

Belief Space Planning under Approximate Hybrid Dynamics

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Abstract—The difficulty of many robot controls tasks stems from stochasticity and partial observability coupled with highly nonlinear dynamics. We propose to approximate nonlinear system dynamics using hybrid dynamics models and extend the POMDP framework to hybrid systems. To do this, we introduce a Bayesian inference based hybrid state evolution model that can be used to develop feasible motion plans under partial observability.

I. INTRODUCTION

One of the biggest challenges in robot motion planning is to develop feasible plans for systems having highly nonlinear dynamics in the presence of partial or noisy observations. However, many robotics tasks have structure that allows them to be described as a hybrid dynamics model using a set of linear dynamics functions, of which only one is active at any given time. While approximating nonlinear dynamics using hybrid models greatly simplifies the motion planning problem, having partial or noisy observations as feedback can often lead to failure while executing these plans. Problems focused on planning under partial observability are modelled as *partially observable Markov decision processes* (POMDP). Planning using POMDPs have been studied in literature for numerous application domains using different approaches [1], [2], [3], [4], [5]. However, most of the approaches consider states that either evolve only in discrete space or in continuous spaces. Brunskill et al. [1] do consider a hybrid dynamics based representation of the system states and have proposed a point-based POMDP planning approach considering hybrid model of the system dynamics. However, the proposed approach places arbitrary constraints on the choice of possible local dynamics models (discrete states) to maintain its closed form solution. Another pomdp planning algorithm considering hybrid system states was proposed by Agha-mohammadi et al. [4] to solve health-aware stochastic motion planning problem for quadrotors. But the proposed solution is

restricted only to the domains in which the discrete and continuous states evolve independently.

We propose to extend the POMDP framework to hybrid dynamics models and present a Bayesian inference-based hybrid state evolution model which can be combined with a POMDP solving algorithm for developing feasible motion plans for complex control tasks under partial observability. In this work, we use the belief space planning algorithm proposed by Platt et al.[3]. Initial plans in belief space are developed using trajectory optimization by assuming maximum likelihood observations and stabilized by designing a belief space LQR controller. A hybrid Bayesian filter is also proposed to reduce uncertainty over states using continuous state observations. The proposed model is used to develop motion plans for an autonomous robot navigating under uncertainty on a spatially varying terrain, results of which are included.

II. HYBRID DYNAMICS UNDER UNCERTAINTY

We extend the POMDP framework to hybrid systems by defining a belief over the hybrid states of the system and developing a belief evolution model using extended deterministic state transition dynamics and a Bayesian state estimator. Two parts of the belief evolution model, prediction and observation based update, are discussed in greater detail in the following sections.

A. Extended Hybrid Belief Dynamics

A belief state over the states of the hybrid system can be defined as $B = \{b^x, b^q\}$, where b^x and b^q correspond to belief distributions over the continuous states $x \in X = \mathbb{R}^N$ and discrete state $q \in Q$ respectively. Transition dynamics for the belief over continuous states can be defined as

$$b_{t+1}^x = p(x_{t+1}|x_t, u_t) \\ = \sum_{q'} p(x_{t+1}|x_t, u_t, q_t = q') p(q_t = q'|x_t) \quad (1)$$

$$\forall q' \in Q$$

where x_{t+1} and x_t represent the continuous states at time $t+1$ and t respectively, while, q_t and u_t represent the discrete state and continuous control input to the system at time t .

Discrete state transitions of the system can be represented as a directed graph with each possible discrete state q corresponding to a node and edges ($e \in E \subseteq Q \times Q$) marking possible transitions between the nodes. In hybrid systems, these transitions are conditioned on the continuous states. A transition from the discrete state q to another state q' happens if the continuous states \mathbf{x} are in the *guard set* $G(q, q')$ of the edge $e_q^{q'}$ where $e_q^{q'} = \{q, q'\}$, $G(\cdot) : E \rightarrow P(X)$ and $P(X)$ is the power set of X . Assuming the transitions of discrete states are given by a directed graph with self-loops, we can define the mode transition matrix Π at time t as $\Pi_t = \{\pi_t(i, j) = p(q_{t+1}^j | q_t^i, b_{t+1}^x) \quad \forall q^i, q^j \in Q\}$ as

$$\begin{aligned} \pi_t(i, j) &= \eta \int_{\mathbb{R}^N} \mathbf{1}_{q^j}^{q^i}(\mathbf{x}) b_{t+1}^x(\mathbf{s}) d\mathbf{s}, \quad \text{if } \exists e_{q^j}^{q^i}, \\ &= \epsilon, \quad \text{otherwise} \end{aligned} \quad (2)$$

where $\mathbf{1}_{q^j}^{q^i}(\mathbf{x})$ is an indicator function defined as

$$\begin{aligned} \mathbf{1}_{q^j}^{q^i}(\mathbf{x}) &= 1, \quad \text{if } \mathbf{x} \in G(q^i, q^j) \\ &= 0, \quad \text{otherwise} \end{aligned} \quad (3)$$

η is a normalization constant, given as $\eta = \sum_{k=1}^{|Q|} \pi(i, k)$ and ϵ is a small probability to handle scenarios in which the received observations do not correspond to any of the discrete states for a considerable amount of time.

