

1. A box contains two coins: a regular coin and one two-headed coin. I choose a coin at random and toss it twice. Define the following events.

A= First toss results in a H

B= Second toss results in a H

C= Regular coin has been selected

Are A and B independent? Are A and B conditionally independent given C?

Find $P(A|C)$, $P(B|C)$, $P(A \cap B|C)$, $P(A)$, $P(B)$, and $P(A \cap B)$, and use them to answer the question.

2. Suppose you have three nickels in a jar, where the first has Heads on both sides, the second has Tails on both sides, and the third is a fair coin. You choose one coin at random and toss it. The toss results in Tails. What is the probability that you chose the fair coin?
3. Pet shelters typically have special adoption events for black cats and dogs, since their adoption rate is lower. One specific pet shelter (which has only cats and dogs) has 40% cats and 60% dogs. Twenty percent of the cats are black, and 10% of the dogs are black. The adoption percentage for black cats is 5%, and for black dogs is 10%. Cats that are not black are equally likely to be adopted as not adopted, and this is the same for dogs that are not black.

For (b)-(f), write the probability statement and then calculate your answer.

- a) Draw the probability tree and label all events and probabilities in it.
- b) What is the probability that a randomly chosen animal is not black, a dog, and is adopted?
- c) What is the probability that a randomly chosen animal is black, a cat, and is not adopted?
- d) What is the probability that a randomly chosen animal is adopted?
- e) Suppose a randomly chosen animal is a cat, what is the probability that it is adopted?
- f) Suppose a randomly chosen animal is adopted, what is the probability that it is a cat?

4. In a certain town, the boy:girl ratio among children is 1:2. Suppose a family has three children. Define events:
A: the family has children of both sexes
B: the family has at most one girl
Are the events A and B independent? Show your work (using probability) to support your answer.