SDS 321 Worksheet 8 (Functions of Random Variables)

1. Consider a random variable X such that

   $p_X(x) = x^2/a$ for x ∈ {−2,−1, 1, 2}

        0   otherwise, where a > 0 is a real parameter.

   (a) Find a.

   (b) What is the PMF for the random variable Y=2X+4?

   (c) What is the PMF for the random variable $Z=X^2$?

2. Suppose a fair coin is tossed 5 times. Determine the PMF for the random variable X that counts the number of heads. What is the PMF of Y=2X?

3. Let X be the number of heads in 5 fair coin tosses. We know that X ~ Bin(5, 0.5). Define Y = X mod 4. What is the PMF of Y?

4. The random variable V has PMF

   $$P_V(v) = \begin{cases} cv^2 & v = 1, 2, 3, 4, \\ 0 & \text{otherwise.} \end{cases}$$

   a) Find the value of the constant c.

   b) What is the probability that V is an even number?

   c) If W = 2V, find P(W < 5).

5.

   Suppose X is a discrete random variable with PMF :

   $P(X = x) = \dfrac{5-x}{10}$; $x \in \{1, 2, 3, 4\}$

   a) Draw the PMF of X

   b) find the CDF of X

   c) Draw the CDF of X