Lecture 5: Geometric Modeling and Visualization

Cellular Structure Models from Thin Section EM

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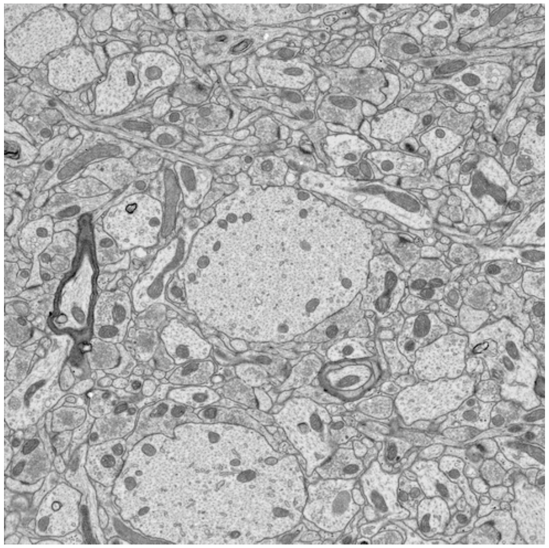
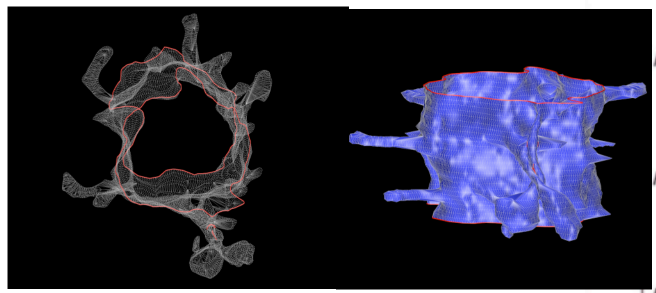
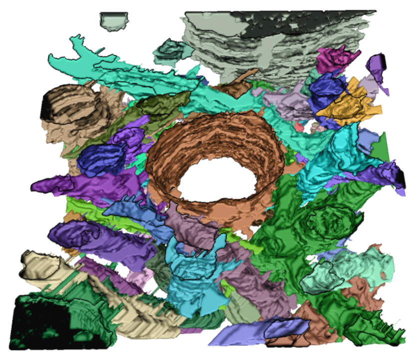
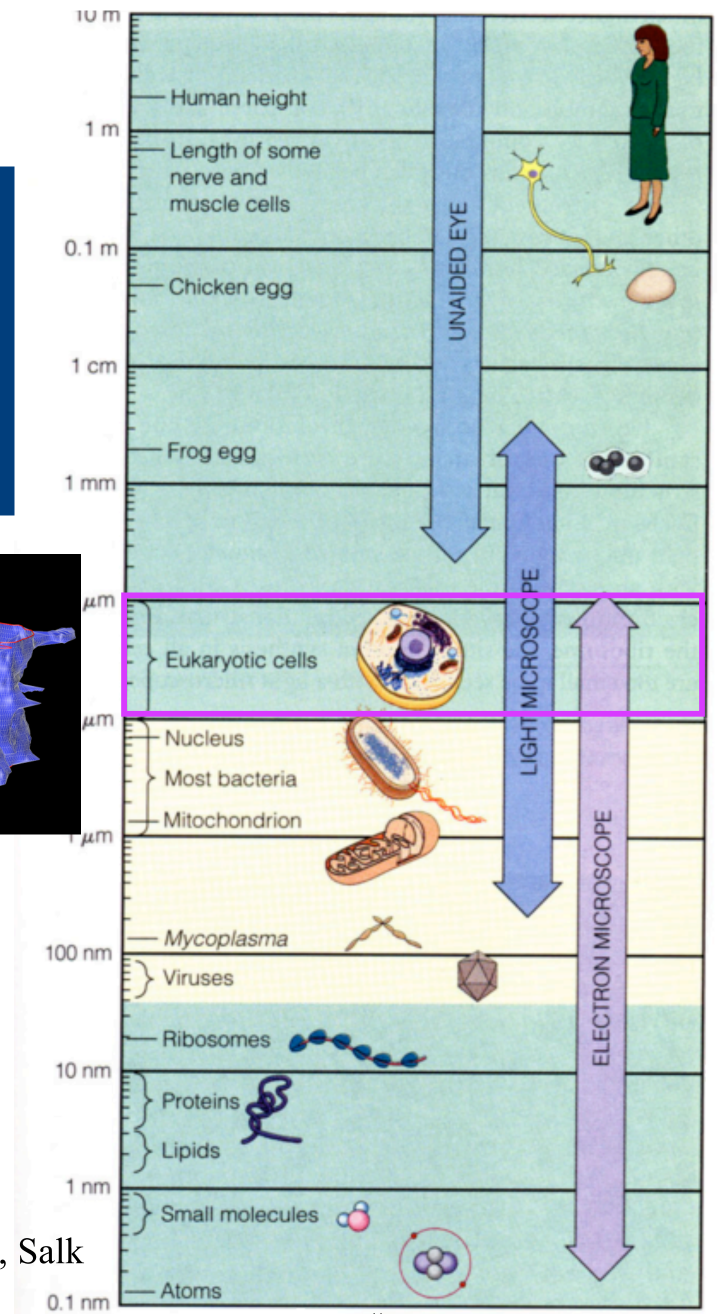
Cell Machinery of Life









