SDS 321 Worksheet 1b

1. What do you know about sets?
   a. $\{1,2\} \cup \{2,3\} =$ {1,2,3}
   b. $\{1,2\} \cap \{2,3\} =$ {2}
   c. True or False: $\{\{1,2\}, \{3\}, \{4,5\}\}$ is a partition of $\{1,2,3,4,5\}$ True

2. Express the following in terms of the events A, B and C using the operations of complement, union and intersection:
   – at least one of the events A, B, C occurs
   $(A \cup B \cup C)$
   – events A and B occur, but not C
   $(A \cap B) \cup B^c$

3. Which of these numbers cannot be a probability?
   (a) - 0.00001
   b) 0.5
   (c) 1.001
   d) 0
   e) 1

4. Let A and B be two sets. Under what conditions is the set $A \cap (A \cup B)^c$ empty?

5. Is it possible to have the following: $P(E) = .3$, $P(F)=.4$, and $P(E \cup F)=.5$? Explain.
   $P(E \cup F) = P(E) + P(F) - P(A \cap B)$

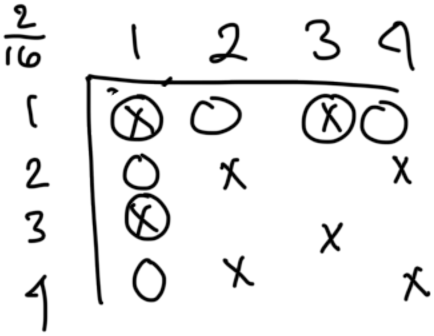
6. Given $P(A) = 0.55$, $P(B^c) = 0.35$, and $P(A \cup B) = 0.75$, find $P(B)$ and $P(A \cap B)$.

7. Given that $P(A^c) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.1$, determine $P(A \cup B)$.

8. Give a mathematical derivation of the formula
   $$P((A \cap B^c) \cup (A^c \cap B)) = P(A) + P(B) - 2P(A \cap B)$$

9. Show that $P(A \cup B \cup C) =$
   $$P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

10. Consider two rolls of a fair four-sided die. Let X be the outcome on the first roll and Y be the outcome on the second. Determine:
   a. $P((X, Y) \text{ is } (1,1) \text{ or } (1,2)) = \frac{1}{8} = \frac{2}{16}$
   b. $P(\{X = 2\}) = \frac{1}{4} = \frac{4}{16}$
   c. $P(X + Y \text{ is even}) = \frac{8}{16} = \frac{1}{2}$



d. P(min(X, Y) = 1) = 7/16

e. P(min(X, Y) > 1) = 9/16

11. If you toss a fair coin until you first see a head, letting your sample space be the number of tosses to reach a head, then the sample space S will be S=*{1, 2, . . .} with* P(n) = $(1/2)^n$, n = 1, 2, . . .

    Find P(the total number of tosses to see a head is even).

Note:

$$a + ar + ar^2 + ar^3 + ar^4 + \cdots = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}, \text{ for } |r| < 1.$$