SDS 321

Lecture 9

Functions of Random Variables and Cumulative Distribution Functions (CDF)

Functions of Random Variables

- A function of a random variable is also a random variable. if $Y = g(X)$ then we have:

$$p_Y(y) = \sum_{\{x | g(x) = y\}} p_X(x).$$

Example

Let X be a discrete random variable with PMF given by the table below:

x	1	2	3	4
px(x)	1/6	1/6	1/6	1/2

Let Y = X - 1. What is the PMF of Y?

Let Z = 2X – 1. What is the PMF of Z?

Functions of Random Variables

Example answer

Let X be a discrete random variable with PMF given by the table below:

x	1	2	3	4
px(x)	1/6	1/6	1/6	1/2

Let Y = X - 1. What is the PMF of Y?

X	1	2	3	4
Y	0	1	2	3
$P_Y(y)$	1/6	1/6	1/6	1/2

Let Z = 2X – 1. What is the PMF of Z?

X	1	2	3	4
Z	1	3	5	7
$P_Z(z)$	1/6	1/6	1/6	1/2

Cumulative Distribution Function

The Cumulative Distribution Function (CDF) of a random variable is the function

$$F(x) = P(X \leq x)$$

The CDF gives us the probability that the variable takes a value less than or equal to x.

Property of a valid CDF: $0 \leq F(x) \leq 1$ for all x.

(1) $F(x) = P(X \leq x)$

(2) $0 \leq F(x) \leq 1$ for all x.

(3) If $x \leq y$ then $F(x) \leq F(y)$.

Cumulative Distribution Function

Example: Obtaining the CDF from the PMF

Suppose the range of a discrete random variable is {0, 1, 2, 3, 4} and its probability mass function is P(X = x) = x/10.
What is its cumulative distribution function?

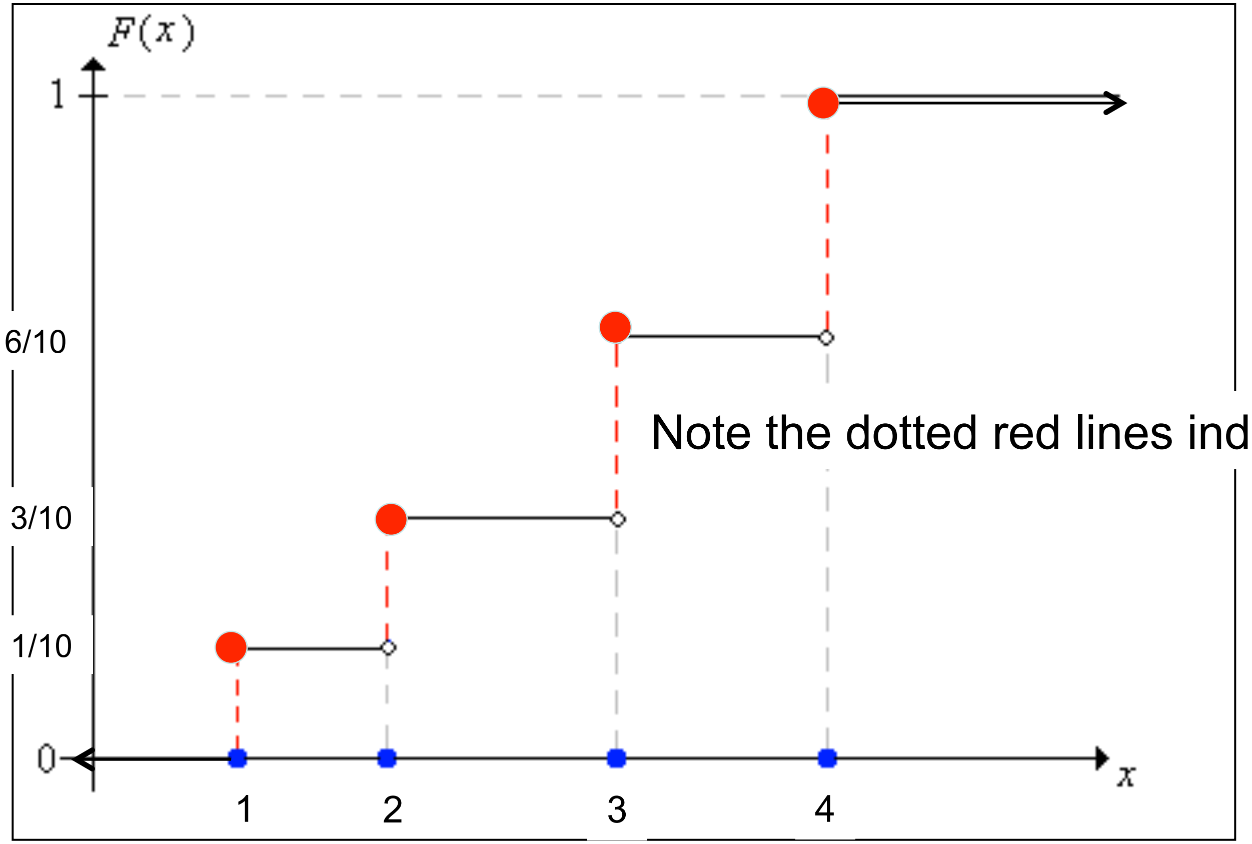
For x < 1, F(x) = $\sum_{x_i \le 0} P(X = x_i)$ = P(X = 0) = 0