B. Belief Evolution Model: Prediction

At each time step t , a priori state estimate for the continuous states is first obtain using Equation 1. Assuming a normal distribution to represent the belief over continuous states b^x , the current belief b_t^x is first propagated through the system dynamics of each discrete state, $\mathcal{F}^{q'}(x_t, u_t)$, individually. A weighted sum of the propagated belief set is then taken to obtain a prior of the belief at next time step b_{t+1}^x

$$\hat{b}_{t+1}^x = \sum_{q'} \mathcal{F}^{q'}(b_t^x, u_t) b_t^{q'} \quad (4)$$

where $b_t^{q'} = p(q_t = q' | x_t)$ is q' -th component of b_t^q . Continuous belief propagation is described in greater detail in Section II-D.

Assuming the belief over discrete state \hat{b}_{t+1}^q is given by a discrete distribution, a prior state estimate is calculated using Equation 2 where the mode transition matrix is updated using the continuous state prediction obtained in the last step \hat{b}_{t+1}^x .

C. Belief Evolution Model: Update

We propose a hybrid estimation algorithm based on Bayesian filters to reduce the uncertainty over states using noisy continuous state observations. The proposed algorithm consists of two layers of filters: first to estimate the continuous states of the system and second to estimate the discrete state of the system. Upon receiving observation z_{t+1} , the continuous state prior is updated by taking a weighted sum of a bank of extended Kalman filters running independently, with each discrete mode having an individual filter. The weights for the sum is determined using the prior for the discrete mode \hat{b}_{t+1}^q . The complete update step for continuous states can be written as

$$b_{t+1}^x = \hat{b}_{t+1}^x + \sum_{q'} \left(\mathbf{K}_{t+1}^{q'} (z_{t+1} - \mathcal{H}_{t+1}^{q'}(\hat{b}_{t+1}^x)) \right) \hat{b}_{t+1}^{q'}$$

where $\mathbf{K}_{t+1}^{q'}$ is the Kalman Gain for discrete state q' at time $t+1$ and $\hat{b}_{t+1}^{q'}$ is q' -th component of \hat{b}_{t+1}^q . Update for the discrete state can be obtained by using a Bayesian filter update given as

$$b_{t+1}^q = \gamma \mathbf{M}_{t+1} \circ \hat{b}_{t+1}^q \quad (5)$$

where $\mathbf{M}_{t+1} = [P(z_{t+1} | q_{t+1} = q')]^T \quad \forall q' \in Q$, \circ is the element-wise multiplication operator, γ is a normalization constant and

$$P(z_{t+1} | q_{t+1} = q') = z_{t+1} \sim \mathcal{H}_{t+1}^{q'}(b_{t+1}^x)$$

where $\mathcal{H}_{t+1}^{q'}(\cdot)$ is the observation function for state q' .

D. Gaussian Mixture Belief Model

The proposed belief evolution model for hybrid states can be implemented for motion planning by considering a suitable distribution over the continuous states of the system. A unimodal Gaussian belief can be effectively used to represent the belief for simple robot control tasks. However, in the case of complex tasks, a unimodal Gaussian might not be sufficient to capture the belief distribution. Here, we explain the state propagation by considering the belief over continuous states of the system as composed of a mixture of Gaussian distributions.

Let the belief over the continuous states of the system is given by a weighted combination of L normal distributions. Let m_t^l and Σ_T^l represent the mean and the co-variance matrix of the l -th Gaussian at time t respectively. If the deterministic state transition dynamics under discrete state q' is given as

$$x_{t+1} = A_t^{q'} x_t + B_t^{q'} u_t + \nu_t^{q'} \quad (6)$$

and the observation function given as

$$z_{t+1} = C_{t+1}^{q'} x_{t+1} \quad (7)$$

where $A_t^{q'}$, $B_t^{q'}$ and $C_{t+1}^{q'}$ are the corresponding system matrices at time t and $t+1$, $\nu_t^{q'}$ is the process noise for discrete state q' , x_t and x_{t+1} are the system states at time t and $t+1$ and u_t is the control input at time t . Then, under hybrid dynamics, the prediction for the l -th Gaussian can be given as

$$\hat{m}_{t+1}^l = \sum_{q'} (A_t^{q'} m_t^l + B_t^{q'} u_t + \nu_t^{q'}) b_t^{q'} \quad (8)$$

$$\hat{\Sigma}_{t+1}^l = \sum_{q'} (A_t^{q'} \Sigma_t^l (A_t^{q'})^T) b_t^{q'} \quad (9)$$

where $b_t^{q'} = p(q_t = q' | x_t)$ is q' -th component of b_t^q , \hat{m}_{t+1}^l and $\hat{\Sigma}_{t+1}^l$ are the priors for the mean and covariance of l -th Gaussian at time $t+1$ respectively. Prior for the discrete state of the system \hat{b}_{t+1}^q is obtained using Equation 2 as described in the previous section. Update in the prior for l -th Gaussian based on the received observations z_{t+1} can be given as