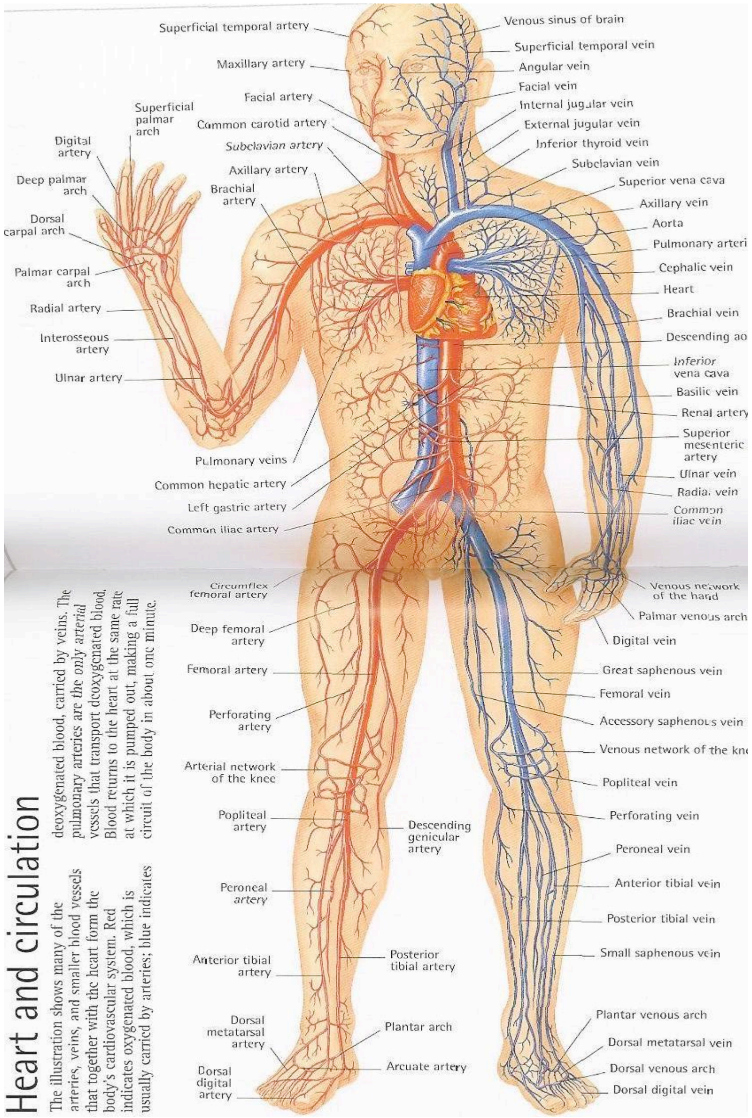
"The World of the Cell", 1996)