For (1 ≤ x < 2), F(x) = $\sum_{x_i \le 1} P(X = x_i)$ = P(X=0) + P(X=1) = 1/10

For (2 ≤ x < 3), F(x) = P(X=0) + P(X=1) + P(X=2) = 3/10, and so on, to get:

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{10} & 1 \le x < 2 \\ \frac{3}{10} & 2 \le x < 3 \\ \frac{6}{10} & 3 \le x < 4 \\ 1 & 4 \le x. \end{cases}$$

Cumulative Distribution Function



Note the dotted red lines indicate P(X=x)

Cumulative Distribution Function

Example: Obtaining the CDF from the PMF

Let X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} 0.3 & \text{for } x = 3 \\ 0.2 & \text{for } x = 5 \\ 0.3 & \text{for } x = 8 \\ 0.2 & \text{for } x = 10 \\ 0 & \text{otherwise} \end{cases}$$

Find and plot the CDF of X.

Cumulative Distribution Function

Example: Obtaining PMF from CDF

A discrete random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{10} & 0 \leq x < 1 \\ \frac{3}{10} & 1 \leq x < 2 \\ \frac{5}{10} & 2 \leq x < 4 \\ \frac{8}{10} & 4 \leq x < 5 \\ 1 & 5 \leq x. \end{cases}$$

Determine the probability mass function of X

Cumulative Distribution Function

The cumulative distribution function only changes value at 0,1,2,4,5. So the range of X is {0,1,2,4,5}.

F(X=0) = 1/10, so P(X=0) = 1/10

F(X=1) = P(X=0) + P(X=1) = 3/10, so P(X=1) = 2/10

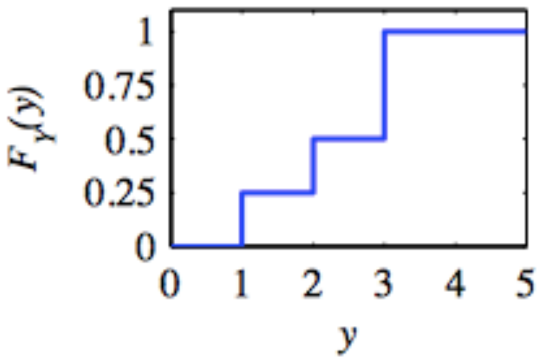
and so on, to get:

x	0	1	2	4	5
P(X=x)	1/10	2/10	2/10	3/10	2/10

Cumulative Distribution Function

Example: Obtaining PMF from CDF

Discrete random variable Y has the CDF $F_Y(y)$ as shown:



Find:

(a) $P[Y < 1]$

(b) $P[Y \leq 1]$

(c) $P[Y > 2]$

(d) $P[Y \geq 2]$

(e) $P[Y = 1]$

(f) $P[Y = 3]$

(g) $P_Y(y)$