Lecture 2b: Geometric Modeling and Visualization

BEM/FEM Domain Models

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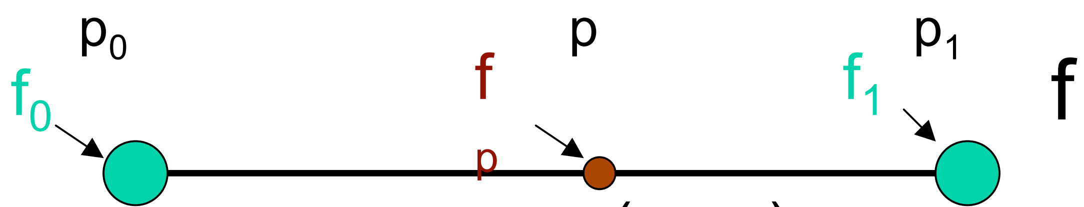
Linear Interpolation on a line segment



The Barycentric coordinates $\alpha = (\alpha_0\ \alpha_1)$ for any point p on line segment $<p_0\ p_1>$, are given by

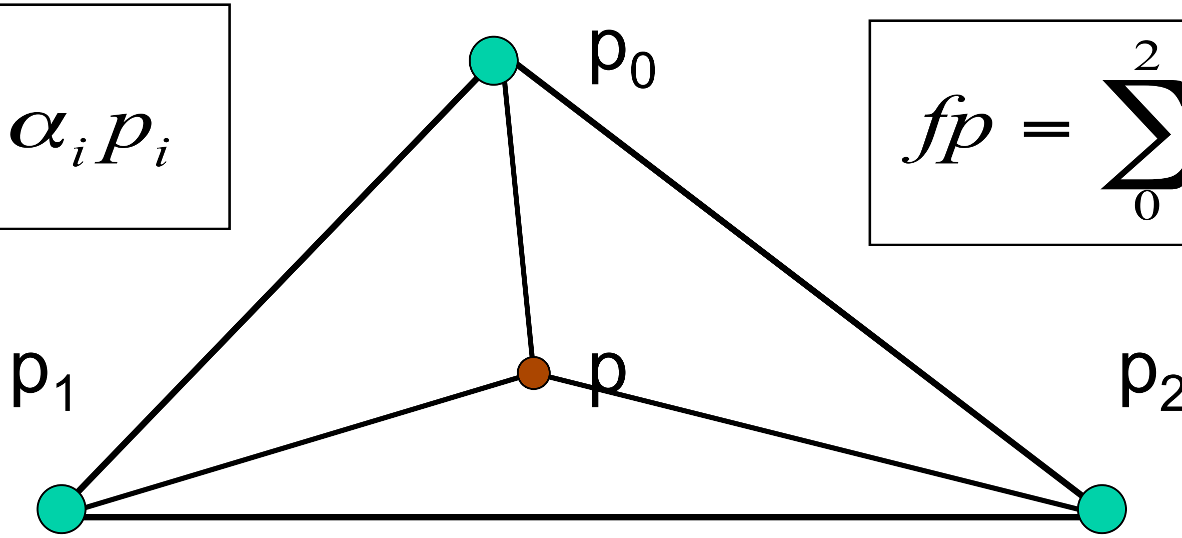
$$\alpha = \left(\frac{dist(p, p_1)}{dist(p_0, p_1)}, \frac{dist(p_0, p)}{dist(p_0, p_1)}\right)$$

which yields $p = \alpha_0\ p_0 + \alpha_1\ p_1$

and $f_p = \alpha_0\ f_0 + \alpha_1\ f_1$

Linear interpolation over a triangle

$$p = \sum_{0}^{2} \alpha_i p_i$$

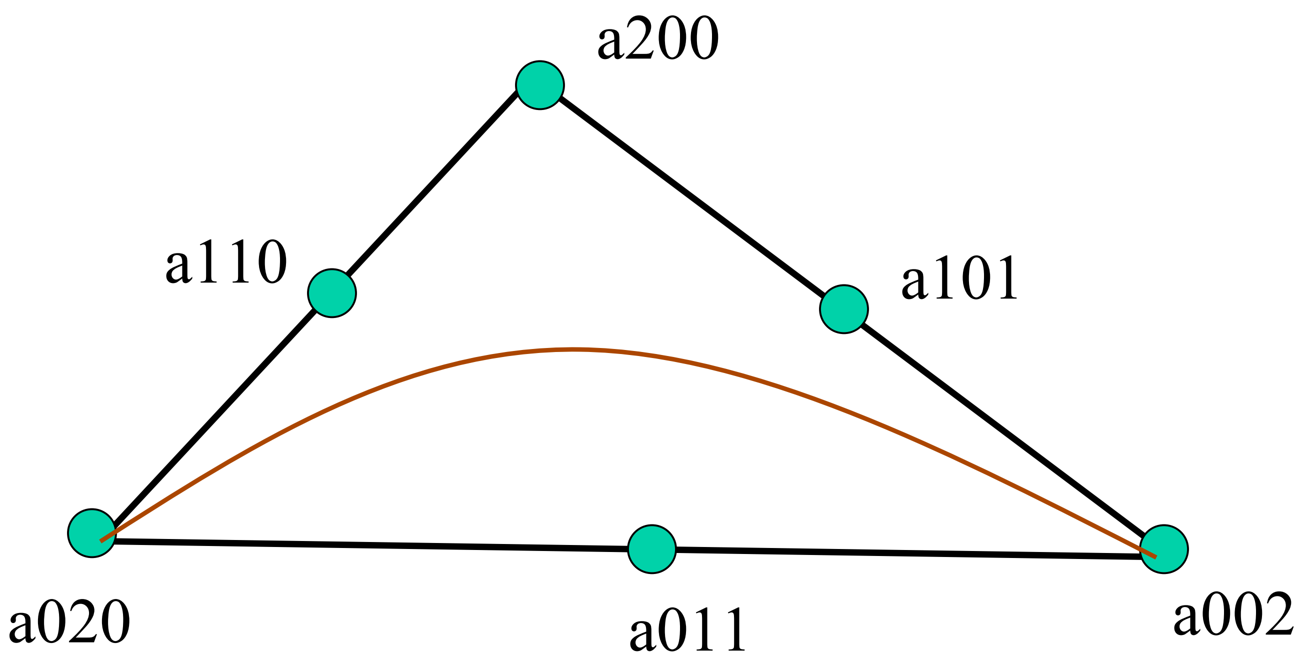


$$fp = \sum_{0}^{2} \alpha_i fp_i$$

For a triangle p_0, p_1, p_2, the Barycentric coordinates $\alpha = (\alpha_0\ \alpha_1\ \alpha_2)$ for point p,

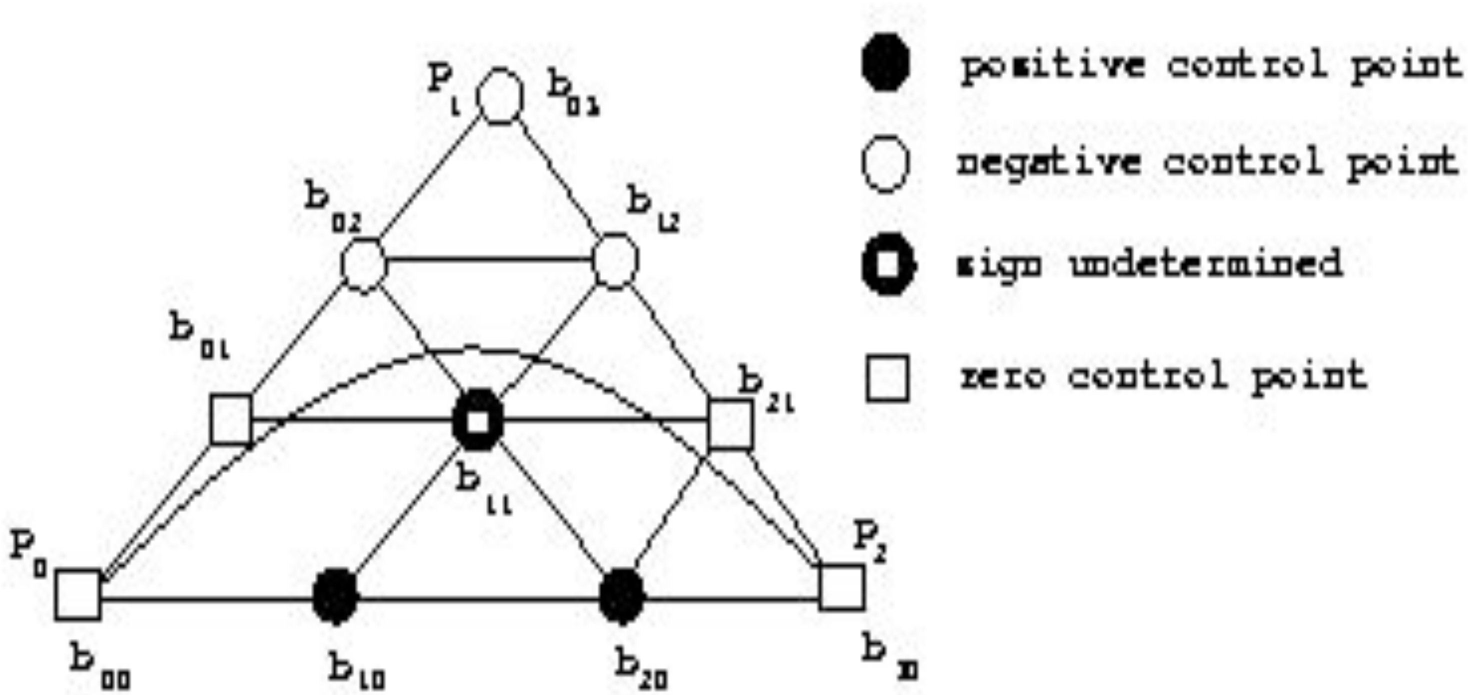
$$\alpha = \left(\frac{area(p, p_1, p_2)}{area(p_0, p_1, p_2)}, \frac{area(p_0, p, p_2)}{area(p_0, p_1, p_2)}, \frac{area(p_0, p_1, p)}{area(p_0, p_1, p_2)}\right)$$

Non-Linear Algebraic Curve and Surface Finite Elements ?

