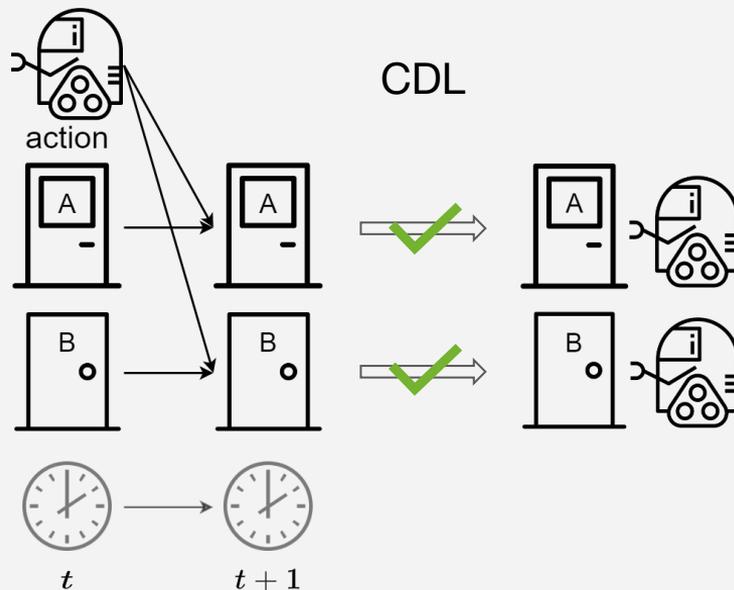
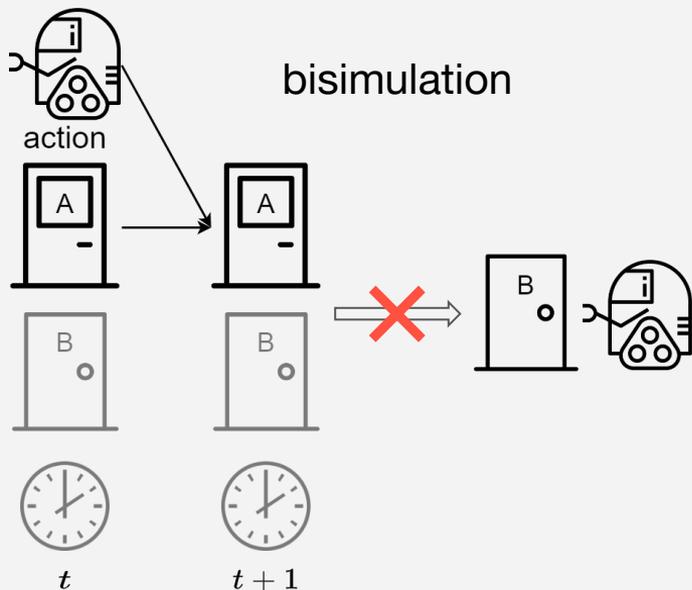
- Bisimulation is reward-specific and thus applicable to **limited** tasks.
In contrast, CDL's abstraction can be applied to a larger range of tasks.



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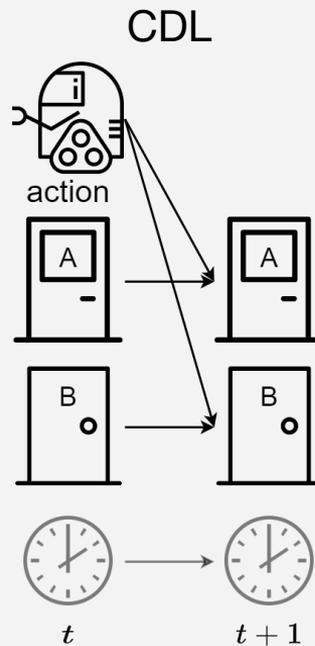
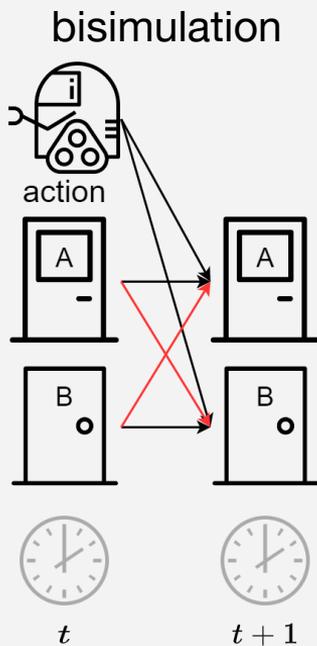
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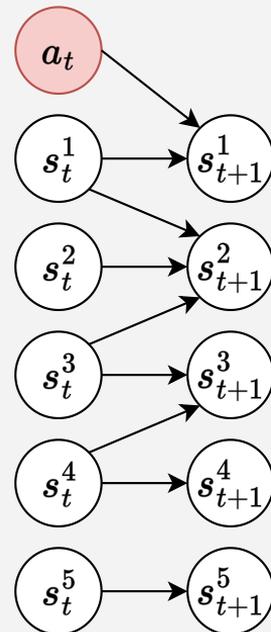
- Bisimulation is reward-specific and thus applicable to **limited** tasks.
- Most bisimulation work still uses dense dynamics, leading to poor generalization.



Method

So far, the key of CDL is to learn a causal dynamics model.

$$\mathcal{P}(s_{t+1}|s_t, a_t) = \prod_{i=1}^{d_s} \mathcal{P}(s_{t+1}^i | \mathbf{PA}_{s^i})$$

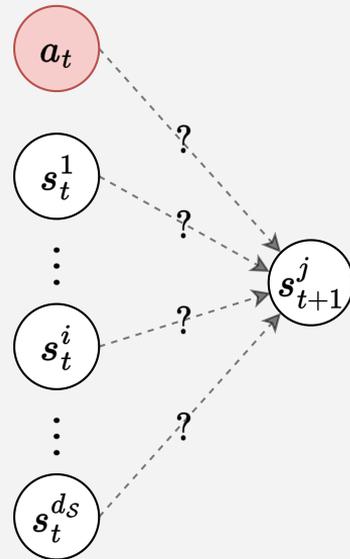


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$$\mathcal{P}(s_{t+1}|s_t, a_t) = \prod_{i=1}^{d_s} \mathcal{P}(s_{t+1}^i | \mathbf{PA}_{s^i})$$

Specifically, for each state variable s^j , how to determine if a state variable s^i is one of its parents?



Method

Key idea: determine if the causal edge $s_t^i \rightarrow s_{t+1}^j$ exists with a conditional independence test.

Skipping assumptions and proofs,

Method

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Skipping assumptions and proofs,

Theorem 1

If $s_t^i \not\perp\!\!\!\perp s_{t+1}^j \mid \{s_t / s_t^i, a_t\}$, then $s_t^i \rightarrow s_{t+1}^j$.

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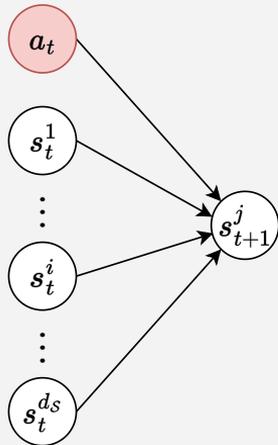
In other words, is s_t^i needed to predict s_{t+1}^j ?

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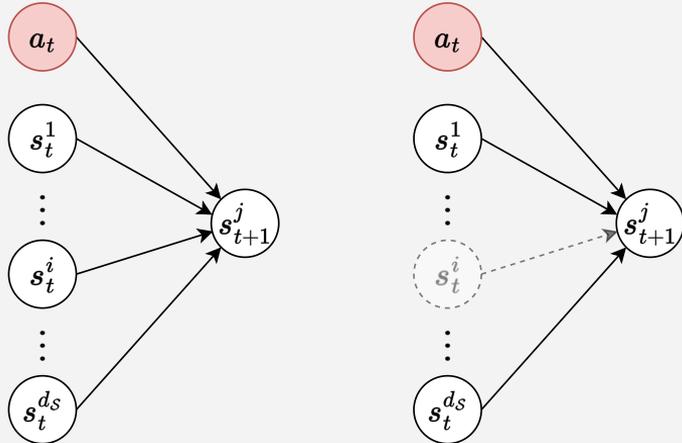


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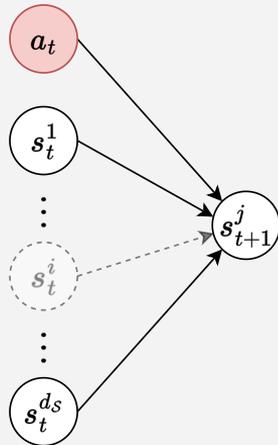
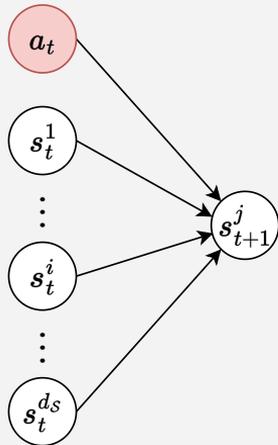


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In other words, is s_t^i needed to predict s_{t+1}^j ?



$$p(s_{t+1}^j \mid s_t, a_t) \stackrel{?}{=} p(s_{t+1}^j \mid \{s_t / s_t^i, a_t\})$$

Method

Learn and predict $p(s_{t+1}^j | s_t, a_t)$ & $p(s_{t+1}^j | \{s/s^i\}_t, a_t)$ using generative models, but there will be d_S^2 models to train...

Method

Learning $p(s_{t+1}^j | s_t, a_t)$ & $p(s_{t+1}^j | \{s/s^i\}_t, a_t)$ needs to train d_S^2 models.

With a mask M_j and an element-wise maximum module, one network can represent all generative models in the form of $p(s_{t+1}^j | \cdot)$.

Method

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With M_j and an element-wise maximum module, one network can represent all models.

For example, to represent $p(s_{t+1}^j | s_t, a_t)$,

inputs

a_t

1

s_t^1

2

⋮

s_t^i

0

⋮

$s_t^{d_S}$

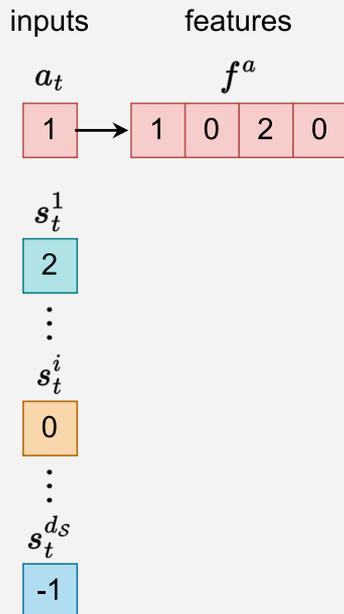
-1

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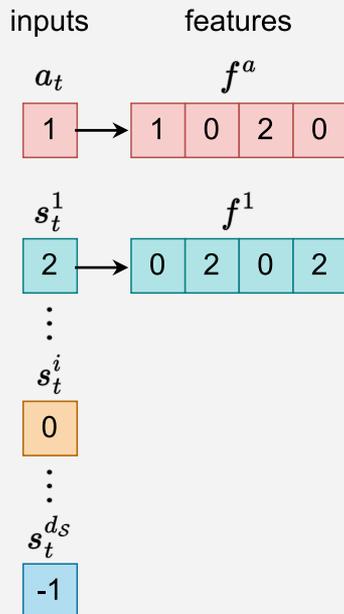


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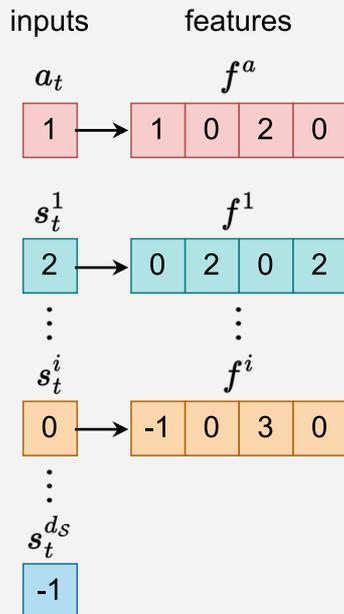


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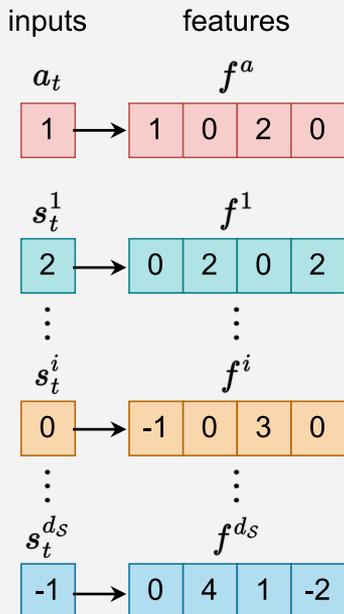