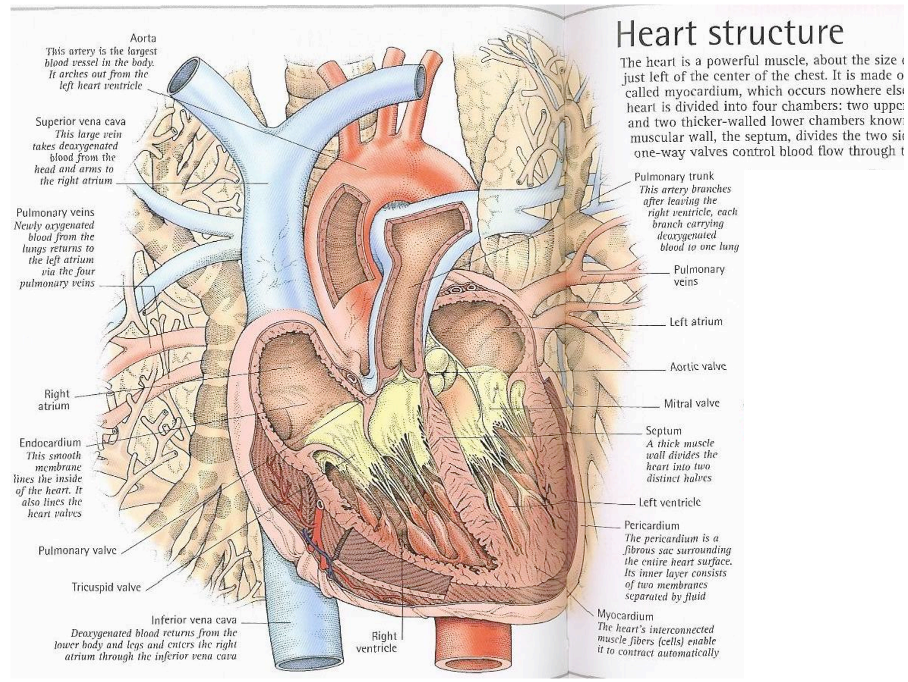
Transmission Electron Microscopy, Thin Sections:

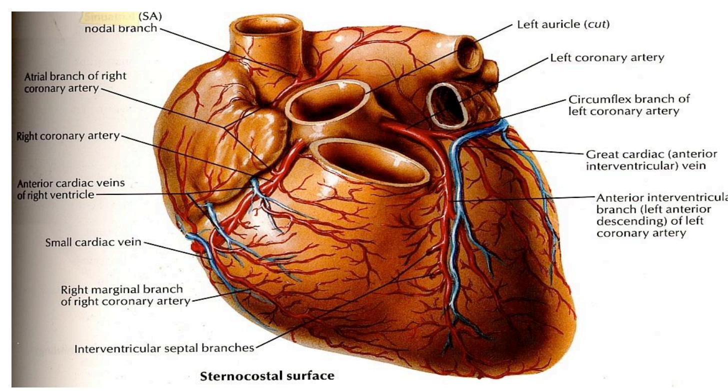
Data Courtesy: Kristen Harris, University of Texas at Austin