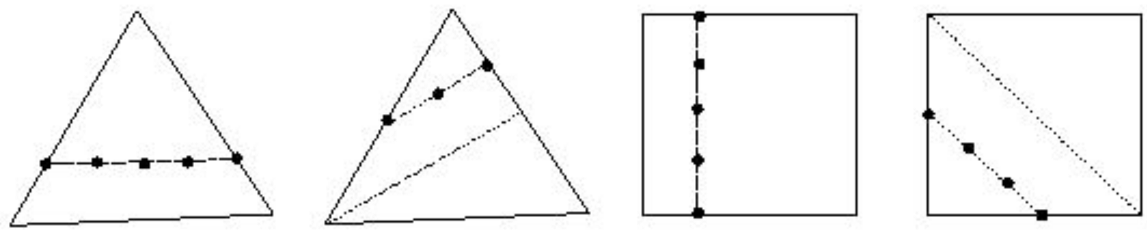


The conic curve interpolant is the zero of the bivariate quadratic polynomial interpolant over the triangle

A-spline segment over BB basis



Regular A-spline Segments



For a given discriminating family $D(R, R_1, R_2)$, let $f(x, y)$ be a bivariate polynomial . If the curve $f(x, y) = 0$ intersects with each curve in $D(R, R_1, R_2)$ only once in the interior of R, we say the curve $f = 0$ is regular(or A-spline segment) with respect to $D(R, R_1, R_2)$.

If $B_0(s)$, $B_1(s)$, … has one sign change, then the curve is

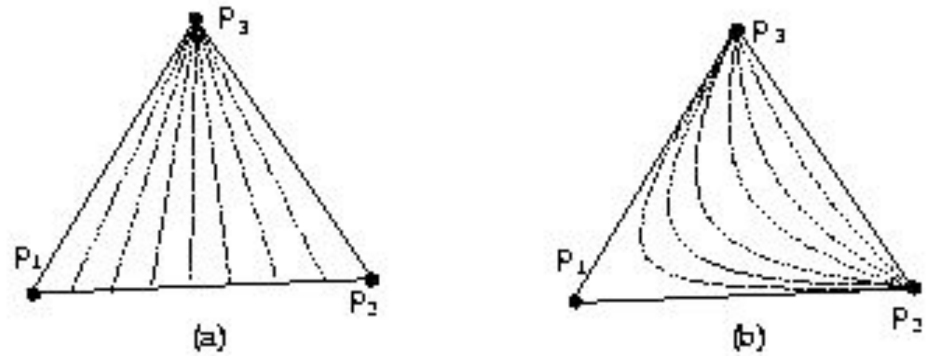
(a) D_1 - regular curve.

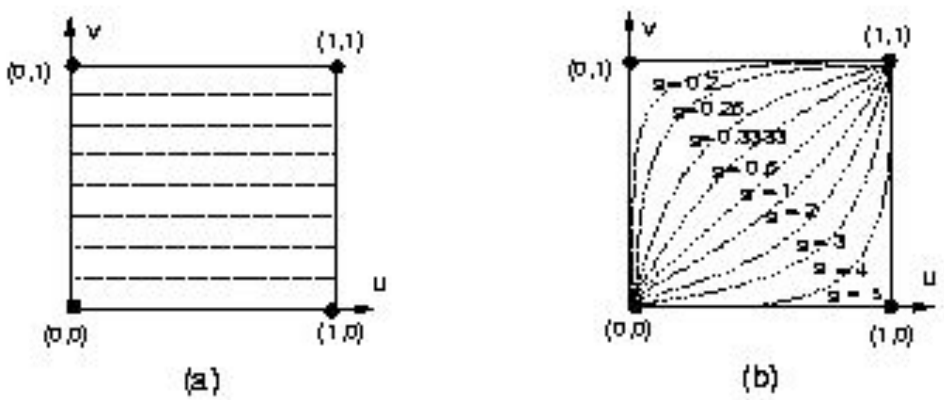
(b) D_2 - regular curve.

(c) D_3 - regular curve.

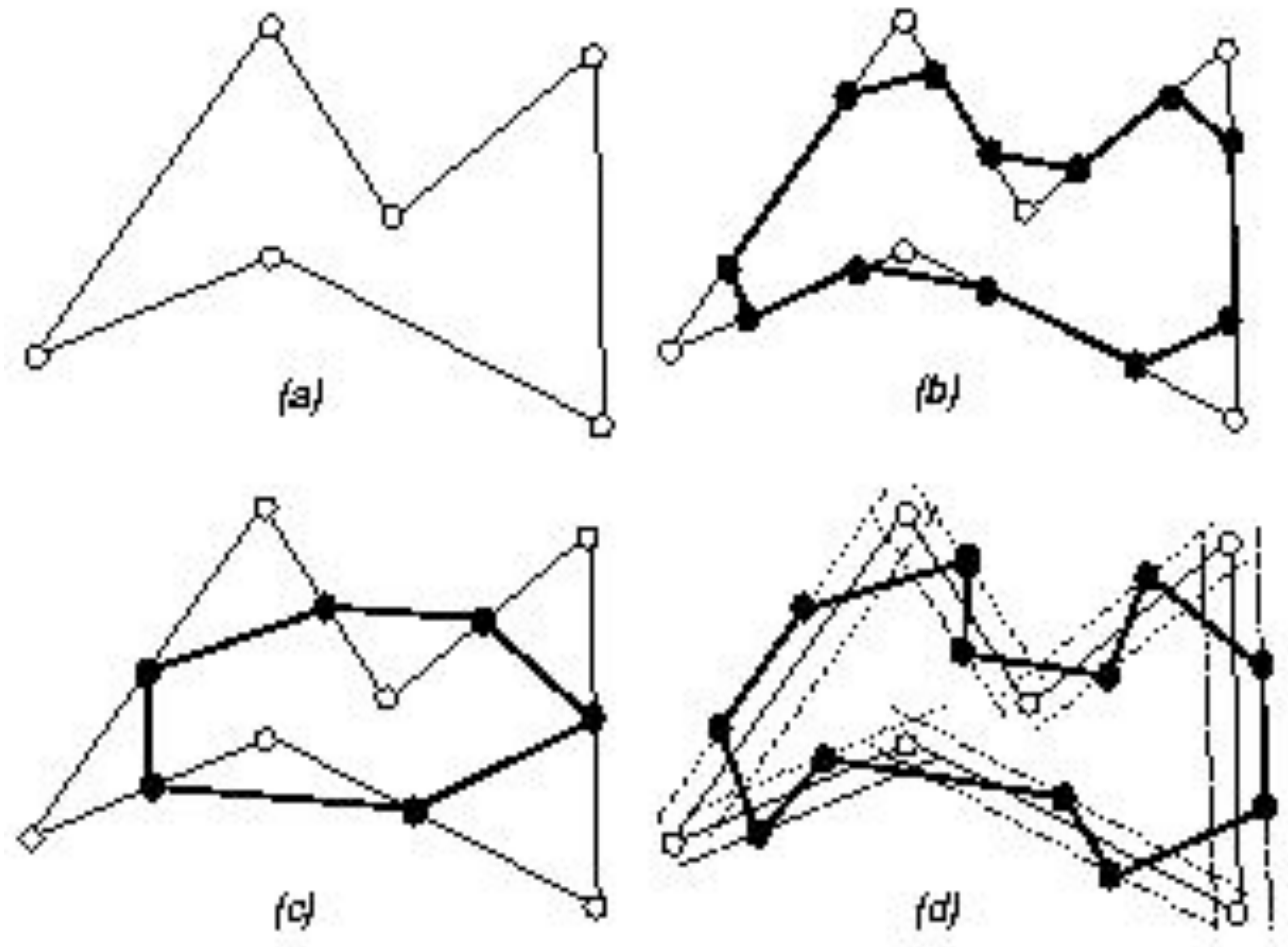
(d) D_4 - regular curve.