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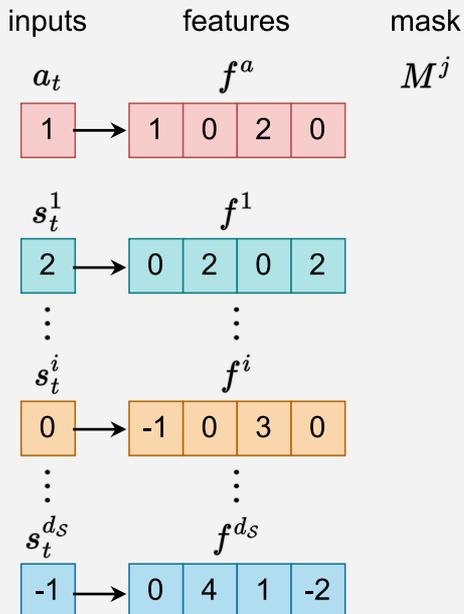


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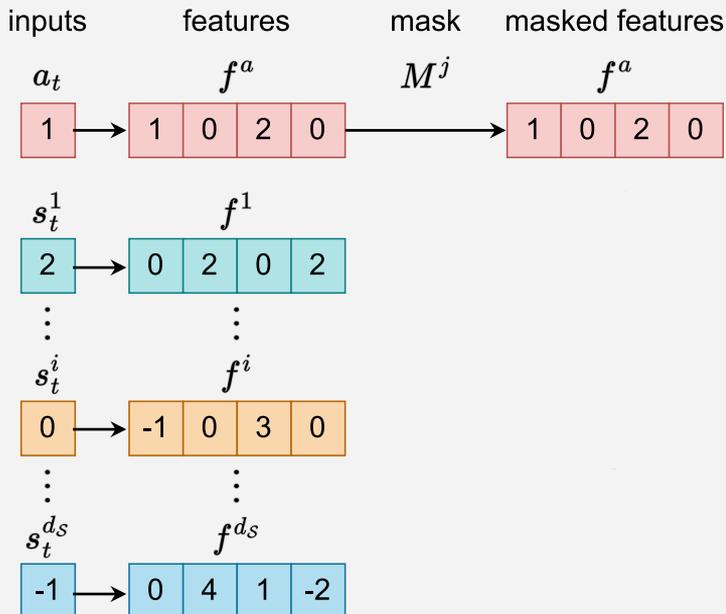


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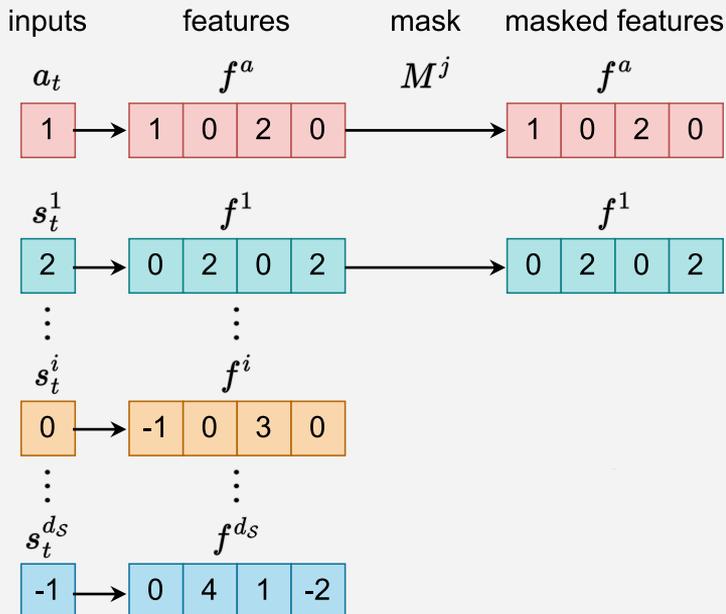


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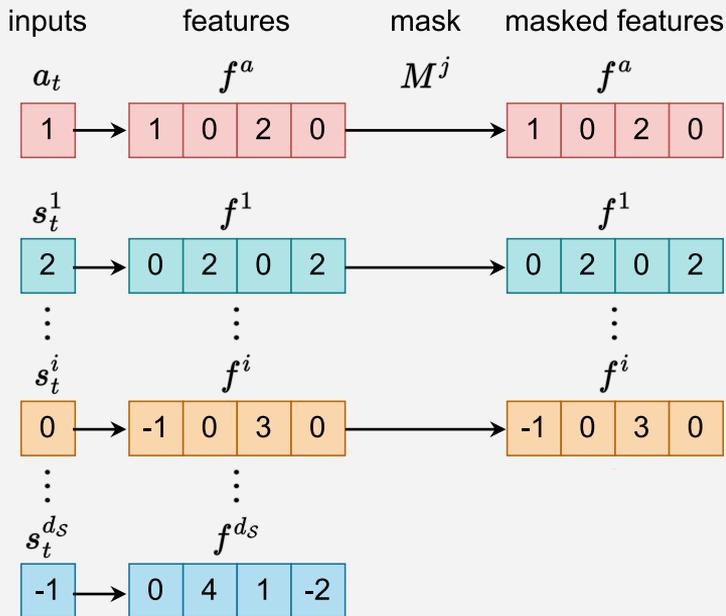


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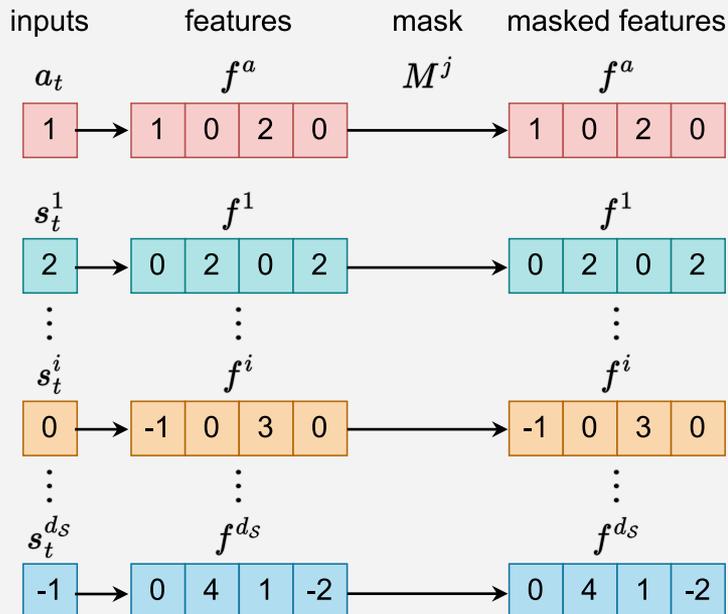


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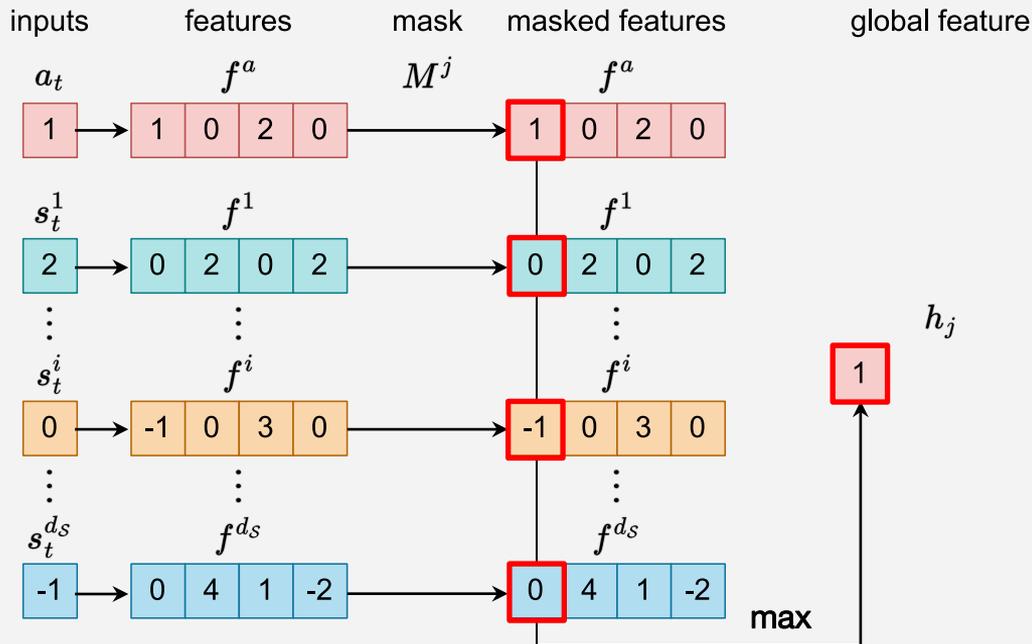


Method

Learning $p(s_{t+1}^j | s_t, a_t)$ & $p(s_{t+1}^j | \{s/s^i\}_t, a_t)$ needs to train d_S^2 models.

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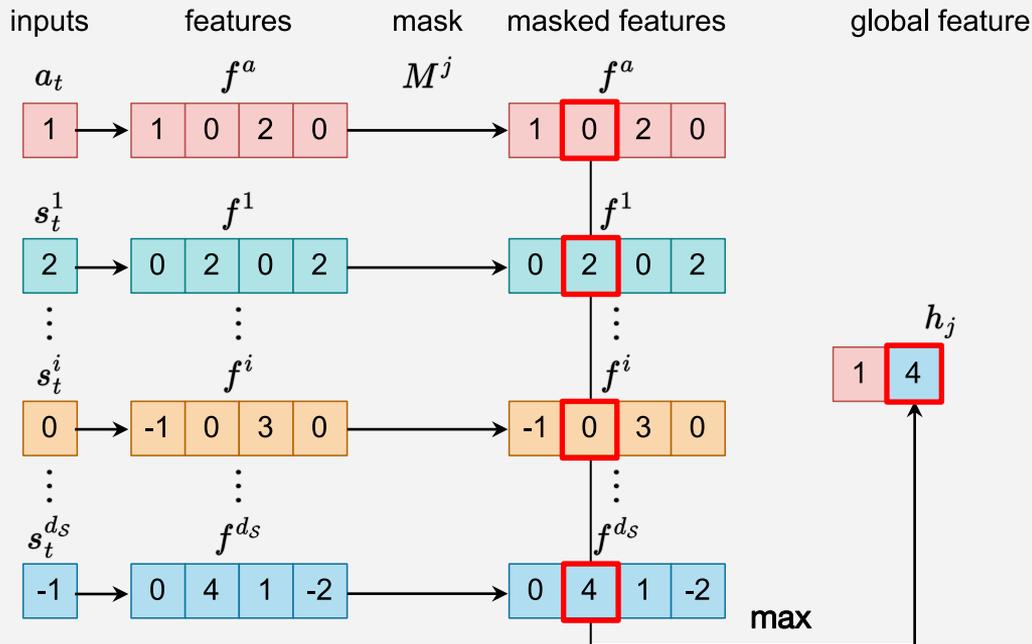


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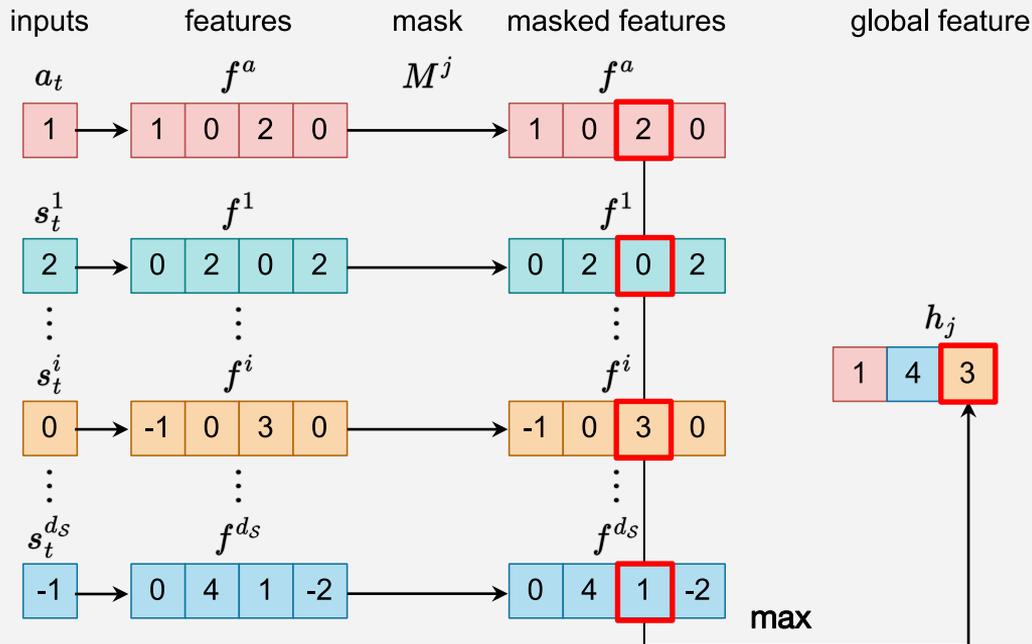