Addtl. Collab: Tom Bartol, Justin Kinney, Terry Sejnowski, Salk

Cardiovascular Anatomy







F. Netter's anatomical charts

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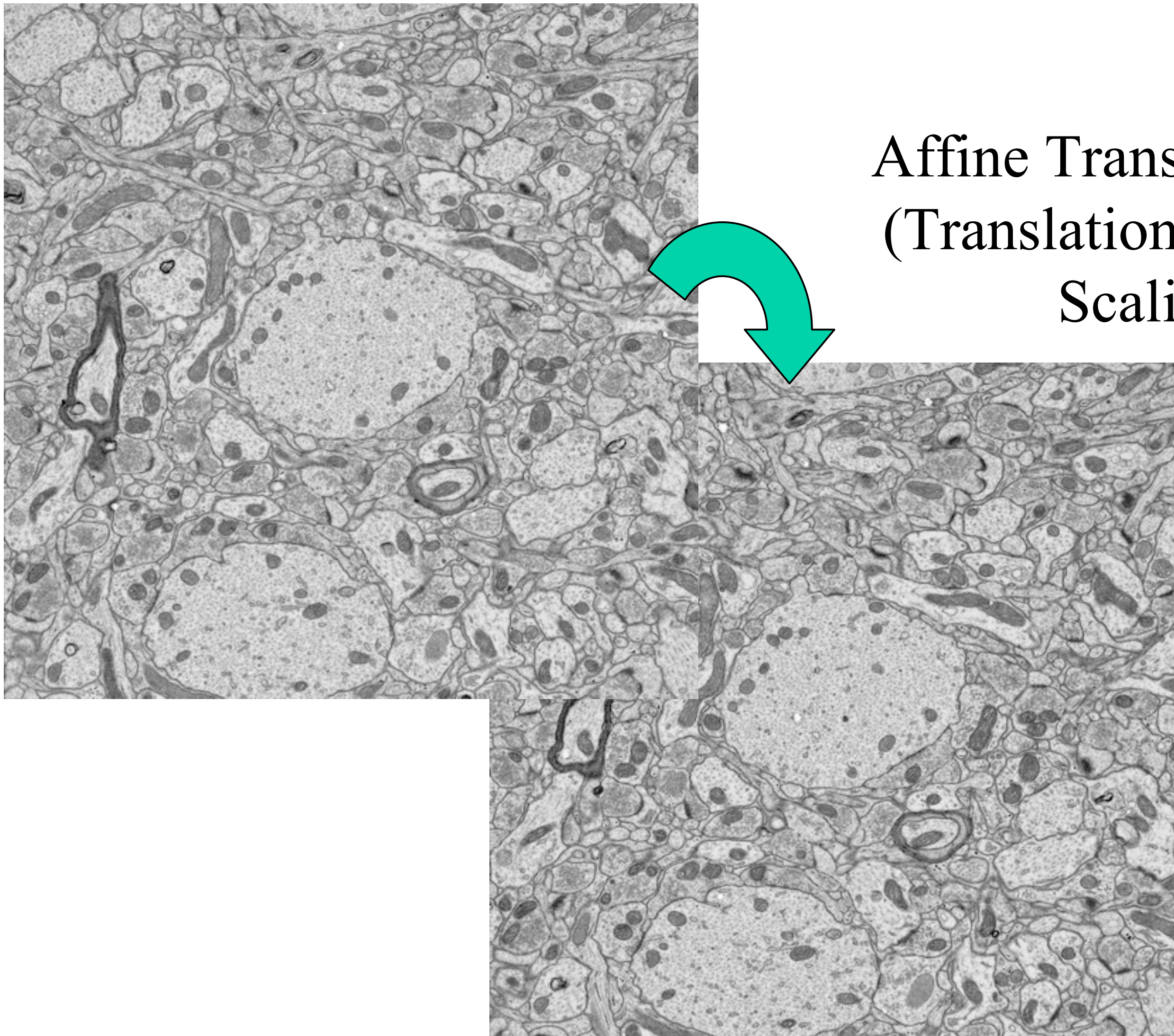
Imaging2Models