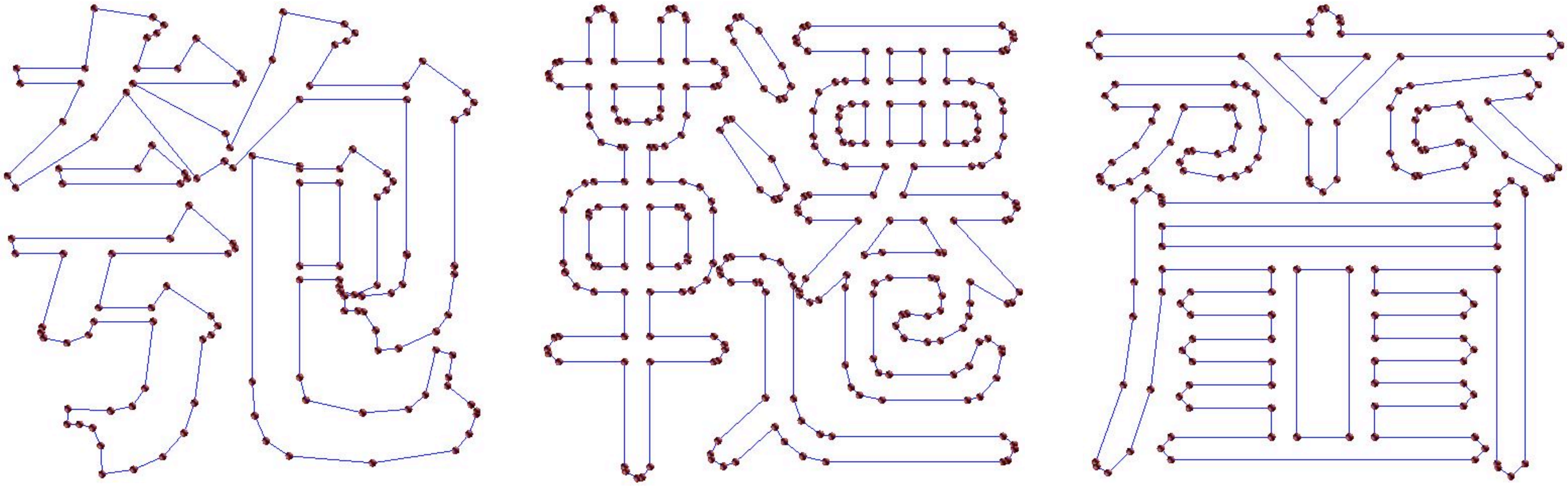
Examples of Discriminating Curve Families