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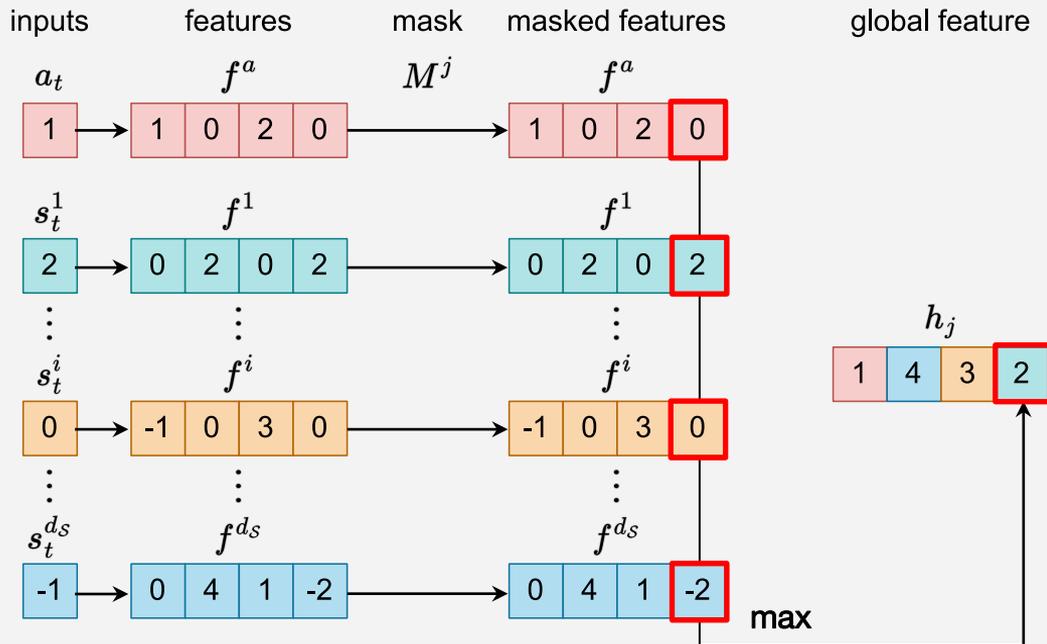


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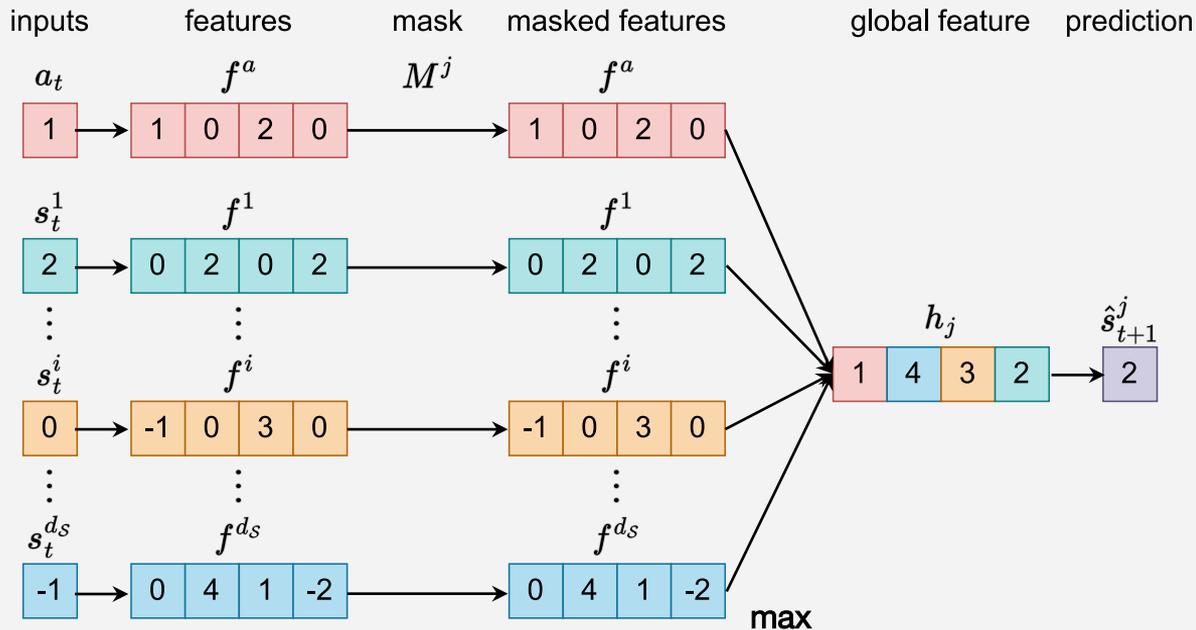


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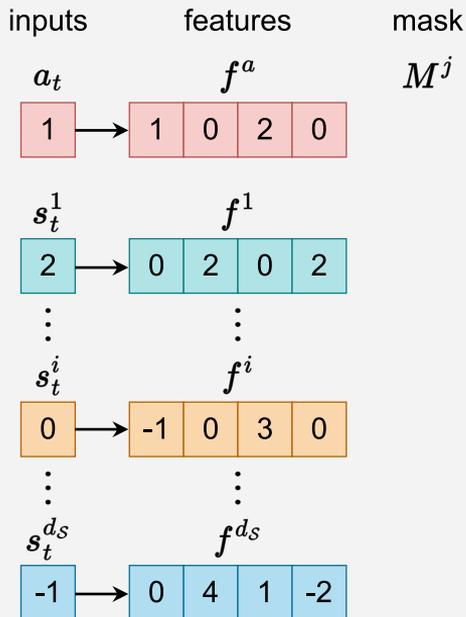


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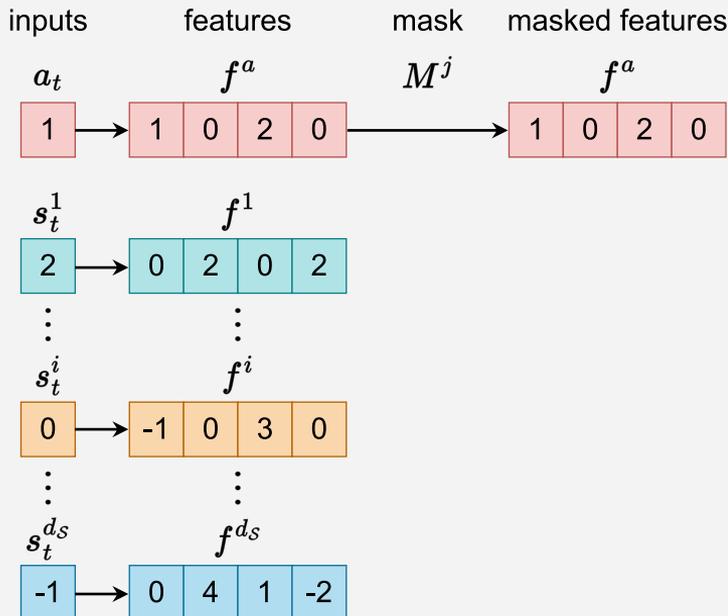


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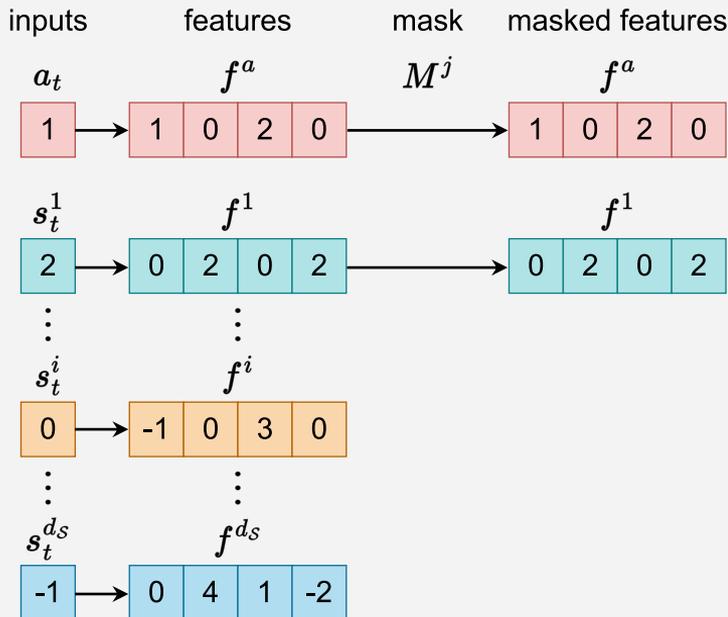


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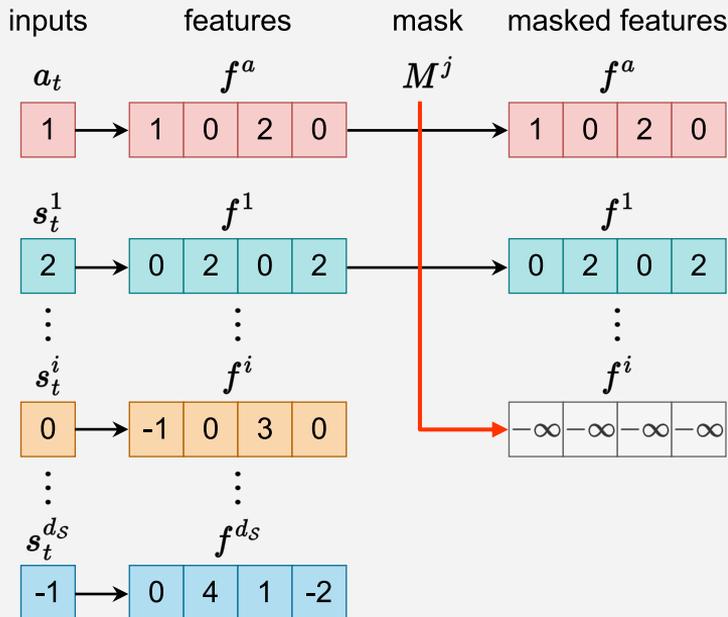


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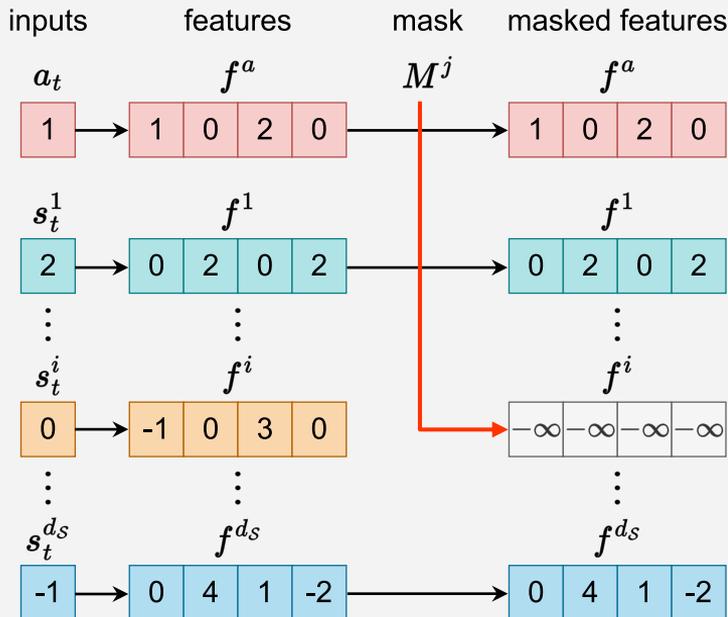


