SDS 321 Worksheet 3

1. A coin is tossed twice. Alice claims that the event of two heads is at least as likely if we know that the first toss is a head than if we know that at least one of the tosses is a head. Is she right?

2. In Vancouver, British Columbia, the probability of rain during a winter day is 0.58, for a spring day is 0.38, for a summer day is 0.25, and for a fall day is 0.53. Each of these seasons lasts one quarter of the year.
   a. What is the probability of rain on a randomly chosen day in Vancouver?
   b. If you were told that on a particular day it was raining in Vancouver, what would be the probability that this day would be a winter day?

3. A manufacturer claims that its drug test will detect steroid use (that is, show positive for an athlete who uses steroids) 95% of the time. What the company does not tell you is that 15% of all steroid-free individuals also test positive (the false positive rate). 10% of the rugby team members use steroids. So if the test is positive, what is the probability that he actually uses steroids? If you were the team manager, would you use this test for your athletes?

4. Suppose a system consists of two components C_1 and C_2. Suppose A_1 is the event that C_1 fails, and A_2 is the event that C_2 fails. $P(A_1) = 0.1$ and $P(A_2) = 0.2$. Suppose the components fail independently of each other.

   a) If C_1 and C_2 are connected in series (for example, batteries in a flashlight where current must pass through both components for it to function), what is the probability that the system works properly?
   b) If C_1 and C_2 are connected in parallel (for a parallel system to fail, both components need to fail, and it will function properly if even one component functions), what is the probability that the system works properly?

5. Suppose candy comes in two flavors: Cherry and Apple. The candy is sold in very large bags and there are known to be 5 kinds, indistinguishable from the outside. The manufacturer advertises that they sell 10% bag type 1, 20% type 2, 40% type 3, 20% type 4, and 10% type 5. The candy composition in each of these bags types is as follows:

   Bag type 1: 100% cherry
   Bag type 2: 75% cherry + 25% apple
   Bag type 3: 50% cherry + 50% apple
   Bag type 4: 25% cherry + 75% apple
   Bag type 5: 100% apple

   a) Suppose you select a bag at random and pick up a piece of candy from that bag. What is the probability that the candy you selected is apple?

   b) Given that you selected an apple candy, what is the probability that you chose bag type i (for $i = 1, \ldots, 5$)?

6. Suppose a box contains four tickets labeled 112, 121, 211, 222. We choose a ticket at random and consider the following events:
   $A_i = \{1 \text{ occurs at the } i^{th} \text{ place}\}$ for $i = 1, 2, 3$
   Are A_1, A_2, and A_3 pairwise independent? Are they mutually independent?