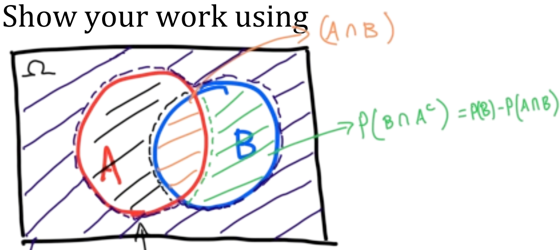
Homework 1

1. If P(A)=.7, P(B)=.4 and P(A∪B)= .9, find the following. Show your work using equations or Venn diagrams.

   d) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
   $-P(A\cup B) \quad + P(A\cap B)$
   $P(A\cap B) = P(A) + P(B) - P(A\cup B)$
   $= .7 + .4 - .9$
   $P(A\cap B) = .2$



$P(A\cap B^c) = P(A) - P(A\cap B)$

$P(A^c\cap B^c) = P(\Omega) - P(A\cup B)$

   a. $P(A^c\cap B^c) = .1$
   b. $P(A^c\cap B) = .2$
   c. $P(A\cap B^c) = .5$
   d. $P(A\cap B) = .2$

2. Suppose you roll a fair six-sided die once.

   a. Write the sample space for this experiment. $\Omega = \{1, 2, 3, 4, 5, 6\}$
   b. For each of the following, tell me whether or not it is a valid partition of the sample space, and if it is not a valid partition, tell me why not.
      i. {{1, 2}, {3, 4, 5}} Not a valid partition since it's missing element "6"; part of the sample space, Ω.
      ii. {{1}, {2}, {3,4}, {5}, {6}} This is a valid partition
      iii. {{1,2,3,6}, {4,5,6}} Not a valid partition, since it has a repeating element "6" and it violates the condition of mutual exclusivity of Ω.
      iv. {{1}, {2,3,4,5}, {6}, {}} Not a valid partition, since it includes an empty set.

3. In a pet shelter for cats only, there were 45 black cats, 30 long-haired cats, and 30 cats who were old (> 10 years old). For this study, cats in the shelter were classified as black/not-black, young/old, and long-haired/short-haired.

~~12 cats were black, young and long-haired~~,
10 were black, short-haired and old,
5 were black, long-haired and old, and
~~none of the cats were long-haired, non-black and old.~~

22   12   5   15
45B   30L   30o

   a. How many total cats were in this shelter (i.e., how many cats were black or long-haired or old)? 73 cats
   b. How many cats were black, short-haired, and young? 18 cats
   c. How many cats were long-haired, not-black, and young? 13 cats
   d. How many cats were short-haired, not-black, and old? 15 cats

Note: Define your events first (cat is black, cat is long-haired, and cat is old), and show your work using equations or Venn diagrams.

B - black
B^c - non-black
L - long-haired
L^c - short-haired
O - old (>10 years)
O' - Young (10> years) = Y

$P(B) = 45$
$P(O) = 30$
$P(L) = 30$
$P(B\cap L\cap O^c) = 12$
$P(B\cap L\cap O) = 5$
$P(B\cap L^c\cap O) = 10$

A. $P(B\cup L\cup O)$
$= P(B) + P(L) + P(O) - P(B\cap L) - P(B\cap O) - P(L\cap O) + P(B\cap L\cap O)$
(17, 15, 5, 5)
$45 + 30 + 30 - 17 - 15 - \cancel{5} + 5 = 73$

B. $P(B\cap L^c\cap O^c) =$
$P(B) - P(B\cap L) - P(B\cap O) + P(B\cap L\cap O)$
$45 - 17 - 15 + 5 = \boxed{18}$

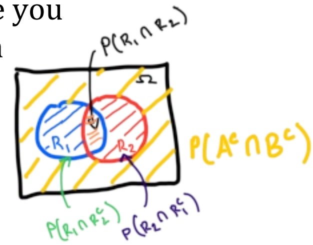
C. $P(B^c\cap L\cap O^c) =$
$P(L) - P(L\cap B) - P(L\cap O) + P(B\cap L\cap O) = 30 - 17 - 5 + 5 = \boxed{13}$

D. $P(B^c\cap L^c\cap O) =$
$P(O) - P(O\cap B) - P(O\cap L) + P(B\cap L\cap O) = 30 - 15 - 5 + 5 = \boxed{15}$

4. A track star runs two races on a certain day. The probability that she wins the first race is 0.7, the probability that she wins the second race is 0.6, and the probability that she wins both races is 0.5. Find the following probabilities.

$P(A) = 0.7$, $P(B) = 0.6$, and $P(A \cap B) = 0.5$

Note: Define your events, and write the probability statement before you calculate answers. You can show your work using equations or Venn diagrams.



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.7 + 0.6 - 0.5$
$= 0.8$

a. She wins at least once race $\rightarrow 0.8$

b. She wins exactly one race $P((A \cap B^c) \cup (A^c \cap B)) = 0.3$

$P(B \cap A^c) = P(B) - P(A \cap B)$
$= 0.6 - 0.5$
$= 0.1$

$P(A \cap B^c) = P(A) - P(A \cap B)$
$= 0.7 - 0.5 = 0.2$

c. She wins neither race $\rightarrow 0.2 = P(A^c \cap B^c)$

$P(A^c \cap B^c) = P(\Omega) - P(A \cup B)$
$= 1 - 0.8$
$P(A^c \cap B^c) = 0.2$

* 5. One card is selected from a deck of 52 cards and placed in a second, similar deck. A card is then drawn from the second deck. What is the probability that this card drawn from the second deck is an Ace? Write the probability statement, and show your work using equations.

$A \rightarrow$ An ace is drawn from the 1st stack of cards

$B \rightarrow$ An ace is drawn from the 2nd stack of cards

$A^c \rightarrow$ The card selected from the 1st deck ~~isn't~~ is not an Ace

$\therefore P(B)$ is equal to $\frac{20}{2756} + \frac{192}{2756} = \frac{212}{2756}$

$\therefore$ The possibility for an ace to be drawn from the deck would be $212/2756$ or 0.0769.

$P(A) = \frac{4}{52} = \frac{1}{13}$

$P(A^c) = \frac{48}{52} = \frac{12}{13}$

The card added is an ace for stack 2

Chance of card not being an ace from 1st stack

Card added is not an Ace in Stack 2.

$P(B) = ? = P(A) \cdot P(B \mid A) + P(A^c) P(B \mid A^c)$

$= \frac{4}{52} \cdot \frac{5}{53} + \frac{48}{52} \cdot \frac{4}{53}$

Card being an ace from the 1st stack

$\frac{20}{2756} + \frac{192}{2756}$