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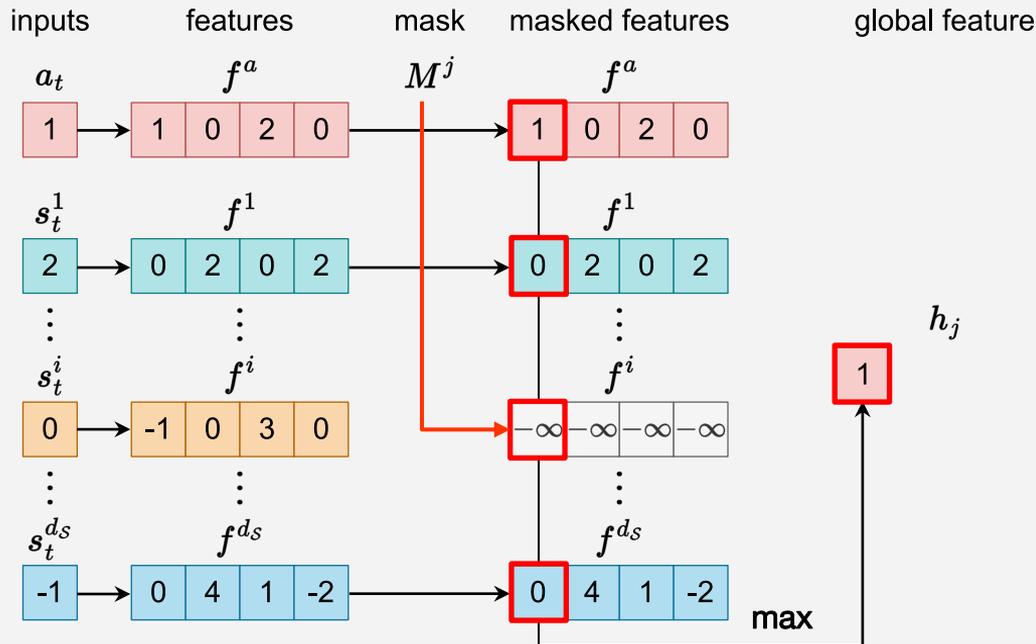


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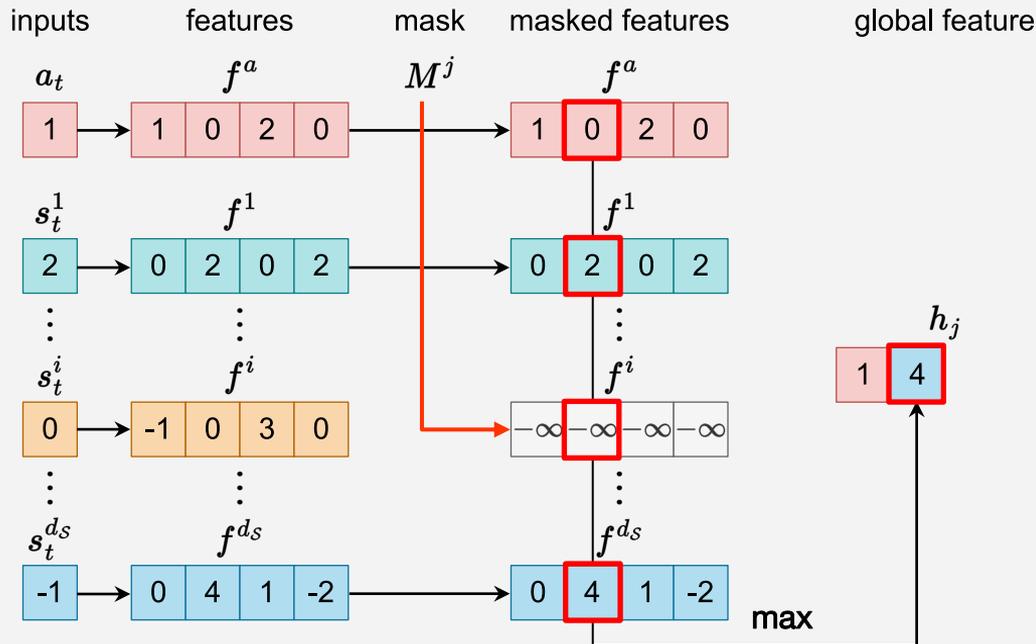


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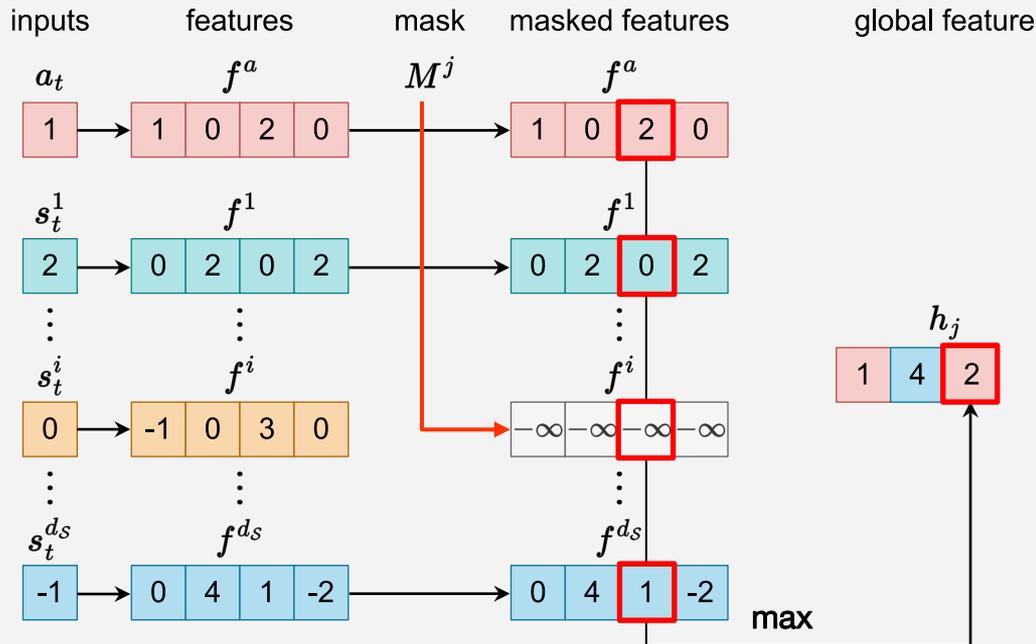


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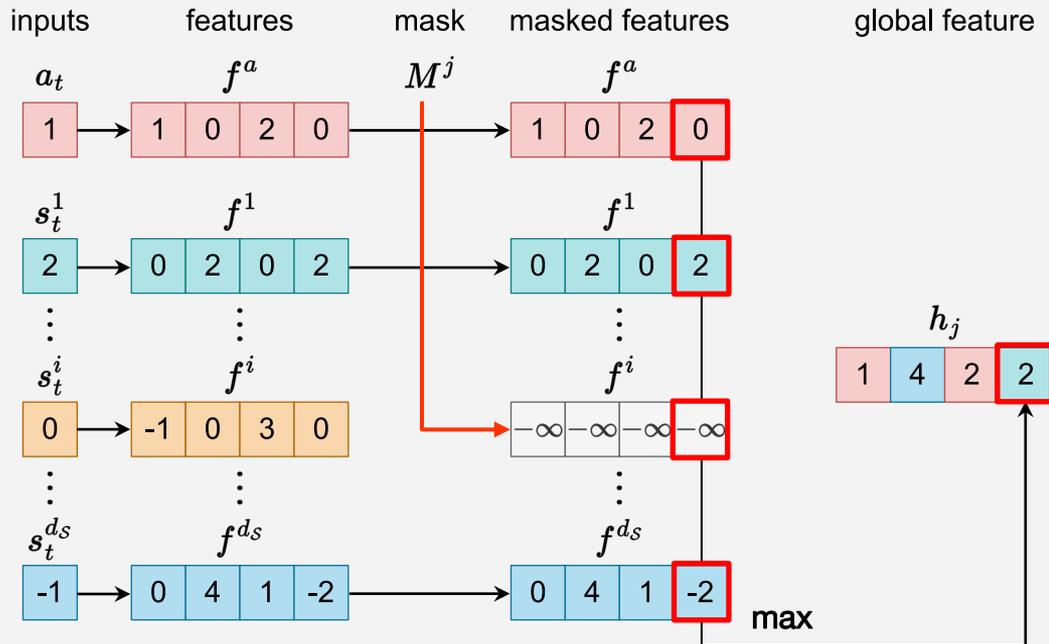


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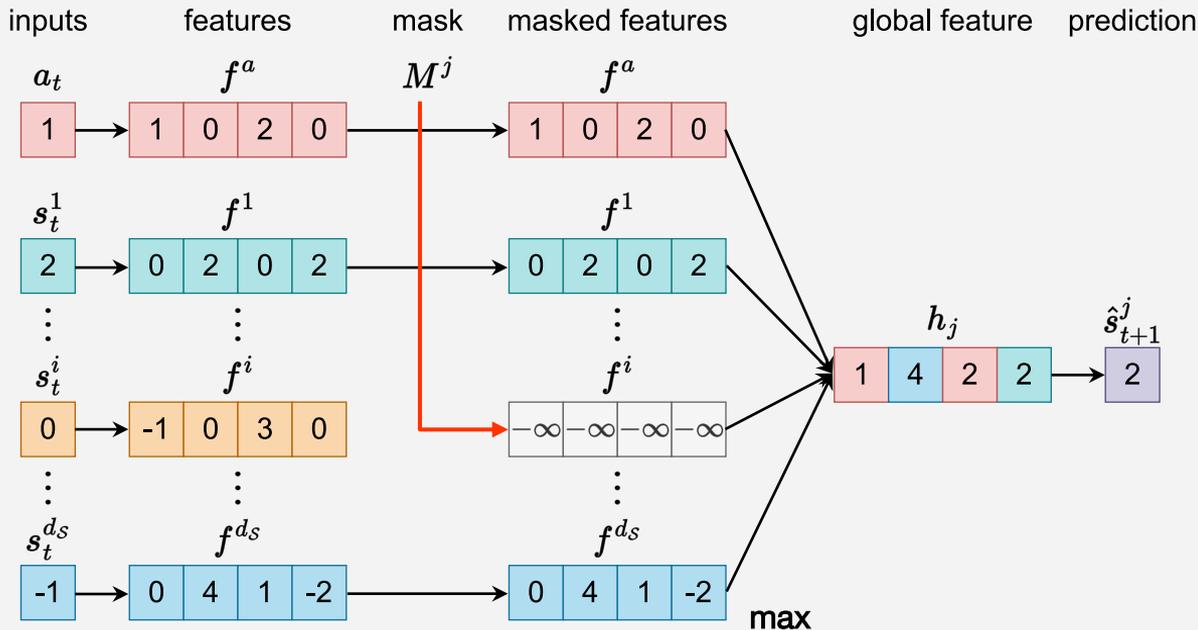


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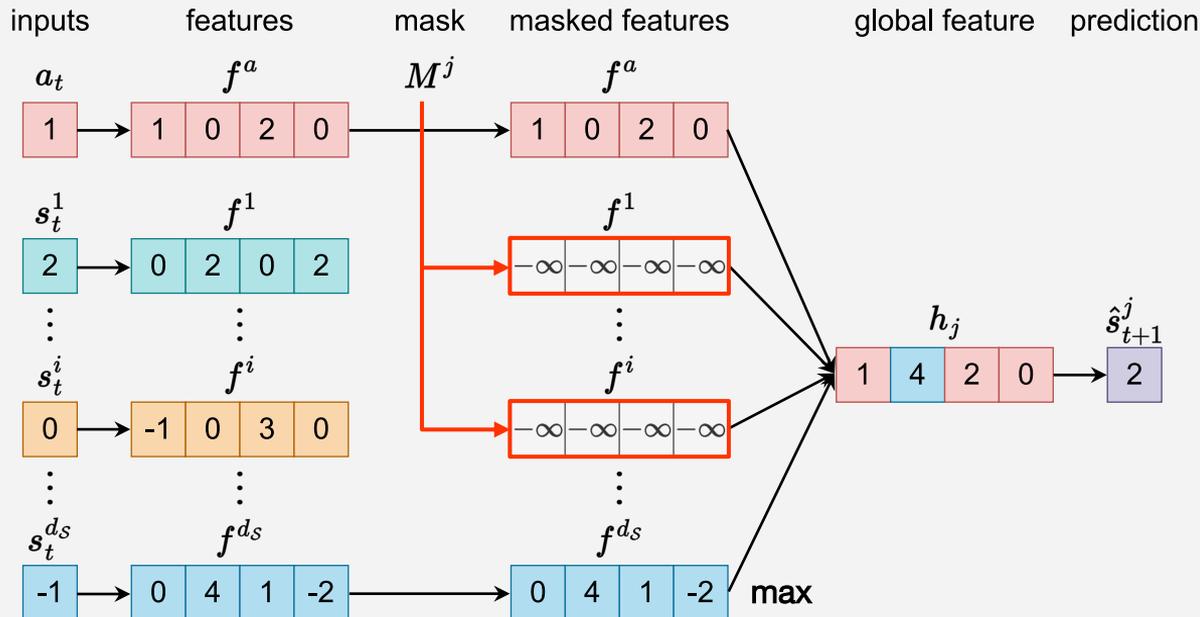
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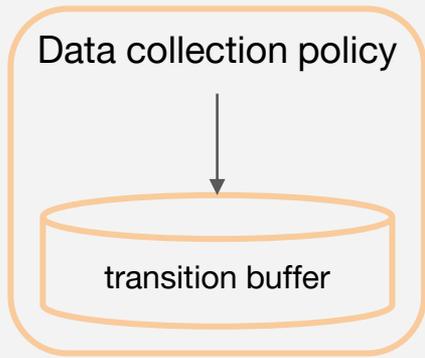
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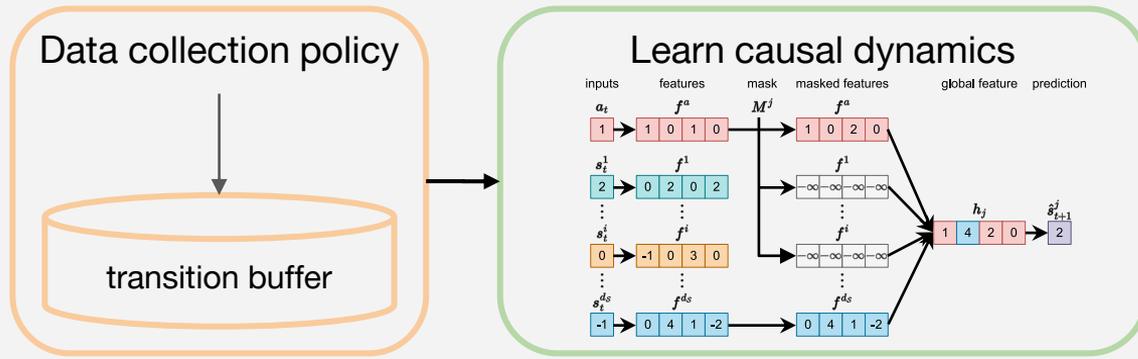
Method

