

SDS 321 Homework 2

1. A family has two children (not twins). Assume that each child is equally likely to be left-handed or right-handed.

Define the following events:

S: family has at least one Right-handed child named Sam

R: child is right-handed

L: child is left-handed

- a) What is the sample space for the family's two children with respect to handedness?
- b) What is the probability that both children are right-handed? Write the probability statement, and calculate your answer.
- c) If we know that at least one of the children is right-handed, what is the probability that both are right-handed? Write the probability statement, and calculate your answer.
- d) If we know that the older child is right-handed, what is the probability that both are right-handed? Write the probability statement, and calculate your answer.
- e) When asked if he has at least one right-handed child named Sam, the father replies, "Yes." What is the probability that both children are right-handed, if we know that the family has at least one right-handed child named Sam?

Assume the following:

- If a child is right-handed, their name will be Sam with probability α .
- It's possible for both children to be named Sam.
- If the child is left-handed, their name cannot be Sam.

Write the probability statement and show your steps as you calculate your answer. Note that your answer will include α in it.

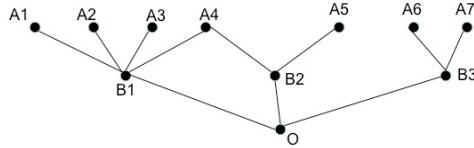
Hint: Bayes Theorem might be useful.

2. A, B and C are events such that $P(A) = 0.1$, $P(B) = 0.3$, and $P(C) = 0.2$. Find $P(A \cup B \cup C)$ under each of the following assumptions:
 - a) If A, B, and C are mutually exclusive.
 - b) If A, B, and C are independent.
3. Consider an urn containing four balls, numbered 110, 101, 011 and 000, from which one ball is drawn at random. For $k = 1, 2, 3$ let A_k be the event of drawing a ball with 1 in the k^{th} position. Show your work in answering the following:
 - a) Are A_1, A_2 , and A_3 pairwise independent?
 - b) Are A_1, A_2 , and A_3 mutually independent?

4. In a large population, 3% of the people are heroin users. A new drug test correctly identifies users 93% of the time and correctly identifies nonusers 90% of the time. Answer the following, showing all work for full credit. Write the probability statement for parts (b) through (e) and show your steps as you calculate your answer.

- a) Define events. Draw the probability tree for this scenario. Label the outcomes and indicate all the probabilities on the tree.
- b) What is the probability that a person who does not use heroin in this population tests positive?
- c) What is the probability that a randomly chosen person from this population is a heroin user and tests positive?
- d) What is the probability that a randomly chosen person from this population tests positive?
- e) If a person tests positive for heroin, what is the probability that he/she is a heroin user?

5. A man starts at the point O on the map below. He chooses a path at random and follows it to point B1, B2, or B3. From that point, he chooses a new path at random and follows it to one of the points A1 – A7.



Suppose the man arrives at point A4, but it is not known which route he took, what is the probability that he passed through each of the three points: B1, B2, B3? Calculate each one.