Introduction to Linear Algebra and Syllabus Rundown

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M 340L
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Lecture 1 & 2

Important Dates for the course!!!

Midterm Exam - June 21st, 2024 (24-hour window, time limit: 1 hour, and 15 minutes)
Final Exam - July 12th, 2024 (3 hours and 40 minutes time limit)

Basic Terminology and notation

$a_1 + a_2x_2 + ... + a_nx_n = b$ (Linear equation)
$a_1, a_2, ..., a_n, b$ (known constants)

System of linear equations

$$a_{1\ 1}x_1 + a_{1\ 2}x_2 + ...a_{1\ n}x_n = b_1$$
$$a_{2\ 1}x_1 + a_{2\ 2}x_2 + ...a_{2\ n}x_n = b_2$$
$$a_{m\ 1}x_1 + a_{m\ 2}x_2 + ...a_{m\ n}x_n = b_m$$

The solution to this system of equations is $(x_1, x_2, ..., x_n)$ which satisfies every equation simultaneously.

Example #1

$$2x + y = -1 \qquad 2(4 + 3y) + y = -1$$
$$x - 3y = 4 \qquad x = 4 + 3y \uparrow$$

Below we will solve for the first system in the system of equation; $2x + y = -1 \equiv 2(4 + 3y) + y = -1$

$$\begin{aligned} 2(4+3y) + y &= -1 \\ 8 + 6y + y &= -1 \\ 7y &= -9 \\ y &= -\frac{9}{7} \end{aligned}$$

We then plug in y into our $x =$ system to solve for x.

$$\begin{aligned} x &= 4 + 3y \\ x &= 4 + 3(-\frac{9}{7}) \\ &= 4 - \frac{27}{7} \\ &= \frac{1}{7} \end{aligned}$$

Note! A ***solution set*** is the set of all solutions to a system

For this example, our solution set would be $\{(x, y) \in^2 \mid x - y = 3\}$

What is a matrix?

A matrix is a rectangular grid of number; it can also mean "womb" in Latin.

$$A = \begin{bmatrix} a_{1\,1} & a_{1\,2} & ... & a_{1\,n} \\ a_{2\,1} & a_{2\,2} & ... & a_{1\,n} \\ ... & ... & ... & ... \\ a_{n\,1} & ... & ... & a_{m\,n} \end{bmatrix} \tag{1}$$

a_{ij} means the element in the ith row and the jth column.

Matrix anatomy

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 8 \\ 5 & 7 \end{bmatrix} \tag{2}$$

$a_{3\,2} = 7$, while $a_{2\,3} =$ not defined. The size of the matrix is size $= 3 \times 2$ matrix.

The size of a matrix $= m \times x$

$=$ num rows $\times$ num columns

Proof of a matrix

$$a_{1\,1}x_1 + a_{1\,2}x_2 + ... + a_{1\,n}x_n = b_1$$
$$a_{2\,1}x_1 + a_{2\,2}x_2 + ... + a_{2\,n}x_2 = b_2$$
$$a_{m\,1}x_1 + ... + a_{m\,n}x_n = b_m$$

$$A = \begin{bmatrix} a_{1\,1} & a_{1\,2} & ... & a_{1\,n} \\ a_{2\,1} & a_{2\,2} & ... & a_{2\,n} \\ ... & ... & ... & ... \\ a_{m\,1} & a_{m\,2} & ... & a_{m\,n} \end{bmatrix} \leftarrow \text{coefficent matrix}$$

$$A = \begin{bmatrix} a_{1\,1} & ... & a_{1\,n} & b_1 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ a_{m\,1} & ... & a_{m\,n} & b_m \end{bmatrix} \leftarrow \text{augmented matrix}$$

Elementary Row Operations (June 7th)

Example #1

$$-2(x+3y=1) \rightarrow -2x - 6y \qquad = -2$$
$$2x + 5y = 1 \qquad 2x + 5y = 1$$
$$x + 3 = 1 \qquad -y = -1$$
$$x = -2 \qquad y = 1$$

Solution:
Solution coordinates: $(-2, 1)$
The matrices below are augmented matrices!s

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 1 \end{bmatrix}$$

The solution matrices are know as ***equivalent systems*** which means *they share the same solution set*

Reduced Row Reduction

1. Swap the two rows
2. Multiply a row by a non-zero number
3. Replace a row by the sum of that row and a multiple of another row

$$\text{This linear system is not allowed to be multiplied simultaneously}$$
$$-3(2x+7y=15)$$
$$2(3x-9y=84)$$

The goal

$$\begin{bmatrix}1 & 3 & 1\\ 2 & 5 & 1\end{bmatrix} \to \begin{bmatrix}1 & 0 & -2\\ 0 & 1 & 1\end{bmatrix}$$

Example #1

This is an augmented matrix (matrices)

$$\begin{bmatrix}1 & 3 & 1\\ 2 & 5 & 1\end{bmatrix} \to R_2 - 2R_1 \begin{bmatrix}1 & 3 & 1\\ 0 & -1 & -1\end{bmatrix} \to -R_2 \begin{bmatrix}1 & 3 & 1\\ 0 & 1 & 1\end{bmatrix} R_1 - 3R_2$$

Multiply the row that is changing

$$\begin{bmatrix}1 & 0 & -2\\ 0 & 1 & 1\end{bmatrix}$$

$$x=-2$$
$$y=1$$

Example #2 (Complex System)

$$\begin{aligned}-4x-5y-6z&=-3\\ -2x-3y-3z&=-2\\ 5x+7y+8z&=5\end{aligned} \to \begin{bmatrix}-4 & -5 & -6 & -3\\ -2 & -3 & -3 & -2\\ 5 & 7 & 8 & 5\end{bmatrix} \to \begin{bmatrix}1 & 0 & 0 & ?\\ 0 & 1 & 0 & ?\\ 0 & 0 & 1 & ?\end{bmatrix}$$

$$\begin{bmatrix}-4 & -5 & -6 & -3\\ -2 & -3 & -3 & -2\\ 5 & 7 & 8 & 5\end{bmatrix} R_1 + R_3 \to \begin{bmatrix}1 & 2 & 2 & 2\\ -2 & -3 & -3 & -2\\ 5 & 7 & 8 & 5\end{bmatrix} \begin{bmatrix}1 & 2 & 2 & 2\\ 0 & 1 & 1 & 2\\ 0 & -3 & -2 & -5\end{bmatrix} R_3 + 3R_2 \begin{bmatrix}1 & 2 & 2 & 2\\ 0 & 1 & 1 & 2\\ 0 & 0 & 1 & 1\end{bmatrix} \leftarrow \text{This is}$$

$$x+2y+2z=2$$
$$y+z=2$$
$$z=1, x=-2, y=1$$

(Row) Echelon Form of a Matrix

1. The leading number (1st non-zero number going left to right) of any row is to the right of the leading entries above it.
2. Everything in a column below a leading entry is zero.
3. Any row of zeros is below all non-zeros rows.

Example #1

$$\begin{bmatrix} 2 & 3 & 8 & -7 & 5 \\ 0 & 0 & 3 & 6 & 4 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix} \leftarrow \text{IN } \textbf{echelon form!}$$

$$\begin{bmatrix} 5 & 7 & 3 & 1 & 2 \\ 0 & 2 & 4 & 8 & -1 \\ 0 & 3 & 6 & 5 & 9 \end{bmatrix} \leftarrow \textbf{NOT} \text{ in echelon form!}$$

Reduced (Row) Echelon Form [Optimal]

Steps 1-3 from REF

1. Every leading entry is 1
2. Everything in a column above a leading entry is zero