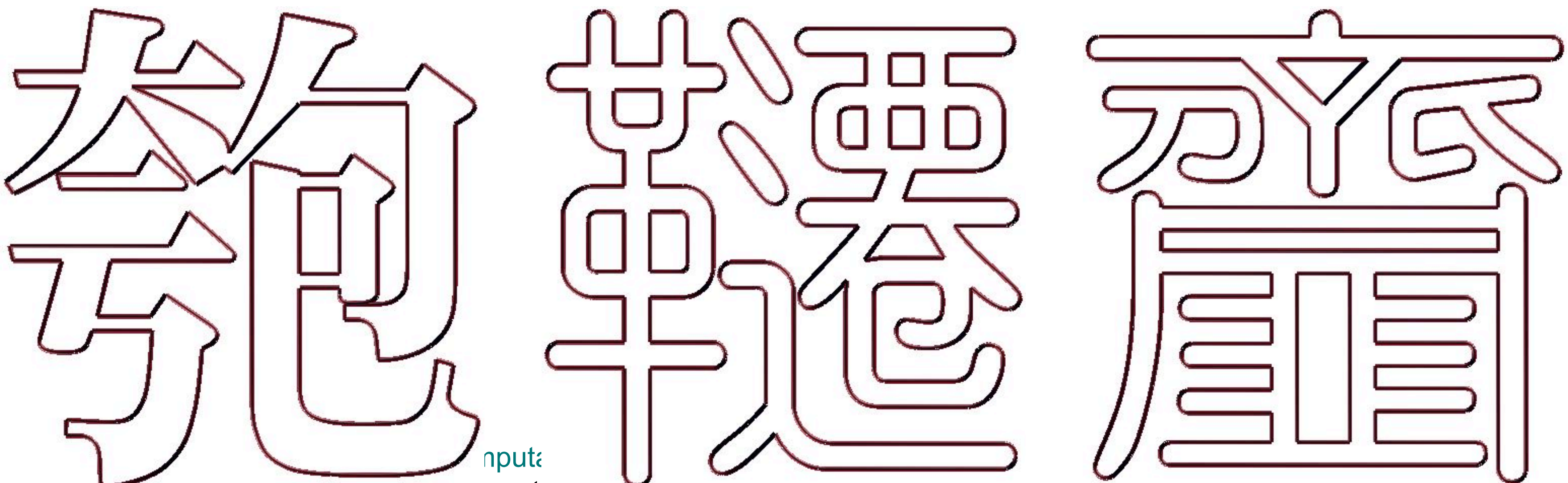




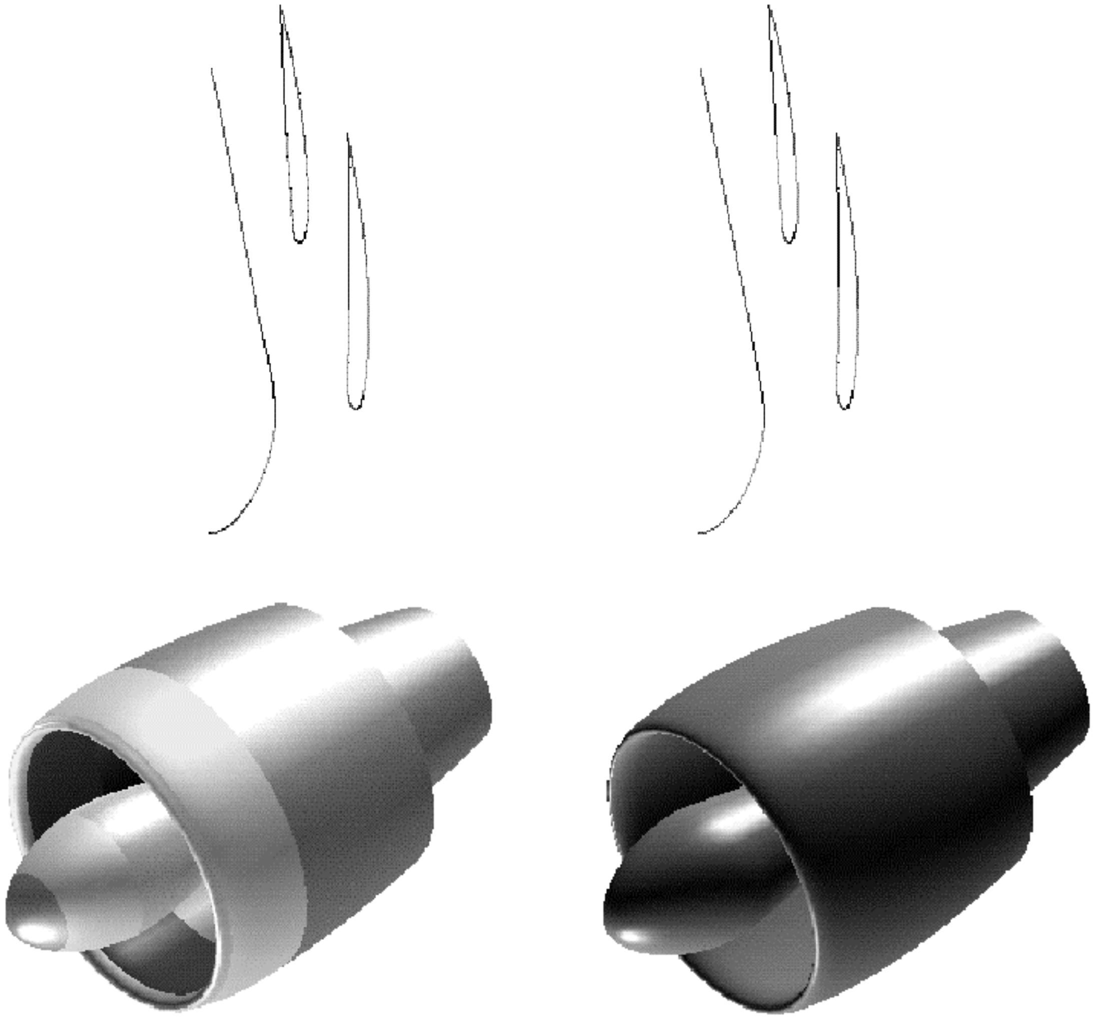
Constructing Scaffolds



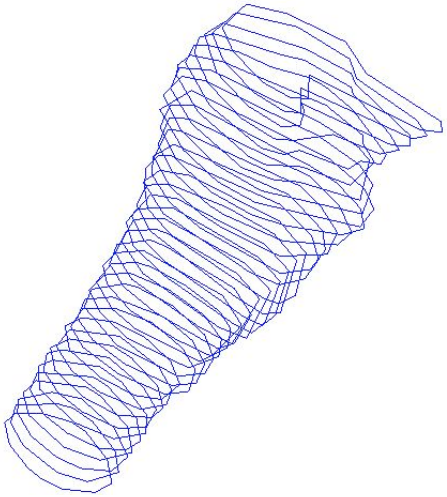


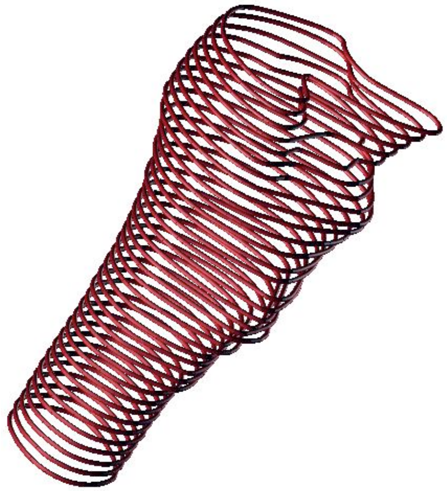


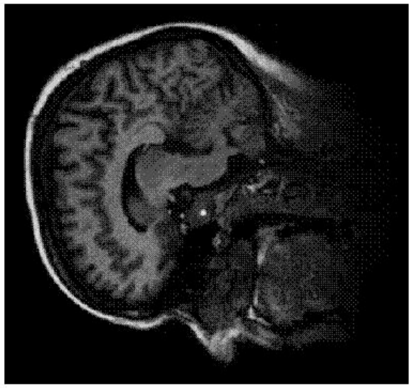
Spline Surfaces of Revolution

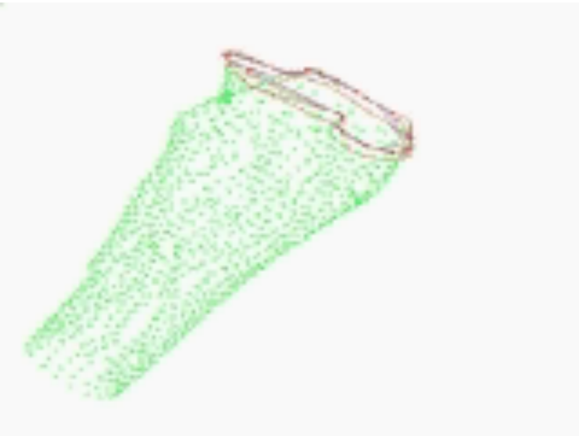


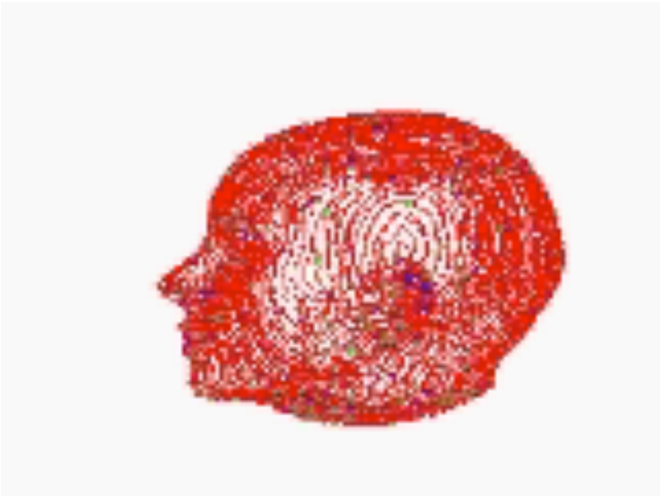
Lofting III : Non-Linear Boundary Elements











Linear interpolant over a tetrahedron

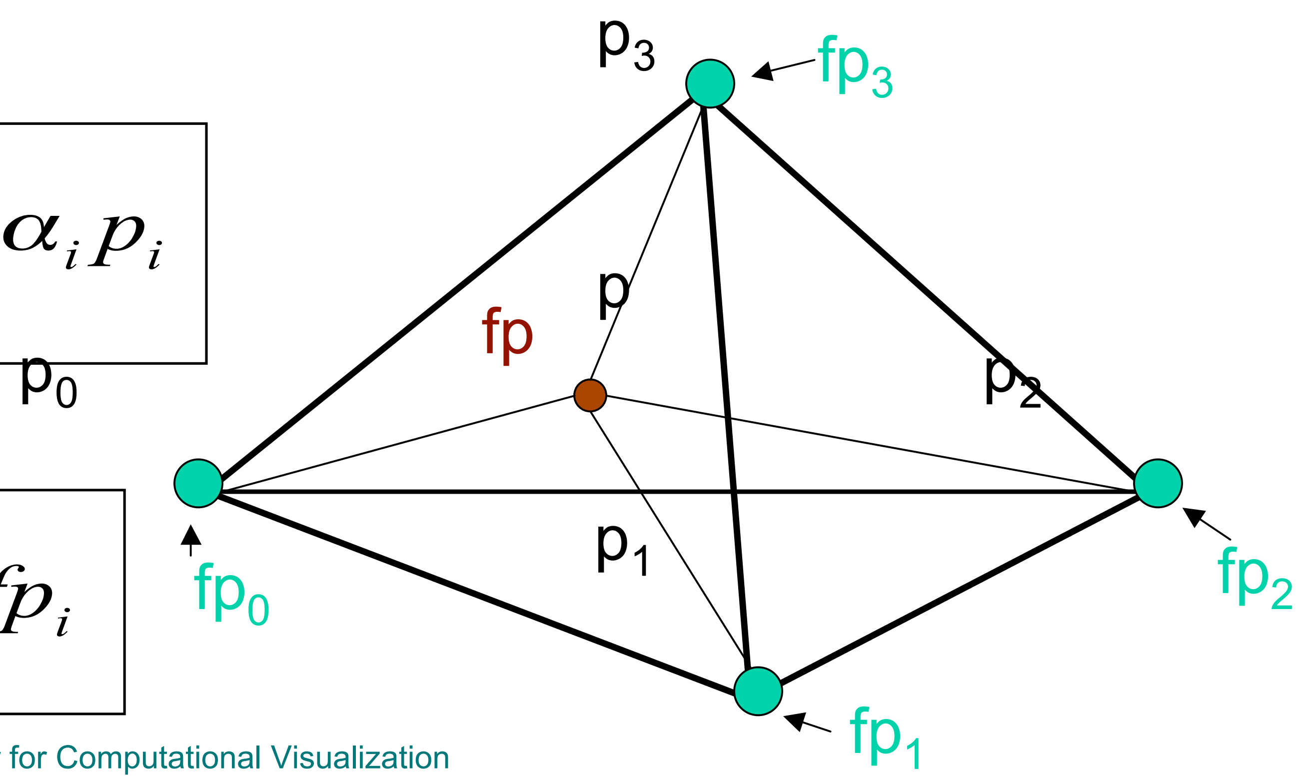
Linear Interpolation within a

- Tetrahedron (p_0,p_1,p_2,p_3)

  $\alpha = \alpha_i$ are the barycentric coordinates of p

$$p = \sum_{0}^{3} \alpha_i p_i$$

$$fp = \sum_{0}^{3} \alpha_i fp_i$$

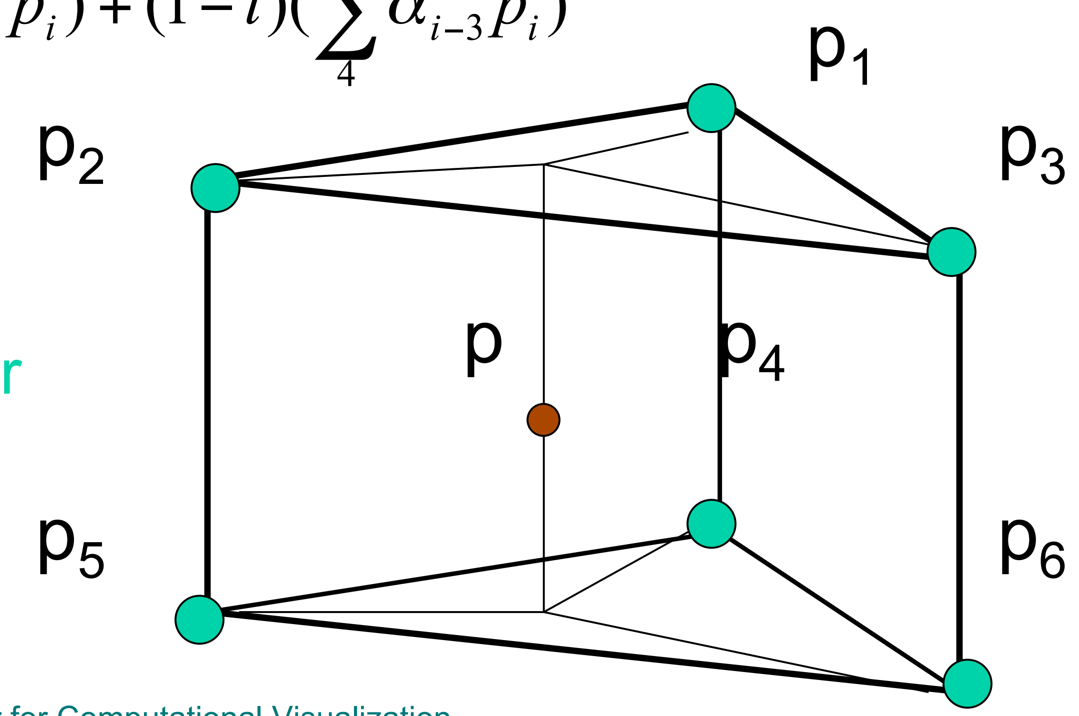


Other 3D Finite Elements (contd)