After training, to represent the causal model $p(s_{t+1}^j | \mathbf{PA}_t^j)$, we can adjust the mask to select causal parents of s^j only.

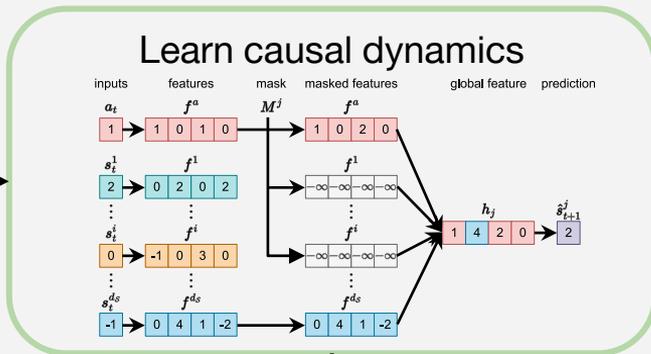
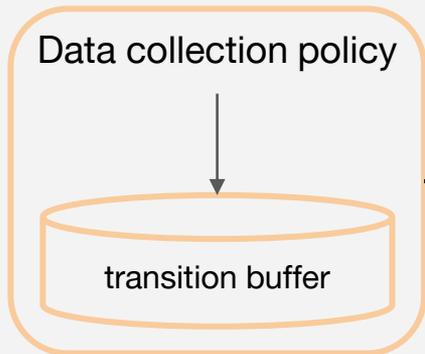




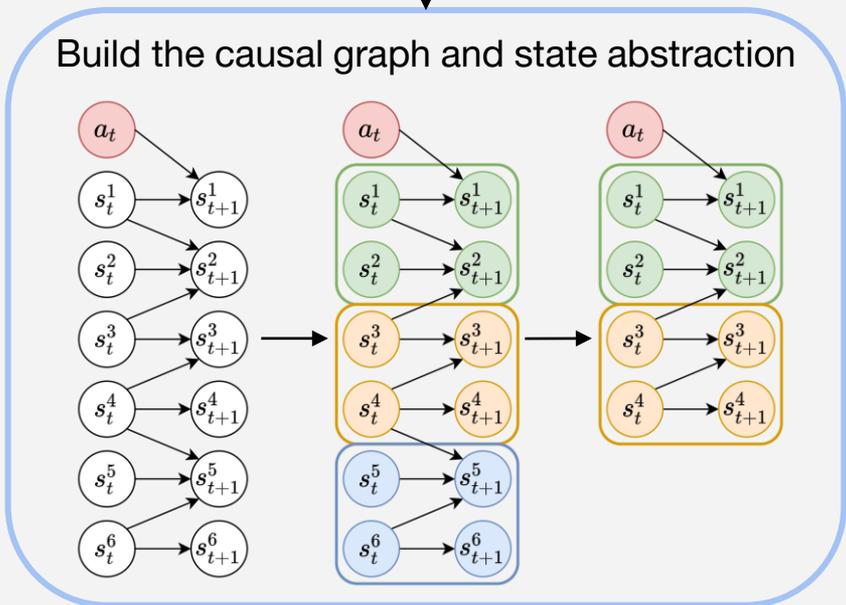
Causal Dynamics Learning (CDL)



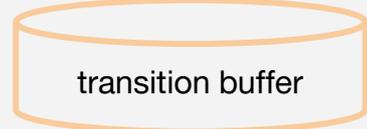
Causal Dynamics Learning (CDL)



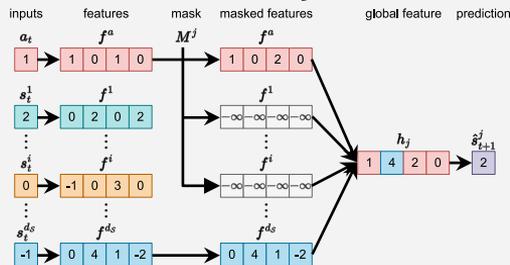
Causal Dynamics Learning (CDL)



Data collection policy

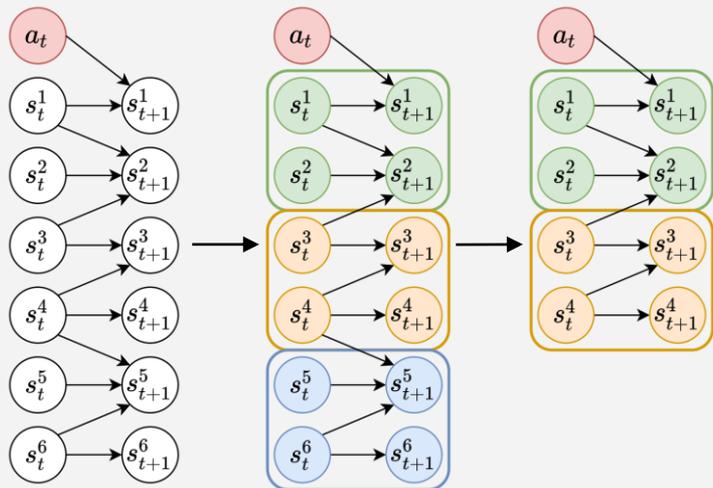


Learn causal dynamics

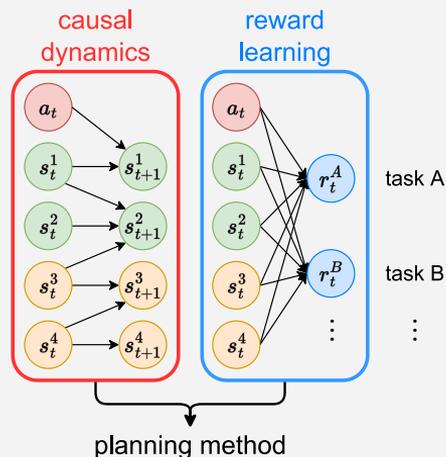


Causal Dynamics Learning (CDL)

Build the causal graph and state abstraction

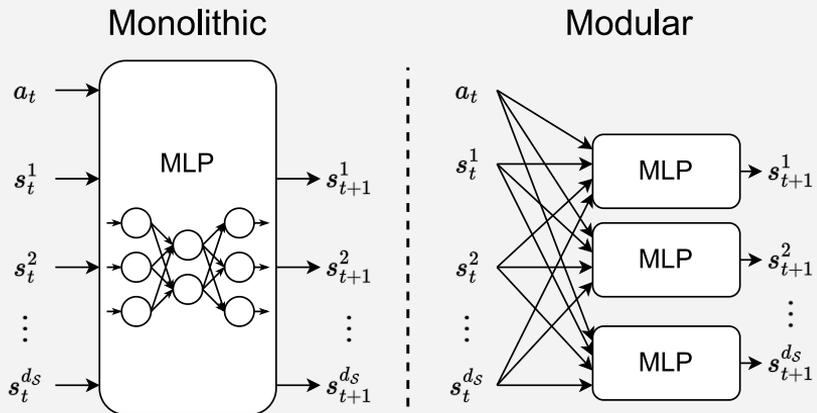


Learn downstream tasks with the abstracted causal dynamics



Experiments

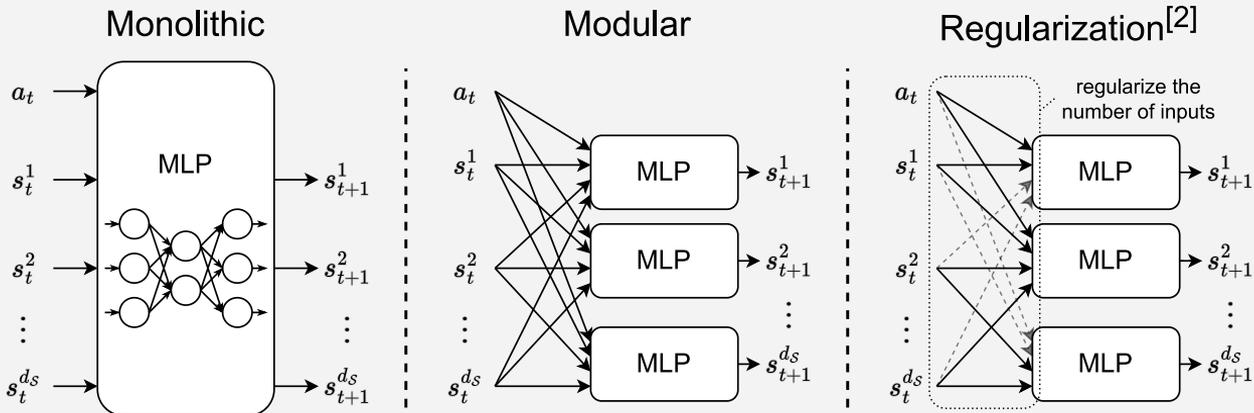
Baselines



MLP: multi-layer perceptron

Experiments

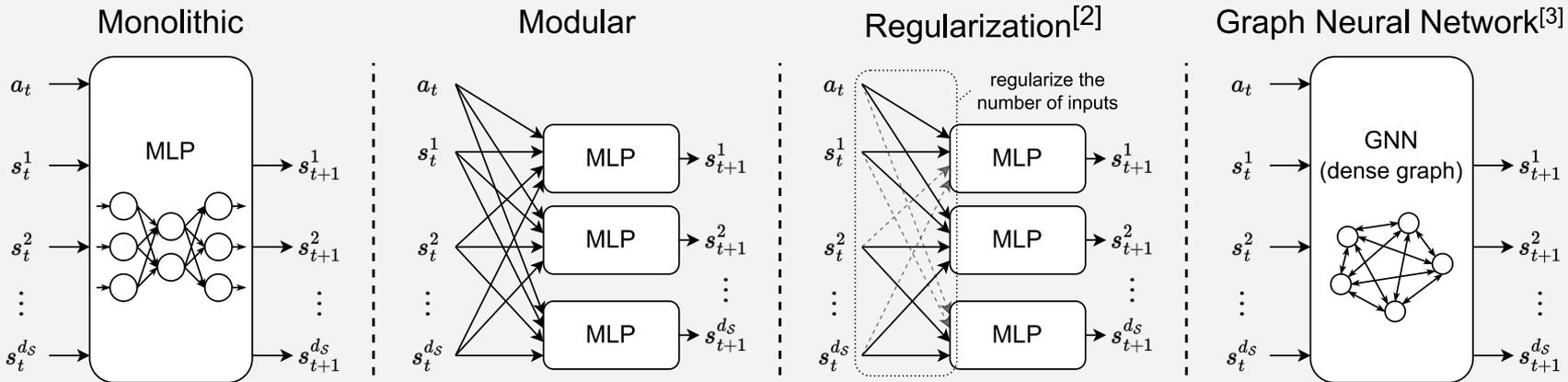
Baselines



MLP: multi-layer perceptron

Experiments

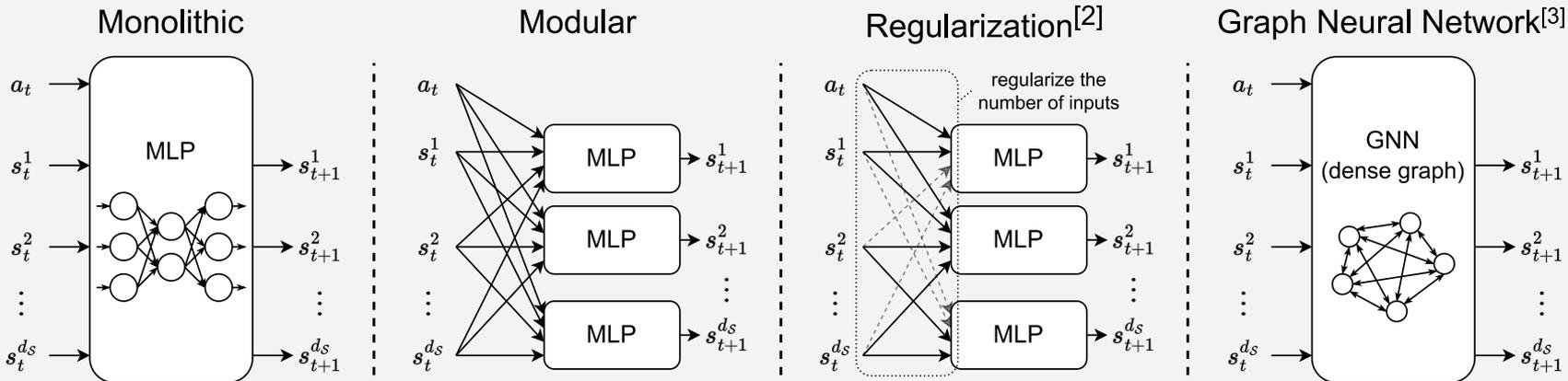
Baselines



MLP: multi-layer perceptron

Experiments

Baselines



MLP: multi-layer perceptron

Does each baseline learn a causal model?