$$m_{t+1}^l = \sum_{q'} \{ \hat{m}_{t+1}^l + \hat{\Sigma}_{t+1}^l (C_{t+1}^{q'})^T (C_{t+1}^{q'} \hat{\Sigma}_{t+1}^l (C_{t+1}^{q'})^T + W_{t+1}^{q'})^{-1} (z_{t+1} - C_{t+1}^{q'} \hat{m}_{t+1}^l) \} \times \hat{b}_{t+1}^{q'} \quad (10)$$

$$\Sigma_{t+1}^l = \sum_{q'} [\hat{\Sigma}_{t+1}^l - \hat{\Sigma}_{t+1}^l (C_{t+1}^{q'})^T (C_{t+1}^{q'} \hat{\Sigma}_{t+1}^l (C_{t+1}^{q'})^T + W_{t+1}^{q'})^{-1} C_{t+1}^{q'} \hat{\Sigma}_{t+1}^l] \times \hat{b}_{t+1}^{q'} \quad (11)$$

where $C_{t+1}^{q'}$ is the observation matrix and $W_{t+1}^{q'}$ is the observation error covariance matrix for q' discrete state at time $t+1$. Updates in the prediction for discrete state can be done using Equation 5.

Mixing weights for the GMM can be updated based on the received observations as

$$\alpha_{t+1}^l = \mathcal{N}(\nu | \mathbf{0}, \Sigma_{t+1}^l) \quad (12)$$

where innovations ν is given as $\nu = z_{t+1} - \hat{z}_{t+1}^l$ and

$$\hat{z}_{t+1}^l = \sum_{q'} C_{t+1}^{q'} m_{t+1}^l \times \hat{b}_{t+1}^{q'}$$

III. MOTION PLANNING IN BELIEF SPACE

The proposed belief evolution model for hybrid dynamics can be combined with a motion planning algorithm to generate motion plans for a control task. In the current work, we use the belief space motion planning algorithm proposed by Platt et al. [3]. Belief space motion planning consists of two steps: developing

an initial plan using trajectory optimization and stabilizing around it by using a belief space linear quadratic Gaussian (LQG) controller [3]. For planning under uncertainty, the algorithm uses an extended Kalman Filter on the *maximum likely observation (MLO)* obtained by propagating the current belief through the system dynamics. In the current implementation of the algorithm, a weighted combination of the dynamics for continuous system states is considered only in the trajectory optimization phase. Trajectory stabilization using B-LQR chooses the most likely discrete state as the governing dynamics for the system.

IV. EXPERIMENTS

Experiments were conducted for navigating a mobile robot in a light-dark domain [3] where the quality of observations improves proportionally with the amount of light. Additionally, we consider the domain to have spatially varying dynamics, which is analogous to a real world scenario of off-road terrain with different surfaces having different ground friction. Domain $(\{x, y\} \in [-10, 10])$ was considered to contain three different linear dynamics functions, given as

$$\begin{aligned} f(\mathbf{x}_t, \mathbf{u}) &= \mathbf{x}_t + 0.5\mathbf{u}, & \text{if } x < 1 \\ f(\mathbf{x}_t, \mathbf{u}) &= \mathbf{x}_t + \mathbf{u}, & \text{if } x \in [1, 4] \\ f(\mathbf{x}_t, \mathbf{u}) &= \mathbf{x}_t + [2\mathbf{u}_1, \mathbf{u}_2]^T, & \text{if } x > 4 \end{aligned}$$

where $\mathbf{x}_t = \{x_t, y_t\}^T$. Observation function was taken as $h(x_t) = x_t + w$ with zero-mean Gaussian observation noise $w \sim \mathcal{N}(\cdot | 0, W(x))$ where

$$W(x) = \frac{1}{2}(5 - x_x)^2 + \text{const}$$

Matrices defining the cost function over error in states, control input, additional cost for final state error and covariance were taken as $Q = \text{diag}(0.5, 0.5)$, $R = \text{diag}(0.5, 0.5)$, $Q_{\text{large}} = 30$ and $\Lambda = 400$ respectively. For continuous state GMM belief, $L = 1$ was taken.

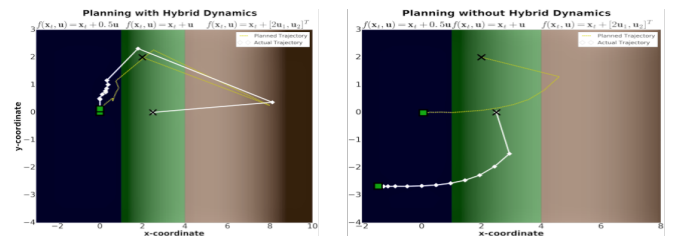


Fig. 1. Plots showing planned and actual robot trajectories with baseline deterministic and our approach. Initial belief $b_t = \{2, 2, 5, 0\}$, True start position: $\{2.5, 0, 0\}$, goal position: $\{0, 0\}$. Color indicates the local dynamics model, while brightness reflects observation noise.

Figure 1 shows that the planning algorithm was able to plan and stabilize around trajectories under uncertainty, while adapting to changing local dynamics.

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