- Unit Prism (p_1,p_2,p_3,p_4,p_5,p_6)

$$p = t\left(\sum_{1}^{3} \alpha_i p_i\right) + (1-t)\left(\sum_{4}^{6} \alpha_{i-3} p_i\right)$$

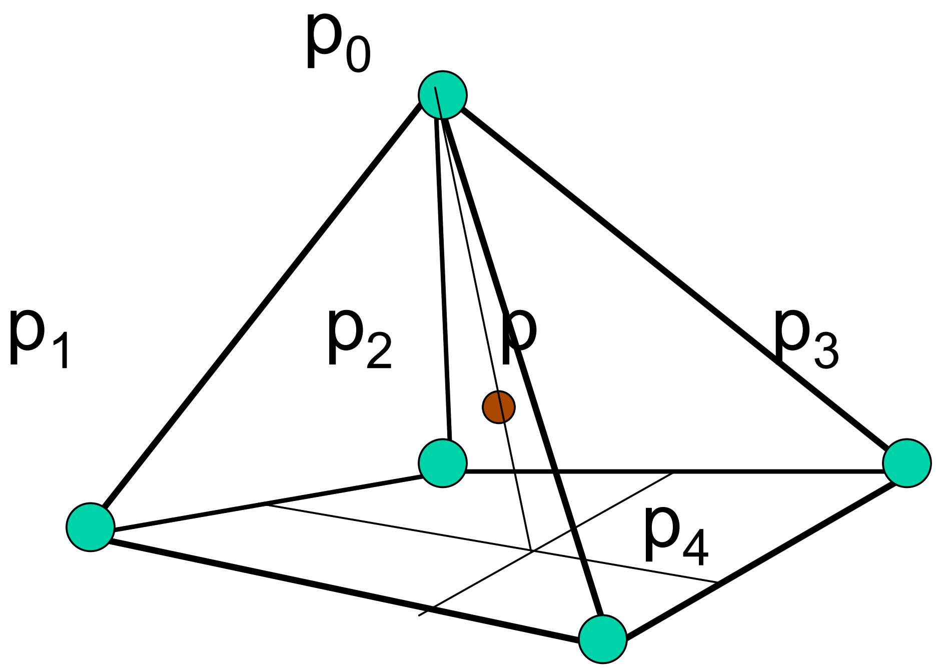
Note: nonlinear



Other 3D Finite elements

- Unit Pyramid (p_0,p_1,p_2,p_3,p_4)

$$p = up_0 + (1-u)(t(sp_1 + (1-s)p_2) + (1-t)(sp_3 + (1-s)p_4))$$



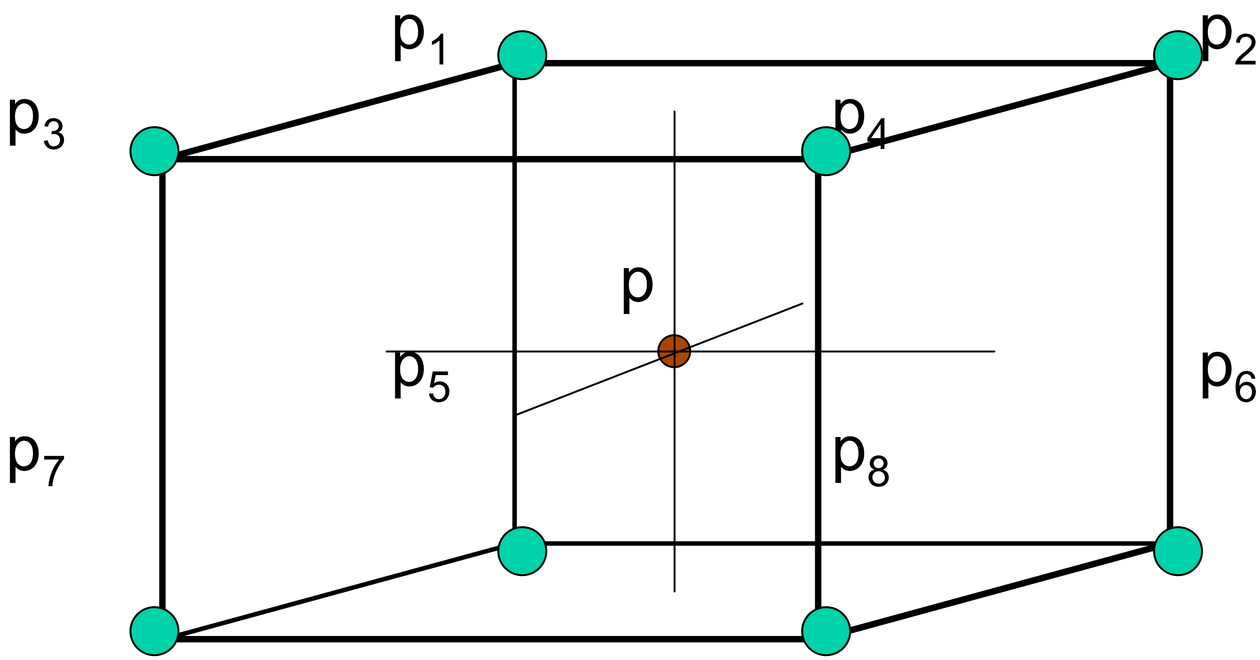
Note:
nonlinear

Other 3D Finite Elements

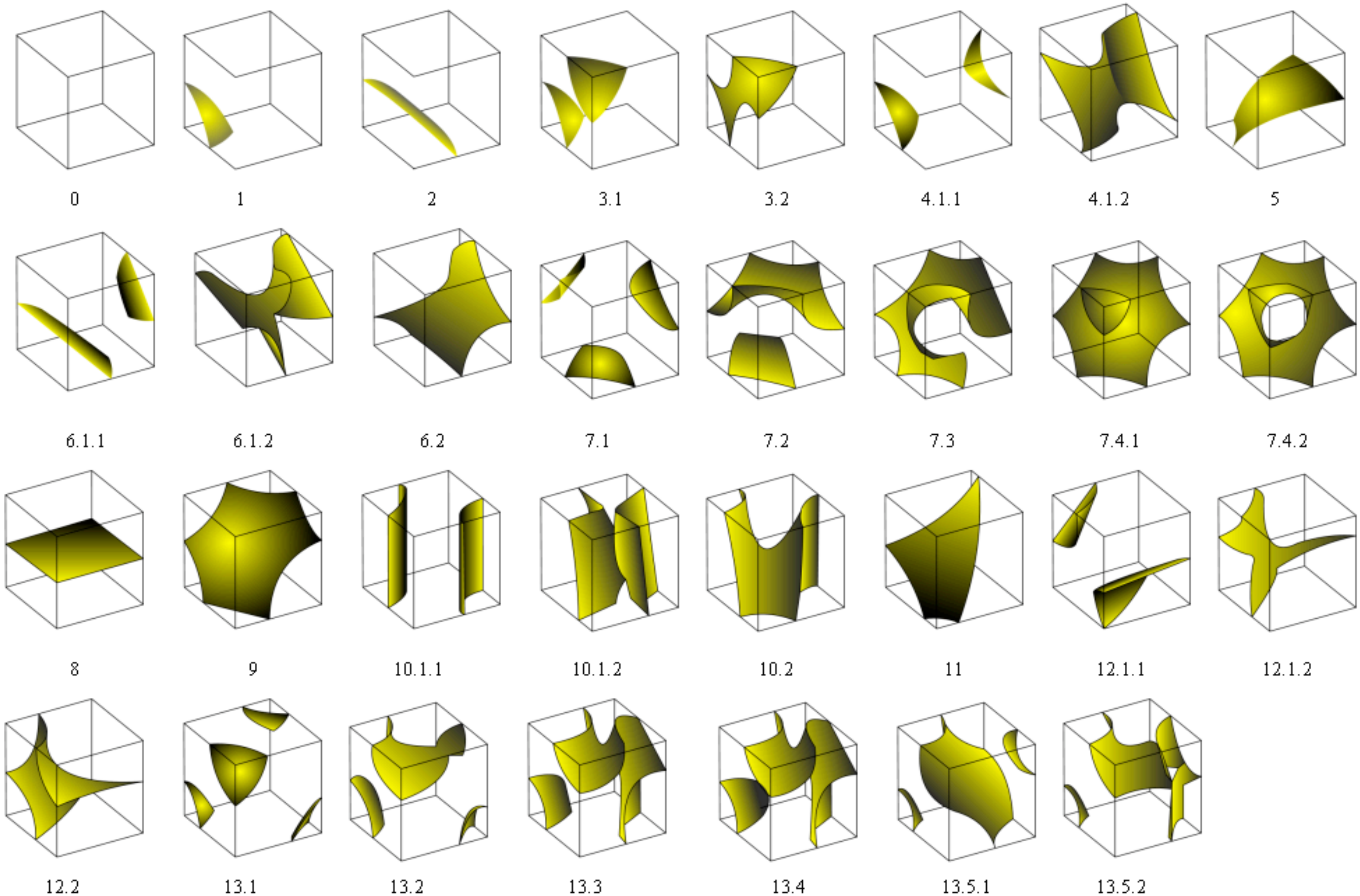
- Unit Cube ($p_1,p_2,p_3,p_4,p_5,p_6,p_7,p_8$)
  - Tensor in all 3 dimensions

$$p = u(t(sp_1 + (1-s)p_2) + (1-t)(sp_3 + (1-s)p_4)) +$$
$$(1-u)(t(sp_5 + (1-s)p_6) + (1-t)(sp_7 + (1-s)p_8))$$