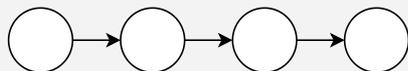


Experiments

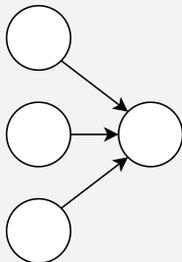
Chemical Environment^[4]

Synthesized environment

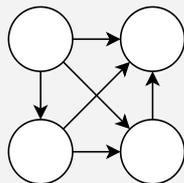
- with different underlying graphs



chain



collider



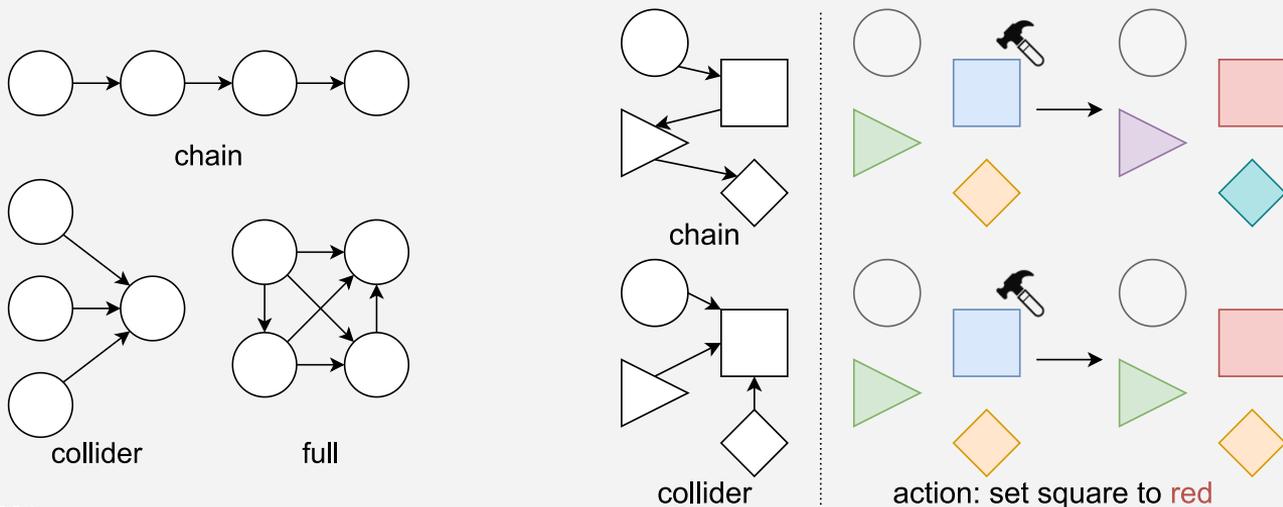
full

Experiments

Chemical Environment^[4]

Synthesized environment

- with different underlying graphs
- as action changes the color of one node, colors of all its descendants will also change.



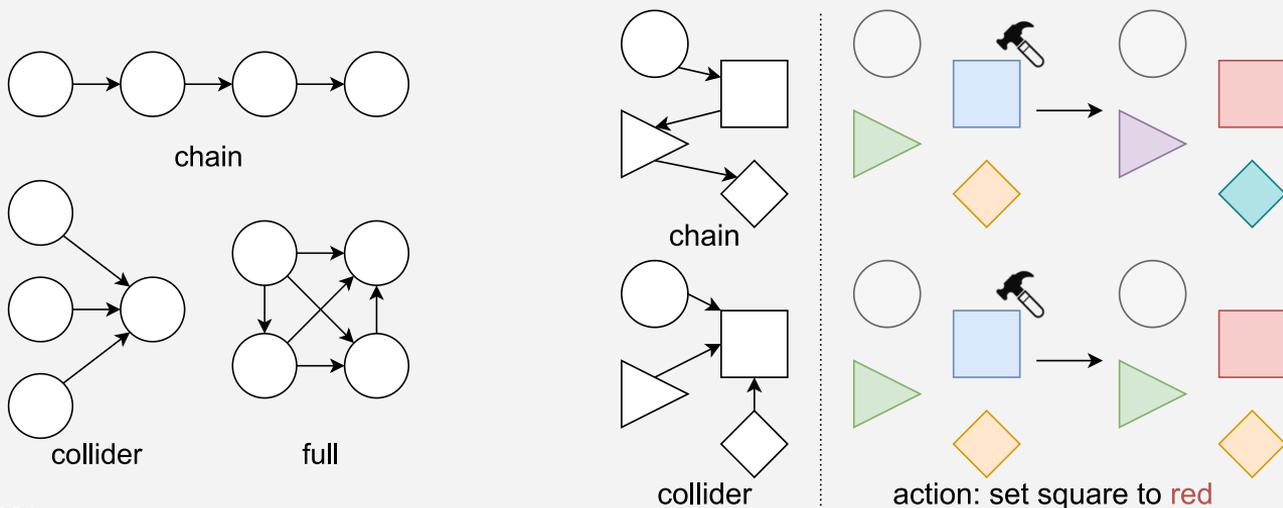
Experiments

Chemical Environment^[4]

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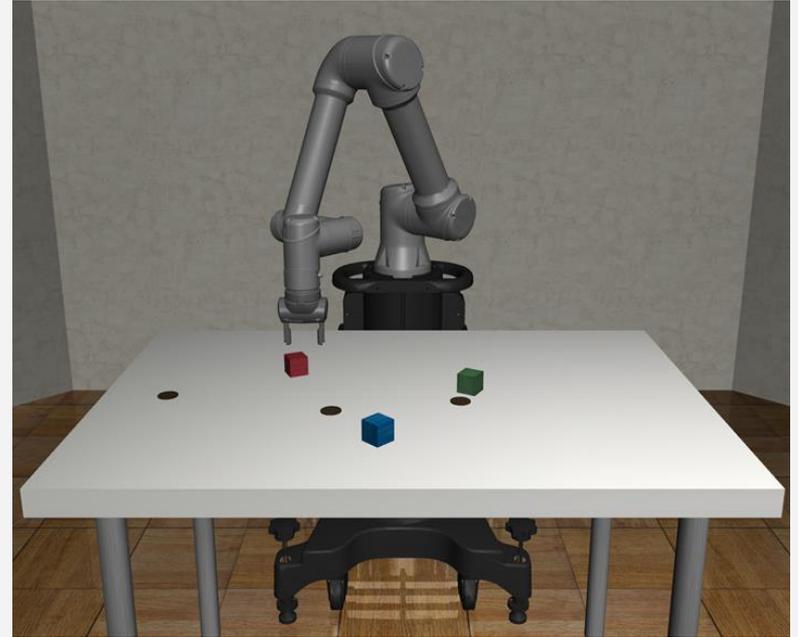
Action-irrelevant variables: positions sampled from $N(0, 0.01)$.



Experiments

Manipulation Environment

State Variables:



Experiments

Manipulation Environment

State Variables:

- end-effector (eef)

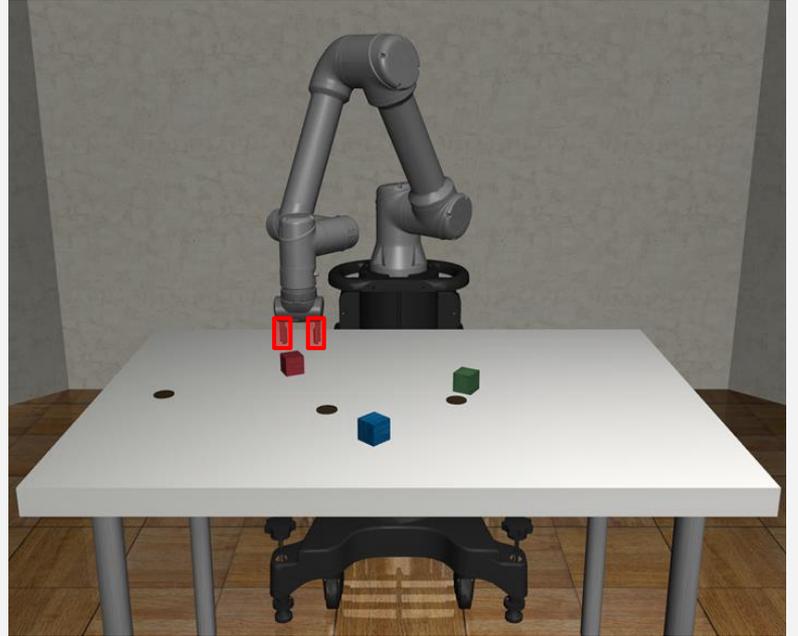


Experiments

Manipulation Environment

State Variables:

- end-effector (eef)
- gripper (grp)

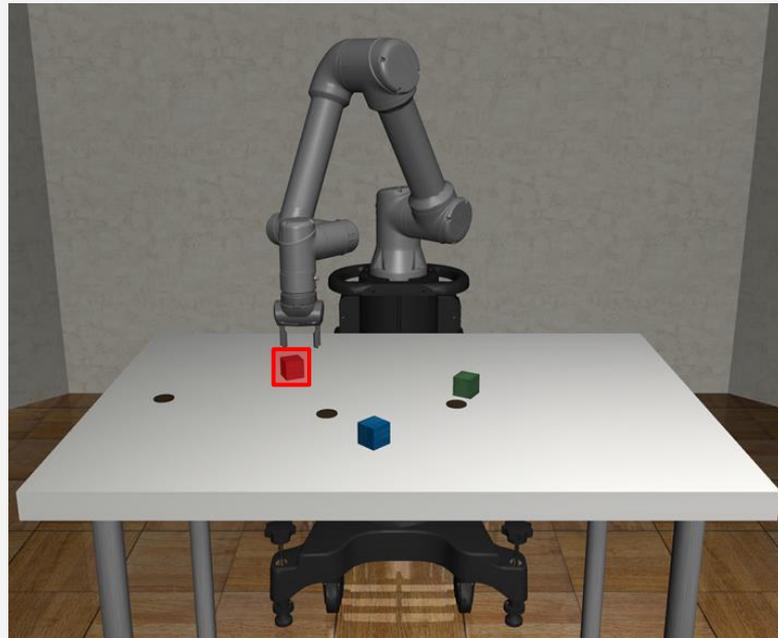


Experiments

Manipulation Environment

State Variables:

- end-effector (eef)
- gripper (grp)
- the **movable** object (mov)

