SDS 321 Worksheet 5 (Counting) Answers

1. There is a basket of fruit containing 3 apples and there are five girls who want to eat one fruit. How many ways are there to give three of the five girls one fruit each and leave two of them without a fruit to eat?

   $C(5,3) = 5! / 3!(5-3)!$

2. There is a basket of fruit containing an apple, a banana and an orange, and there are five girls who want to eat one fruit. How many ways are there to give three of the five girls one fruit each and leave two of them without a fruit to eat?

   $5! / (5-3)! = 60$

3. An hexadecimal number is a number whose digits can take sixteen different values: either one of the ten numbers from 0 to 9, or one of the six letters from A to F. How many different 8-digit hexadecimal numbers are there, if an hexadecimal number is allowed to begin with any number of zeros?

   16^8

4. From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed?

   $C(5,2)$ x $C(7,3) = (5$ x 4 x 7 x 6 x $5) / (2$ x 3 x $2) = 350$

5. How many different arrangements can be made from the letters: (a) FLUKE (b) PROPOSE? The arrangements don't need to be real words.

   (a) 120   (b) 1260

6. In #4, what if two of the men are feuding and refuse to serve on the committee together?

   $C(5,2)$ for the women
        $C(2,0)C(5,3)$ for neither of the feuding men
        $C(2,1)C(5,2)$ for exactly one of the feuding men
   So $[C(2,0)C(5,3) + C(2,1)C(5,2)]$ for the men
   Combining the men and women,
   Number of combinations = $C(5,2)$ x $[C(2,0)C(5,3) + C(2,1)C(5,2)] = 300$

7. A police department in a small city consists of 10 officers. If the department policy is to have 5 of the officers patrolling the streets, 2 of them working full-time at the station, and 3 of them on reserve at the station, how many different divisions of the 10 officers into the 3 groups are possible?

   $10! / 5!2!3! = 2520$

8. Ten children are to be divided into an A team and a B team of 5 each. The A team will play in one league and the B team in another league. How many different divisions are possible?

This is a partition of the 10 (distinguishable) children into two (distinguishable) teams. Since the teams play in different leagues, they will never play each other.
The number of ways of selecting 5 children out of 10 is C(10, 5).
So the answer is C910, 5) = 10! / 5!5! = 252

9. In order to play a game of basketball, 10 children at a playground divide themselves into two teams of 5 each. How many different divisions are possible?

This is similar to #8, except the two teams always play each other.

So for example, the following two ways of dividing the children has the same result since they play each other: (A: 1, 2, 3, 4, 5), (B: 6, 7, 8, 9, 10) and
(B: 1, 2, 3, 4, 5), (A: 6, 7, 8, 9, 10).

Every such grouping appears 2! times, we have to divide the answer in #8 by 2! to remove the repetitions.

So the answer is (10! / 5!5!)/2! = 126

10. Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

This is unordered sampling, without replacement. So it would be C(20, 2) = 190

Another way of thinking about this:

\# handshakes = 19 + 18 + 17 + … + 2 + 1
(note: The sum $1 + 2 + … + n = n(n+1)/2$)
So this sum is 19*20/2 = 190

11. In how many ways can 8 people be seated in a row if (a) there are no restrictions, (b) A and B must sit together (c) A and B cannot sit next to each other?

(a) 8! = 40,320   (b) 2x7! = 10080   (c) 8! – 2x7! = 30240