Trilinear interpolant

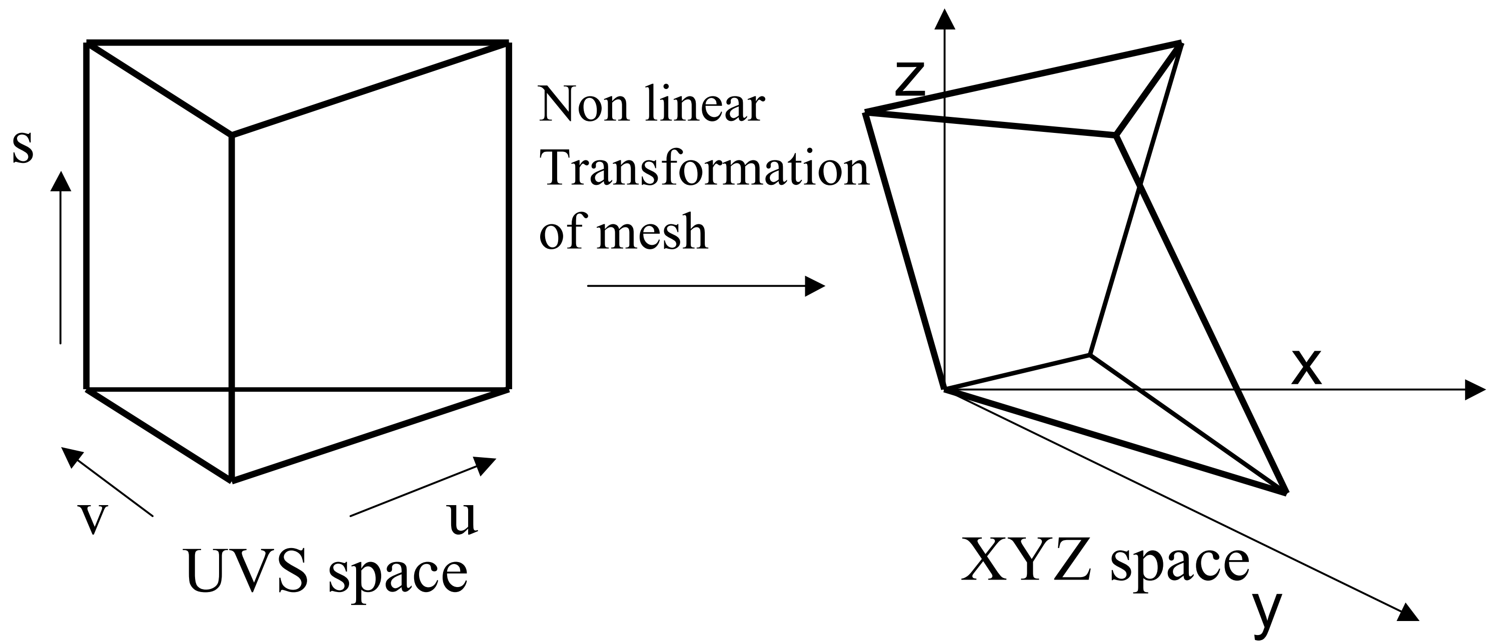


Topology of Zero-Sets of a Tri-linear Function



Non-linear finite elements-3d

- Irregular prism
  - Irregular prisms may be used to represent data.



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = ((1-u-v)\overrightarrow{\mathbf{p}_0} + u\overrightarrow{\mathbf{p}_1} + v\overrightarrow{\mathbf{p}_2})(1-s) + ((1-u-v)\overrightarrow{\mathbf{p}_3} + u\overrightarrow{\mathbf{p}_4} + v\overrightarrow{\mathbf{p}_5})(s)$$