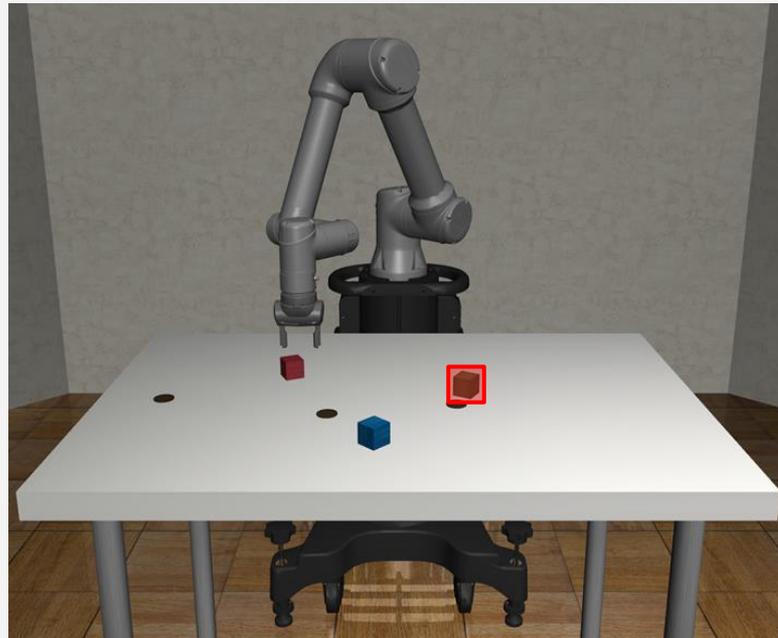


Experiments

Manipulation Environment

State Variables:

- end-effector (eef)
- gripper (grp)
- the **movable** object (mov)
- the **unmovable** object (unm)

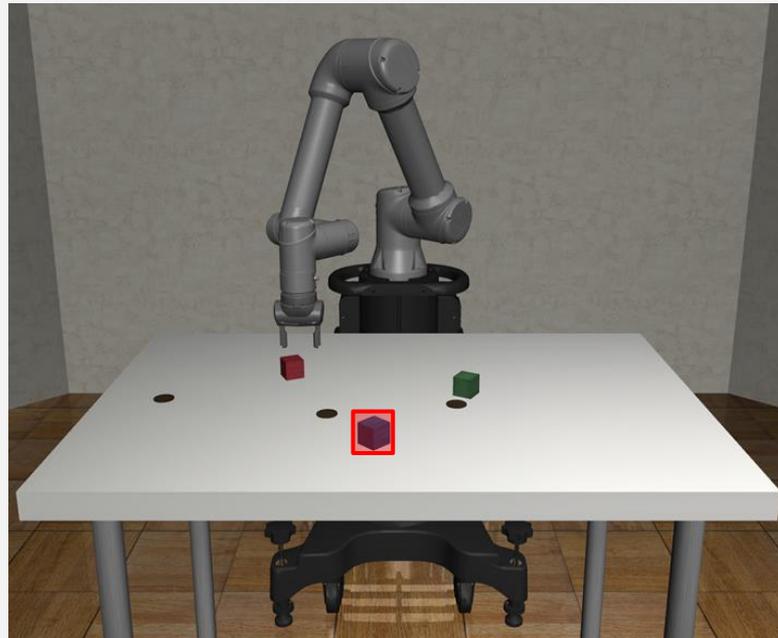


Experiments

Manipulation Environment

State Variables:

- end-effector (eef)
- gripper (grp)
- the **movable** object (mov)
- the **unmovable** object (unm)
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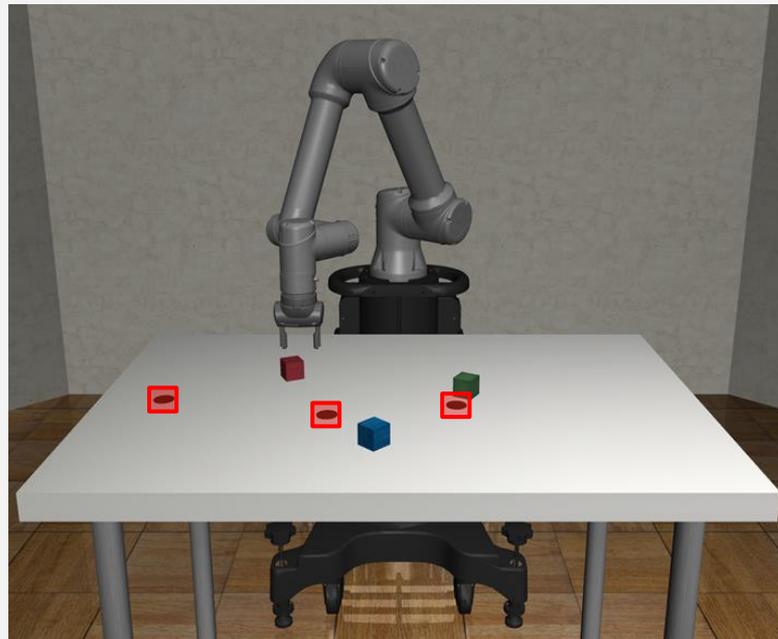


Experiments

Manipulation Environment

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- end-effector (eef)
- gripper (grp)
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- non-interactable markers (mkr¹⁻³)



Experiments

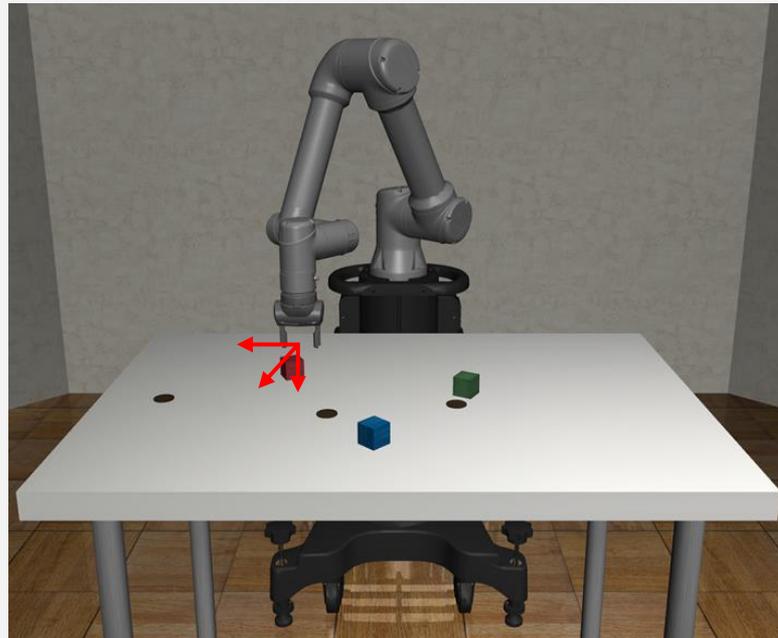
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Action dimensions:

- end-effector target



Experiments

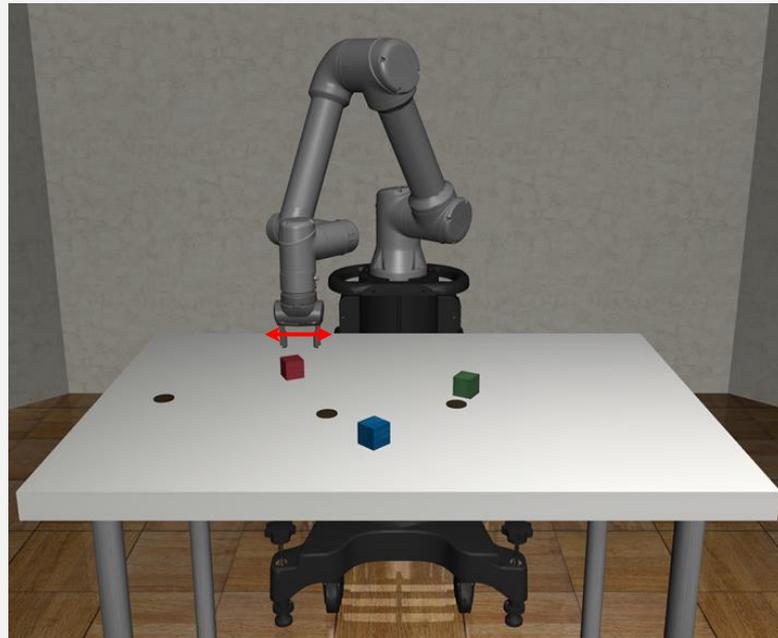
Manipulation Environment

State Variables:

- end-effector (eef)
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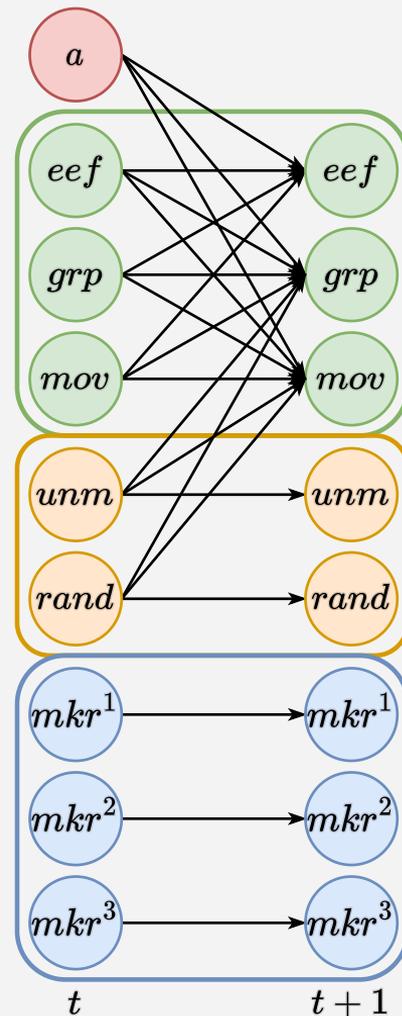
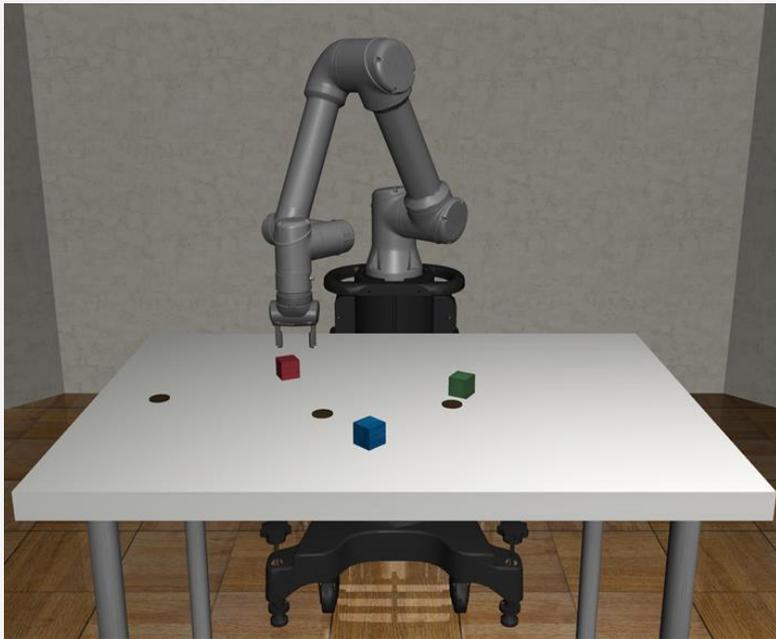
- end-effector target
- gripper open/close



Results

Causal Graph Accuracy

At the object level, the learned dependence is (subjectively) reasonable.



Results

Causal Graph Accuracy

Table 1. Causal Graph Accuracy (in %) for CDL and Reg

Environment	CDL (Ours)	Reg
Chemical (Collider)	100.0 \pm 0.0	99.4 \pm 0.4
Chemical (Chain)	100.0 \pm 0.1	99.7 \pm 0.1
Chemical (Full)	99.1 \pm 0.1	97.7 \pm 0.4
Manipulation	90.2 \pm 0.3	84.4 \pm 0.5

Results

Dynamics Generalization

Causal dynamics generalizes best in unseen states.