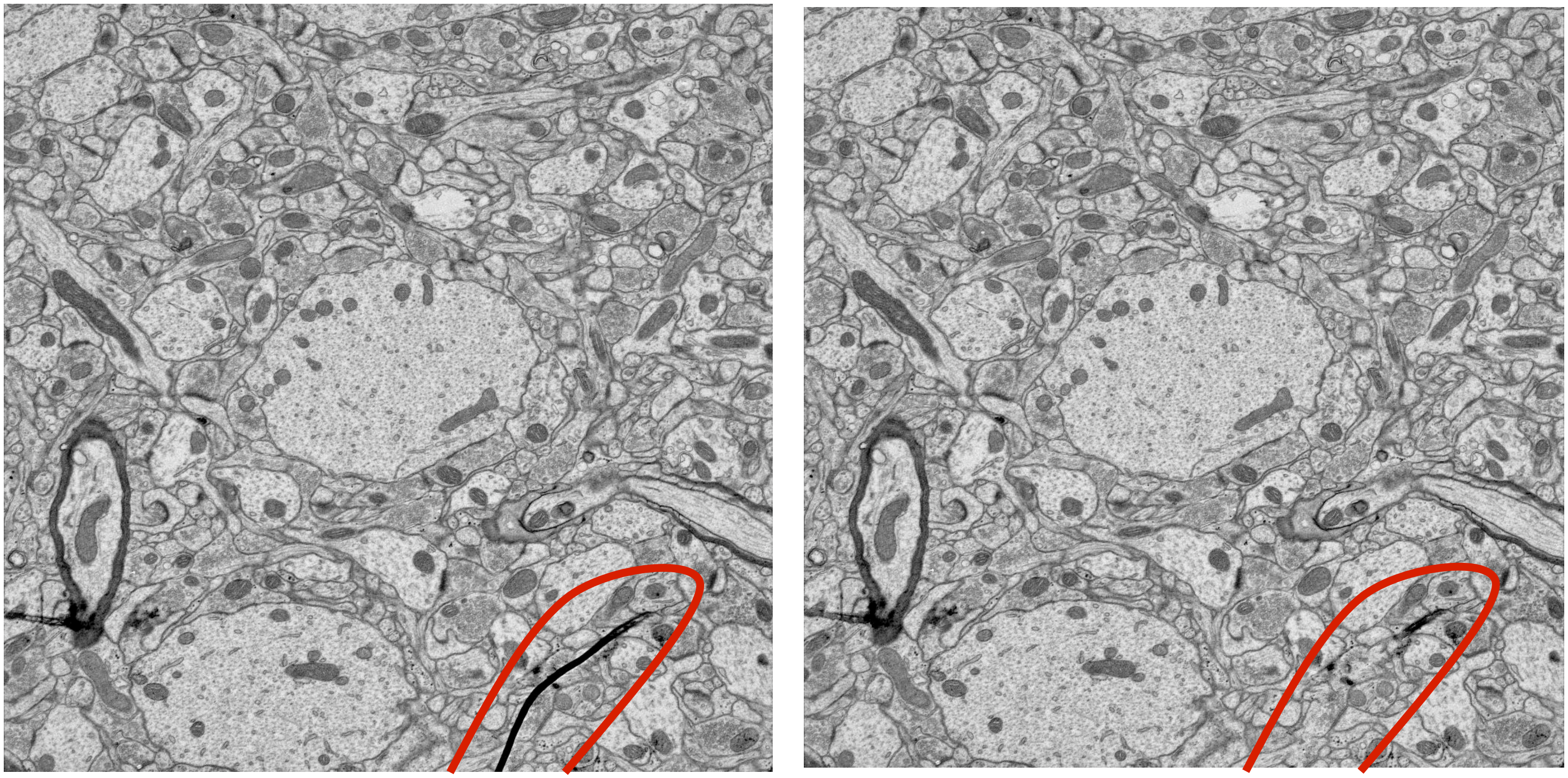
1. X-ray Crystallography → 2D Image Processing → Atomic Centers/Bonds (PDB) → FCC → Surface, Volume Processing → BEM/FEM/Shells
2. Single Particle Cryo-EM → 2D Image Processing → 3D Reconstruction → 3D Image Processing → Symmetry, Surfaces, Volume Processing→ BEM/FEM/Shells
3. Single-section EM/Anisotropic CT/MRI → 2D Image Processing → Planar X-section Contour Stack → BEM/FEM/Shells
4. Tomographic EM/MicroCT/CT/MRI → 3D Image Processing → Higher Order 3D Reconstructions, Surfaces, Skeletons → BEM/FEM/Shells
5. Time Dependent Mesh Maintenance