C^1 Interpolant

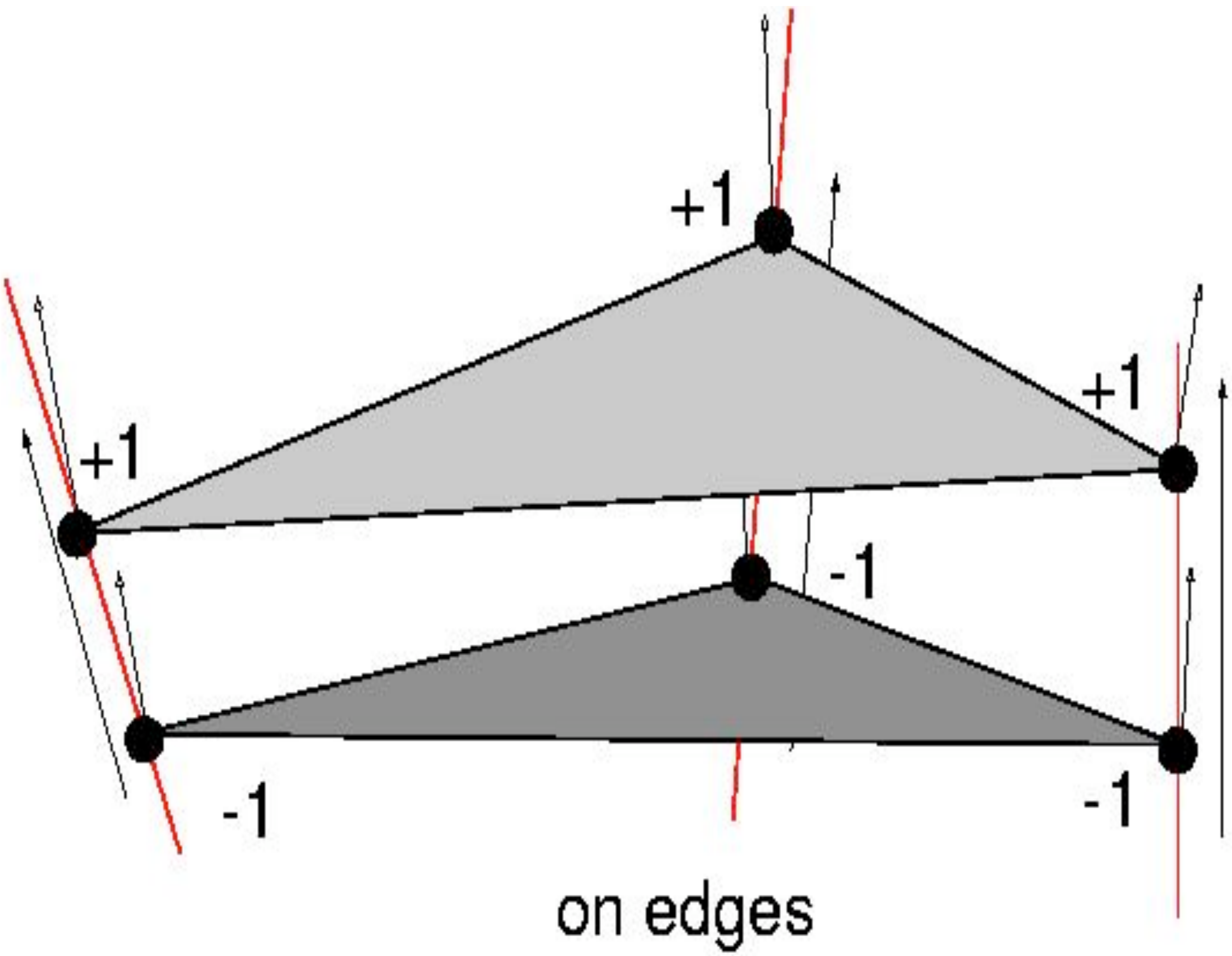
Hermite interpolation

$$f(t) = f_0 H_0^3(t) + f_0^0 H_1^3(t) + f_1 H_2^3(t) + f_1^0 H_3^3(t)$$

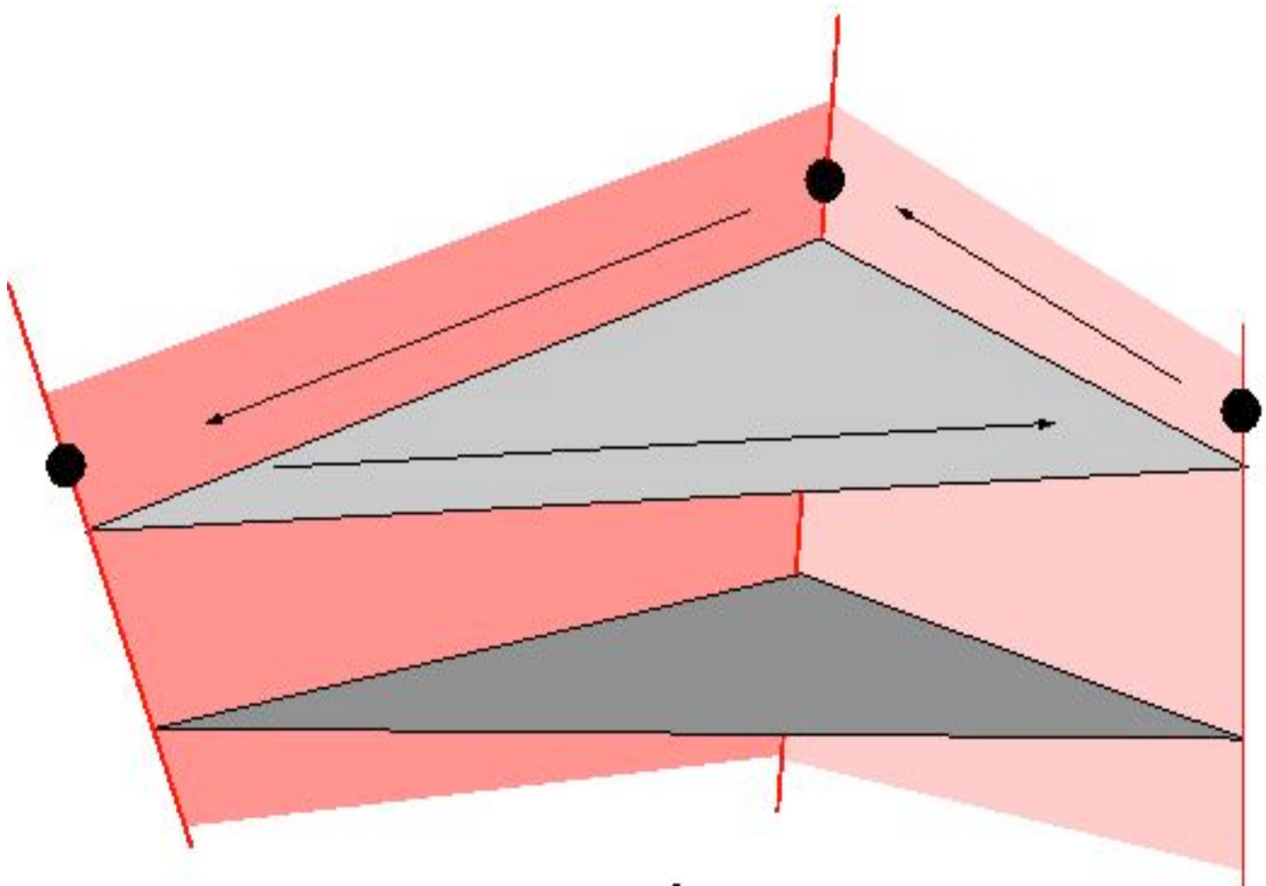
Incremental Basis Construction

- Define functions and gradients on the edges of a prism
- Define functions and gradients on the faces of a prism
- Define functions on a volume
- Blending

Hermite Interpolant on Prism Edges

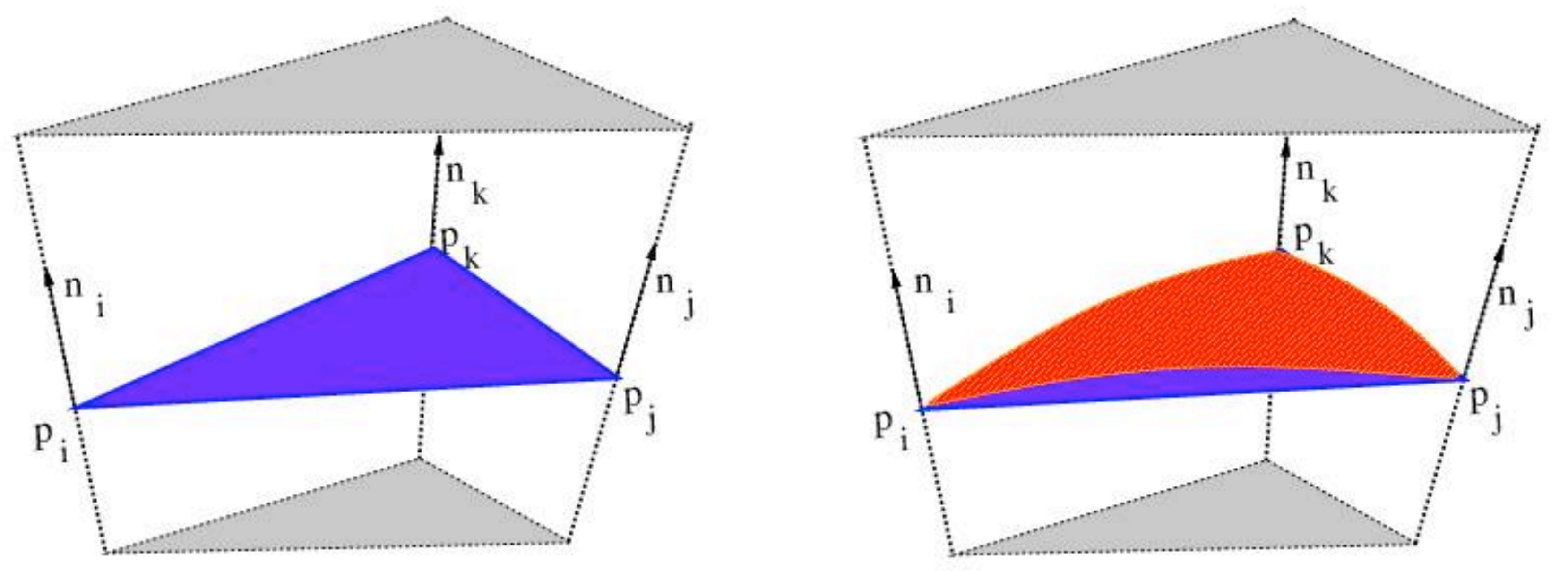


Hermite Interpolation on Prism Faces



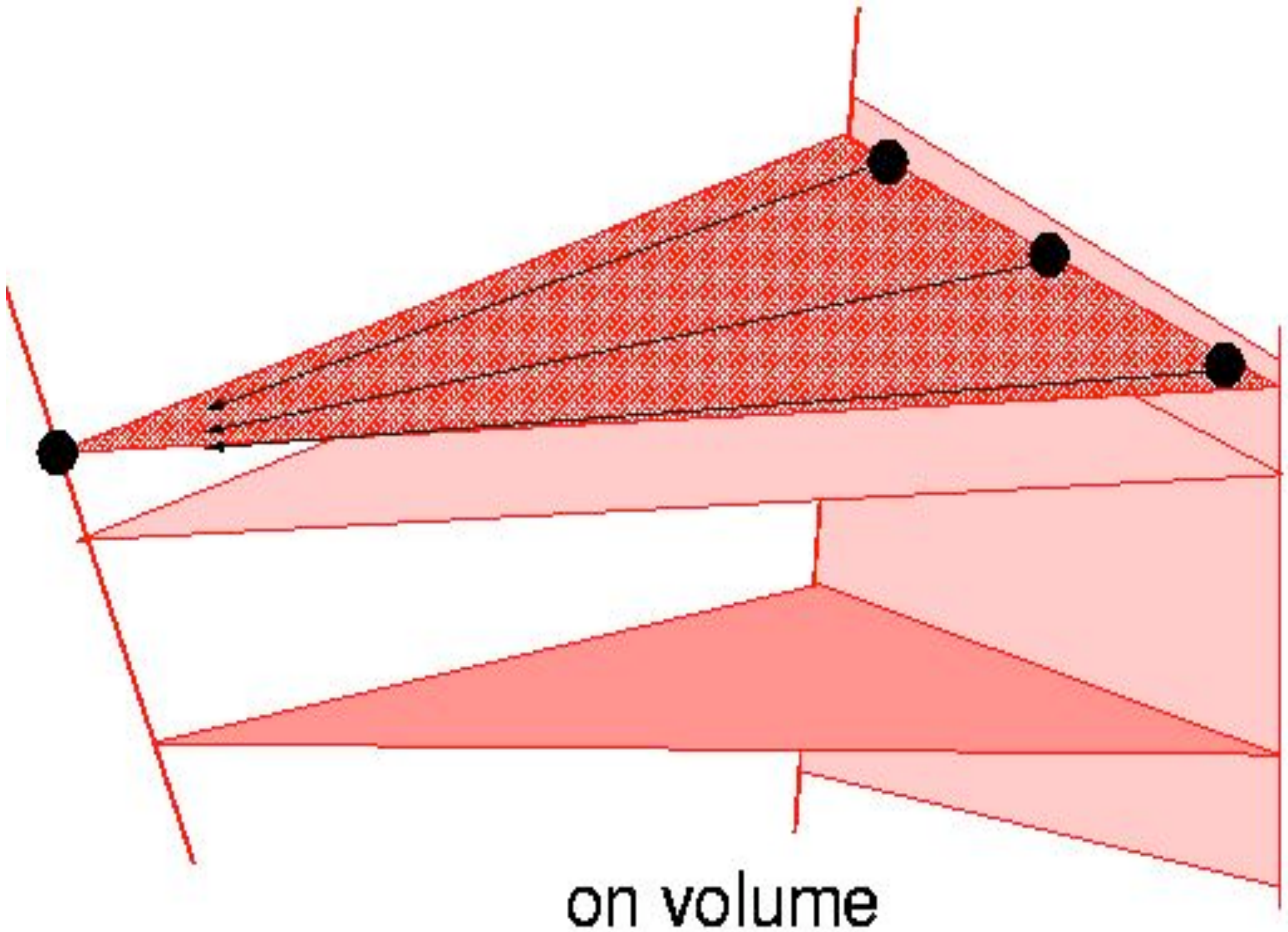
on faces

Shell Elements (contd)