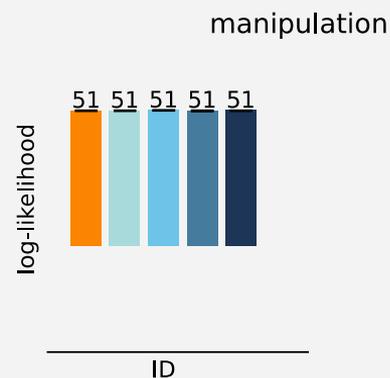
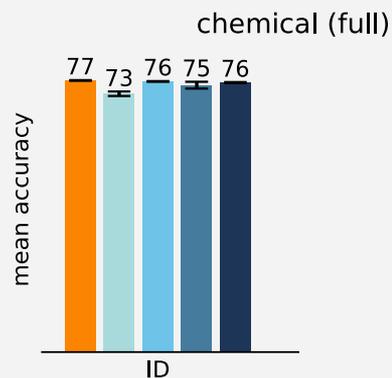
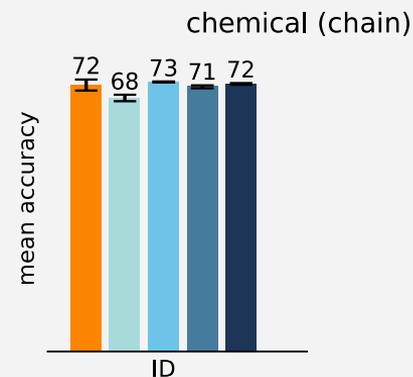
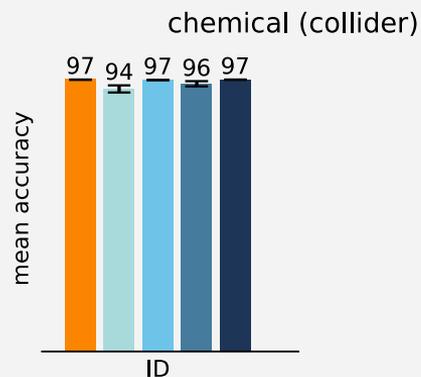
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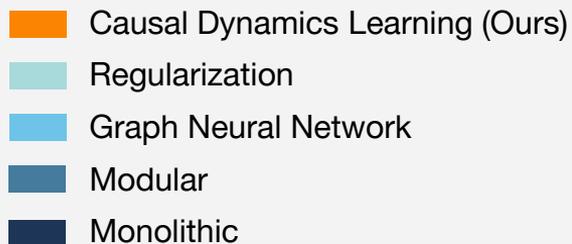
ID: in-distribution states



Results

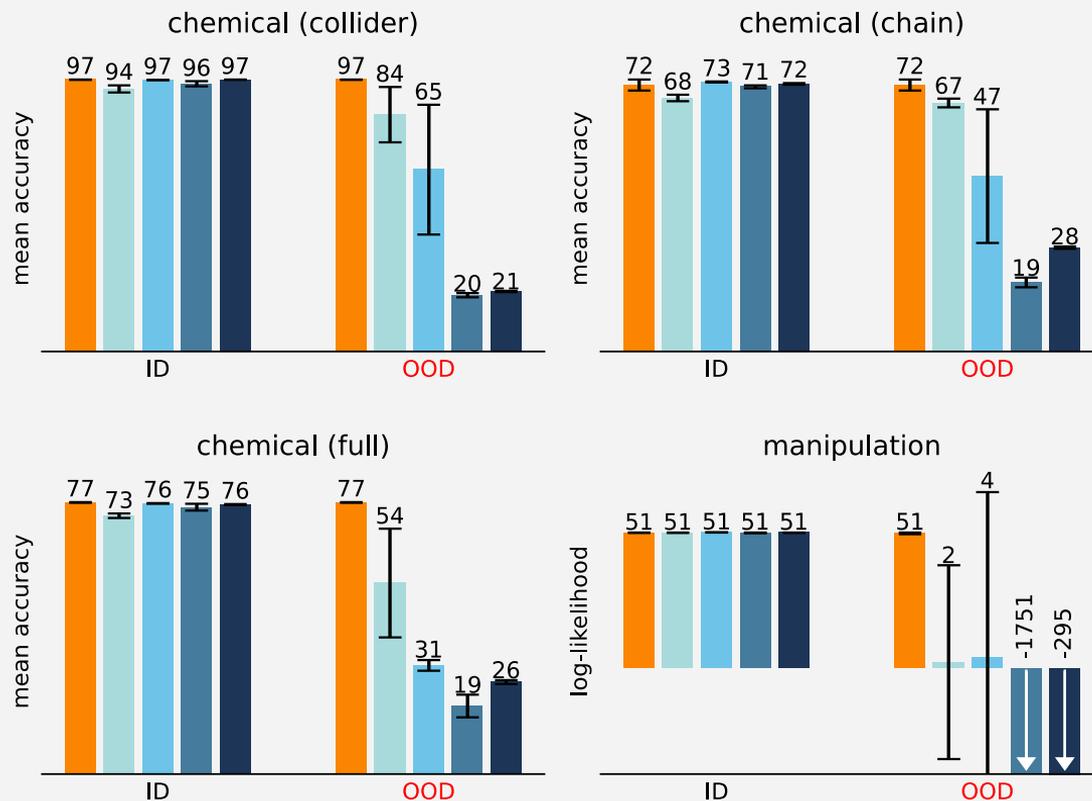
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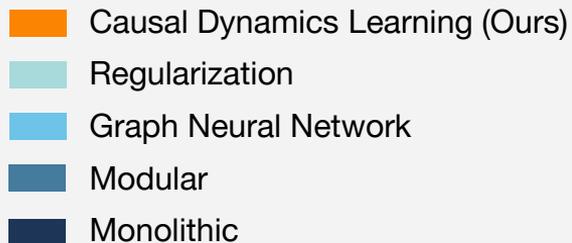
ID: in-distribution states

OOD: out-of-distribution states



Results

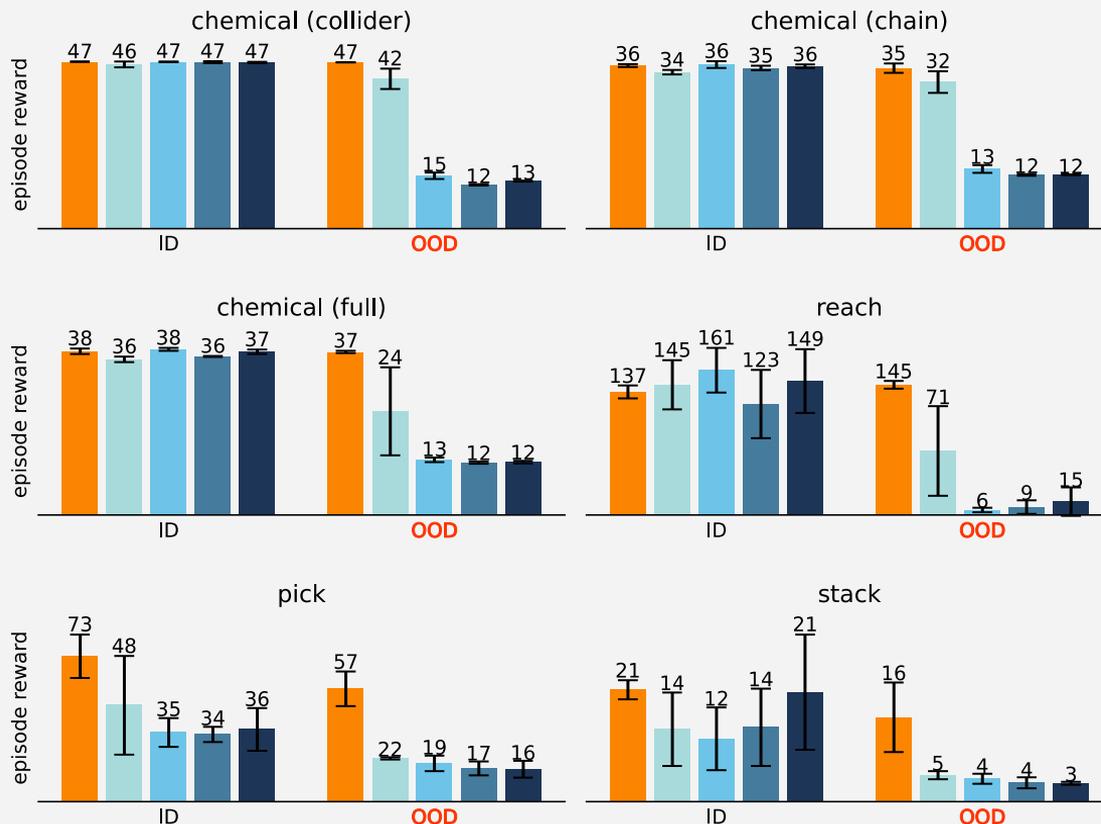
Causal dynamics generalizes best in unseen states.



ID: in-distribution states

OOD: out-of-distribution states

Task Generalization



Limitations and Future Directions

Scale to high-dimensional observations (e.g. images)?

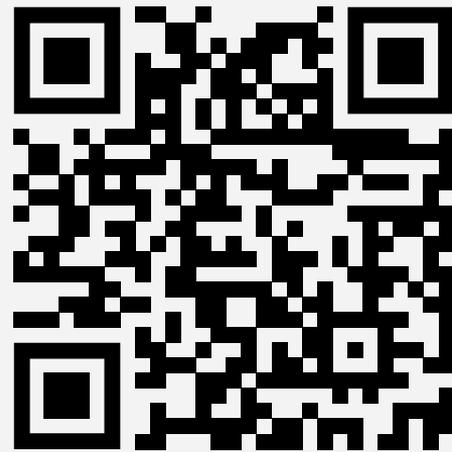
- Learn disentangled representations, then learn dynamics in the representation space

Causal dependencies are learned globally only.

- Learning local independencies to further sparsify the dynamics.

Causal Dynamics Learning for Task-Independent State Abstraction

Zizhao Wang, Xuesu Xiao, Zifan Xu, Yuke Zhu, and Peter Stone



Scan to read the paper

Contact Information:

Zizhao Wang: zizhao.wang@utexas.edu

Link to the Paper: <https://arxiv.org/pdf/2206.13452.pdf>



Sony AI

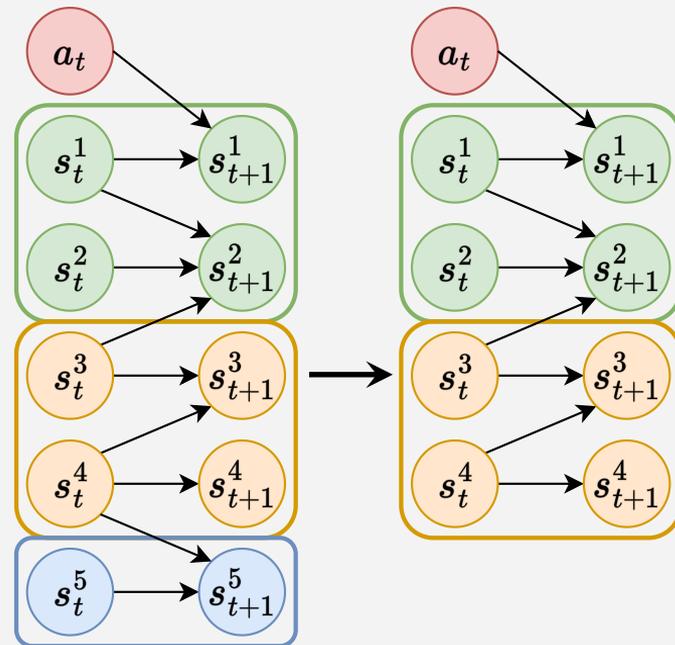
Problem Setup

CDL's state abstraction omits **action-irrelevant** variables.

What tasks can this state abstraction solve?

- ✓ Tasks whose rewards are defined by **controllable** and **action-relevant** state variables
- ✗ Tasks with rewards involving **action-irrelevant** state variables

Solving any task (learning any reward) means no abstraction.

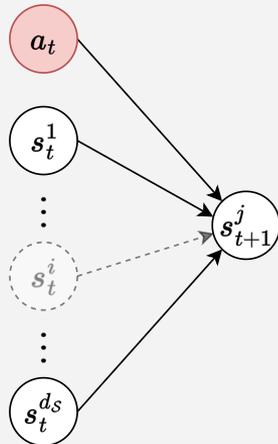
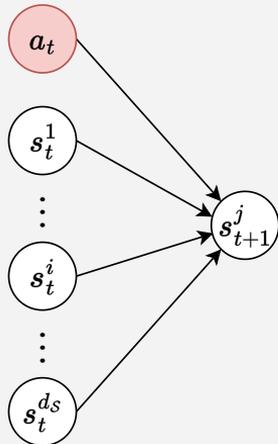


Method

Key idea: determine if the causal edge $s_t^i \rightarrow s_{t+1}^j$ exists with a conditional independence test.

Theorem 1 If $s_t^i \not\perp\!\!\!\perp s_{t+1}^j \mid \{s_t / s_t^i, a_t\}$, then $s_t^i \rightarrow s_{t+1}^j$.

In other words, is s_t^i needed to predict s_{t+1}^j ?



$$p(s_{t+1}^j | s_t, a_t) \stackrel{?}{=} p(s_{t+1}^j | \{s_t / s_t^i, a_t\})$$



$$\text{CMI}^{ij} = \mathbb{E} \left[\log \frac{p(s_{t+1}^j | s_t, a_t)}{p(s_{t+1}^j | \{s / s^i\}_t, a_t)} \right] \geq \epsilon$$