Step #1: Automatic Image Alignment

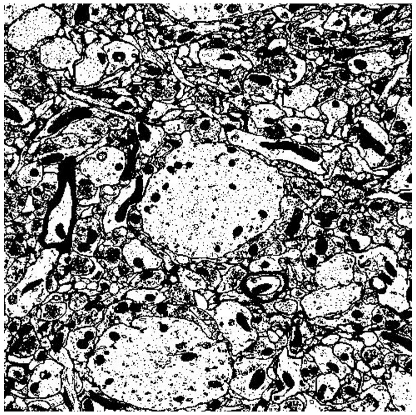
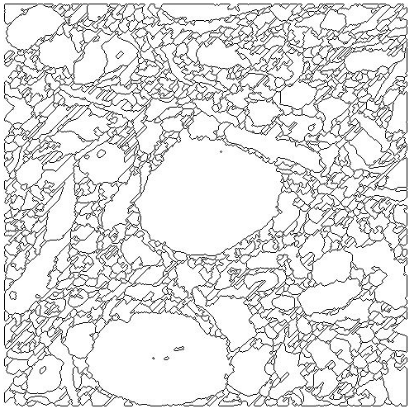
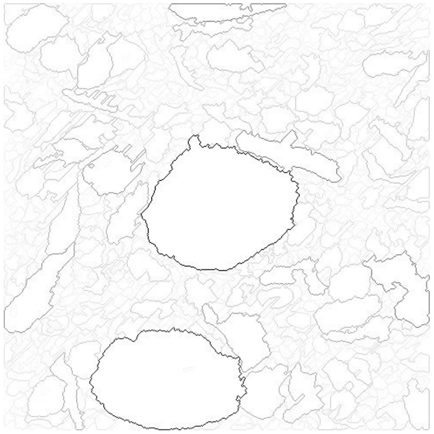
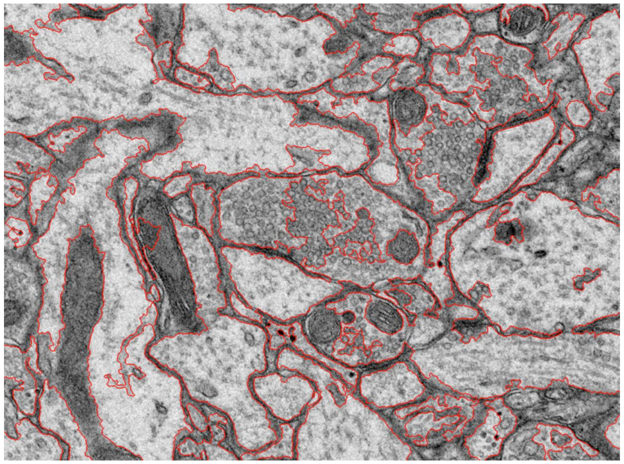
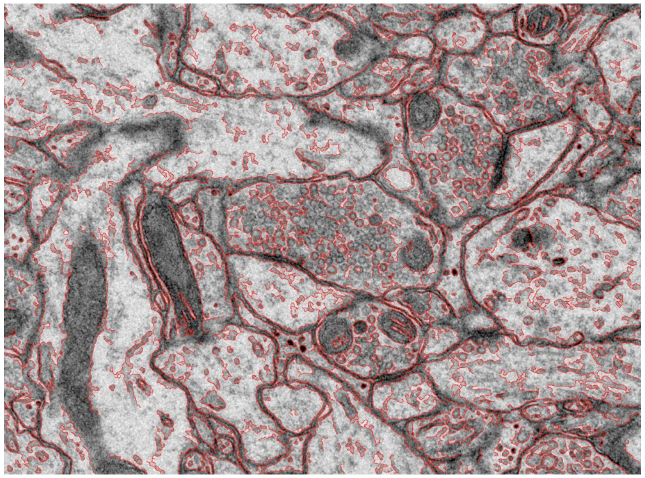


Affine Transformations (Translation, Rotation, Scaling)