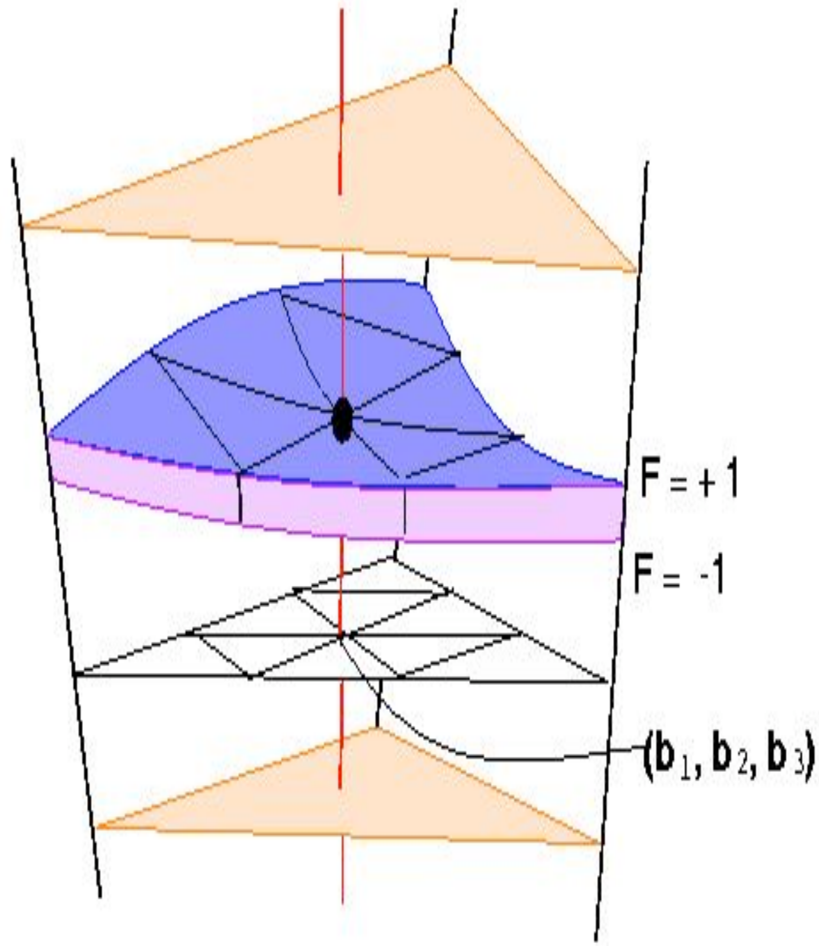


- The function F is C^1 over $\sum$ and interpolates C^1 (Hermite) data
- The interpolant has quadratic precision

Side Vertex Interpolation

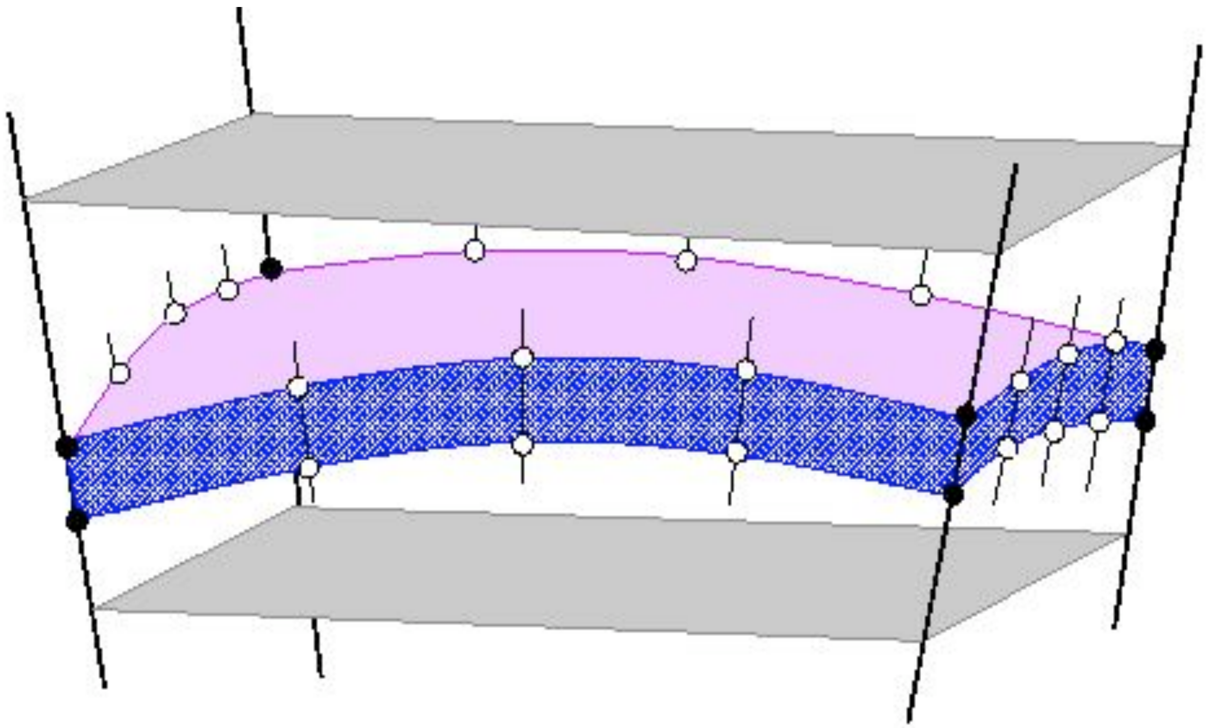


C^1 Shell Elements

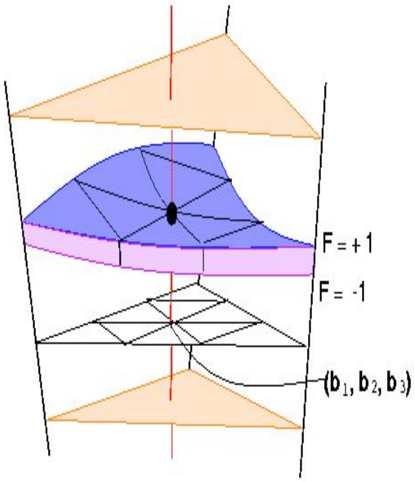


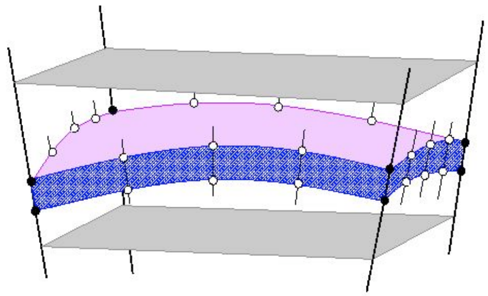
C^1 Shell Elements within a Cube

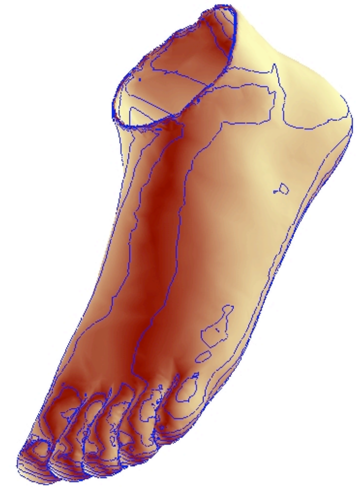
C^1 Quad Shell Surfaces can be built in a similar way, by defining functions over a cube

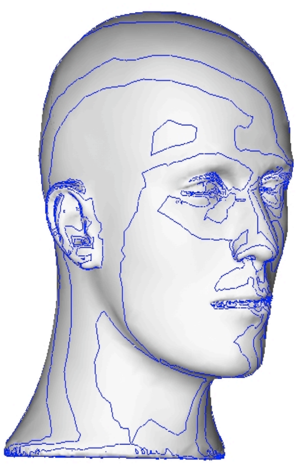


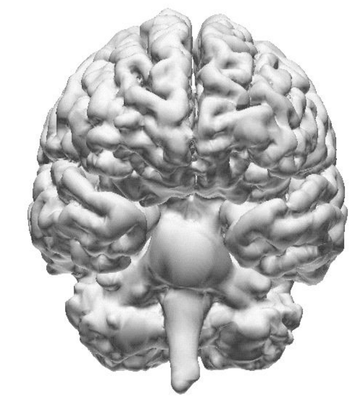
Shell Finite Element Models

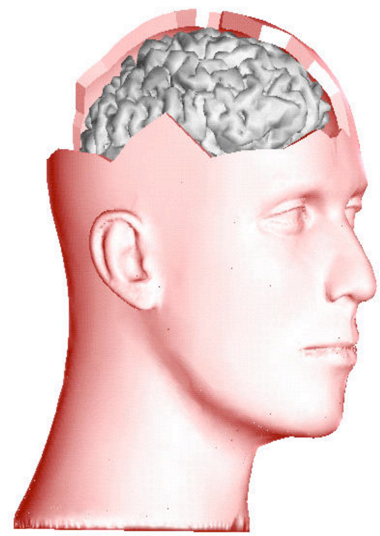












Also see my algebraic curve/surface spline lectures 7 and 8 from

http://www.cs.utexas.edu/~bajaj/graphics07/cs354/syllabus.html