This print-out should have 10 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

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001 10.0 points

If the augmented matrix for a system of linear equations in variables x_1, x_2, and x_3 is row equivalent to the matrix

$$B = \begin{bmatrix} 1 & 2 & -1 & -5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & -4 \end{bmatrix},$$

determine x_1.

1. $x_1 = -2 + t, \quad t$ arbitrary

2. $x_1 = -1 + t, \quad t$ arbitrary

3. $x_1 = 0$

4. $x_1 = -1$ **correct**

5. $x_1 = -2$

6. system inconsistent

Explanation:

The linear system

$$x_1 + 2x_2 - x_3 = -5,$$
$$x_2 - x_3 = -1,$$
$$2x_3 = -4,$$

associated with

$$B = \begin{bmatrix} 1 & 2 & -1 & -5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

is equivalent to the original system. Thus $x_3 = -2$. But B is in echelon form, so the remaining variables can be determined by back substitution. Consequently, $x_2 = -3$ and

$$\boxed{x_1 = -1}.$$

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002 10.0 points

If the augmented matrix for a system of linear equations in variables x_1, x_2, and x_3 is row equivalent to the matrix

$$B = \begin{bmatrix} 3 & 9 & 6 & -15 \\ -3 & -9 & -9 & 18 \\ -1 & -3 & -5 & 8 \end{bmatrix},$$

determine x_1.

1. $x_1 = -1 - 3t, \quad t$ arbitrary

2. $x_1 = 2$

3. $x_1 = -3 - 3t, \quad t$ arbitrary **correct**

4. $x_1 = -3$

5. $x_1 = -1$

6. system inconsistent

Explanation:

By row reduction

$$B = \begin{bmatrix} 3 & 9 & 6 & -15 \\ -3 & -9 & -9 & 18 \\ -1 & -3 & -5 & 8 \end{bmatrix}$$
$$\sim \begin{bmatrix} 3 & 9 & 6 & -15 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & -3 & 3 \end{bmatrix}$$
$$\sim \begin{bmatrix} 3 & 9 & 6 & -15 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

which is now in echelon form. But the system

$$3x_1 + 9x_2 + 6x_3 = -15$$
$$-3x_3 = 3$$
$$0x_1 + 0x_2 + 0x_3 = 0,$$

associated with this matrix has a free variable $x_2 = t$, say, and by back substitution, we see that

$$x_3 = -1, \quad x_1 = -3 - 3t,$$

Consequently,

$$\boxed{x_1 = -3 - 3t \quad t \text{ arbitrary}}.$$

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003 10.0 points

If the augmented matrix for a system of linear equations in variables x_1, x_2, and x_3 is row equivalent to the matrix

$$B = \begin{bmatrix} 1 & -1 & 5 & 2 \\ 0 & -3 & 9 & -3 \\ 0 & 1 & -3 & 4 \end{bmatrix},$$

determine x_1.

1. $x_1 = 2$

2. system inconsistent **correct**

3. $x_1 = 2 + t, \quad t$ arbitrary

4. $x_1 = 0$

5. $x_1 = 1$

6. $x_1 = 1 + t, \quad t$ arbitrary

Explanation:

By row reduction

$$B = \begin{bmatrix} 1 & -1 & 5 & 2 \\ 0 & -3 & 9 & -3 \\ 0 & 1 & -3 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 5 & 2 \\ 0 & -3 & 9 & -3 \\ 0 & 0 & 0 & 3 \end{bmatrix},$$

which is now in echelon form. But the system

$$x_1 - x_2 + 5x_3 = 2,$$

$$-3x_2 + 9x_3 = -3,$$

$$0x_3 = 3,$$

associated with this matrix is inconsistent because there is no solution x_3. Consequently, the original system is

$$\boxed{\text{inconsistent}}.$$

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004 10.0 points

Determine the Reduced Row Echelon Form of the matrix

$$A = \begin{bmatrix} 1 & -1 & -3 \\ -3 & 4 & 6 \\ -1 & 0 & 6 \end{bmatrix}.$$

1. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ **correct**

4. $\text{rref}(A) = \begin{bmatrix} 1 & 2 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

5. $\text{rref}(A) = \begin{bmatrix} 1 & 1 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

Explanation:

After performing elementary row operations downwards on the columns we see that

$$A \sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & -3 \\ 0 & -1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Next perform elementary row operations upwards. Then

$$A \sim \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Consequently,

$$\boxed{\text{rref}(A) = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}}.$$

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005 10.0 points

Determine the Reduced Row Echelon Form of the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -3 \\ 2 & 2 & -5 \end{bmatrix}.$$

1. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2. $\text{rref}(A) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ **correct**

3. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

4. $\text{rref}(A) = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

5. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Explanation:

After performing elementary row operations downwards and row interchanges on the columns we see that

$$A \sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Next perform elementary row operations upwards. Then

$$A \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Consequently,

$$\boxed{\text{rref}(A) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}.$$

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006 10.0 points

Determine the Reduced Row Echelon Form of the matrix

$$A = \begin{bmatrix} 3 & 3 & -15 \\ -2 & 0 & 6 \\ -2 & -4 & 14 \end{bmatrix}.$$

1. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ **correct**

2. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

3. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. $\text{rref}(A) = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

5. $\text{rref}(A) = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Explanation:

After performing elementary row operations downwards we see that

$$A \sim \begin{bmatrix} 1 & 1 & -5 \\ -2 & 0 & 6 \\ -2 & -4 & 14 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -5 \\ 0 & 2 & -4 \\ 0 & -2 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -5 \\ 0 & 1 & -2 \\ 0 & -2 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}.$$

Next perform elementary row operations upwards:

$$A \sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}.$$

Consequently,

$$\boxed{\text{rref}(A) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}}.$$

007 10.0 points

Determine the Reduced Row Echelon Form of the matrix

$$A = \begin{bmatrix} 3 & -3 & -3 & 6 \\ 3 & -1 & -5 & 8 \\ -1 & -1 & 3 & -2 \end{bmatrix}.$$

1. $\text{rref}(A) = \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. $\text{rref}(A) = \begin{bmatrix} 1 & -2 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

4. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ **correct**

5. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Explanation:

After performing elementary row operations downwards we see that

$$A \sim \begin{bmatrix} 1 & -1 & -1 & 2 \\ 3 & -1 & -5 & 8 \\ -1 & -1 & 3 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 2 & -2 & 2 \\ 0 & -2 & 2 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

Next perform elementary row operations upwards:

$$A \sim \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Consequently,

$$\boxed{\text{rref}(A) = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}.$$

008 5.0 points

Every matrix is row equivalent to a unique matrix in echelon form.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

Every matrix is row equivalent to a unique matrix in *reduced* echelon form; however the echelon form of a matrix need not be unique. Consider the following counter-example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.$$

Then A is row equivalent to B and C, both of which are in echelon form.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

009 5.0 points

If [0 0 0 0 1] is one row in an echelon form of an augmented matrix, then the associated linear system is inconsistent.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:

If [0 0 0 0 1] is one row in an echelon form of an augmented matrix, then the associated linear system contains the equation

$$0x_5 = 1\,,$$

which does not have a solution for x_5.

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$

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010 5.0 points

If a system of linear equations has two different solutions, it must have infinitely many solutions.

True or False?

1. TRUE **correct**

2. FALSE

Explanation:

Because the linear system has at least one solution it must be consistent, and if it has more than one solution, it must have a free variable. But if it has a free variable, it must have infinitely many solutions. Hence, any linear system must have no solutions, unique solutions, or infinitely many solutions, so if it has two different solutions, it must have infinitely many solutions.

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$