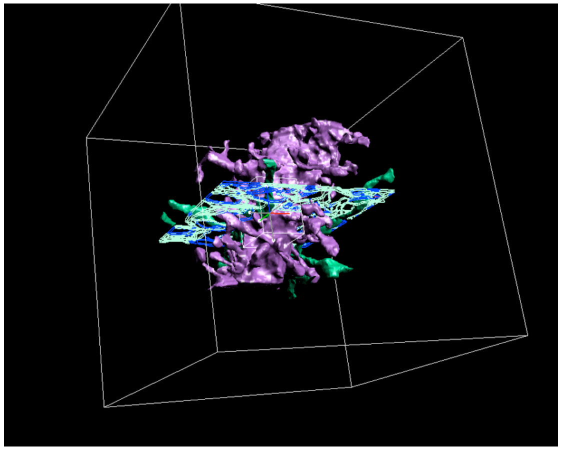
Step #2: Semi-Automatic Image Restoration

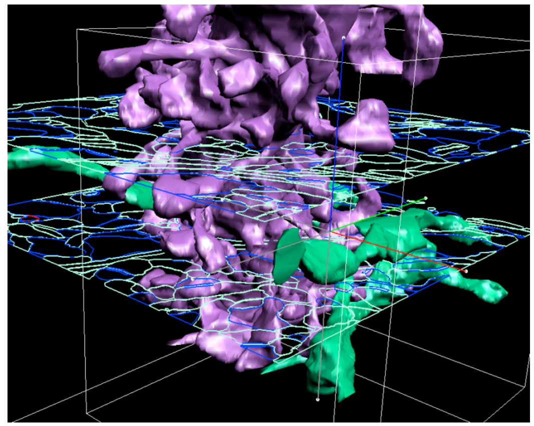


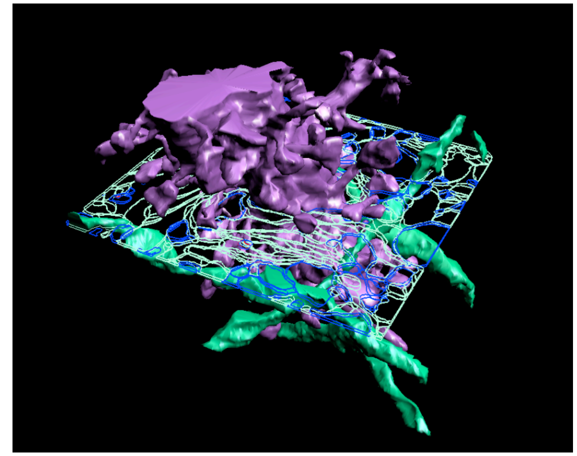
Step #3: Automatic Filtered Segmentation

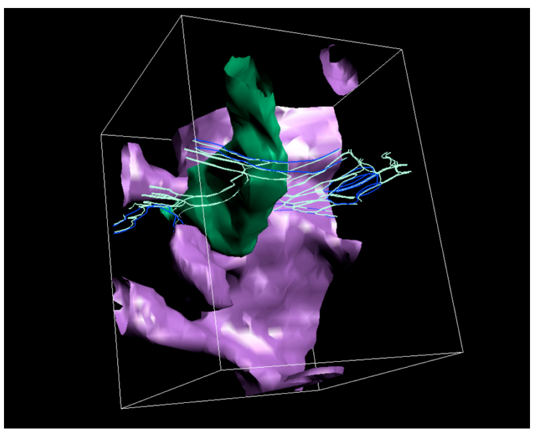






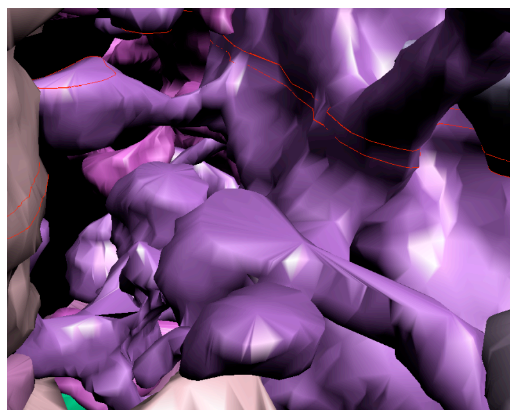
Step #4: Hippocampal Neuron Model Reconstruction

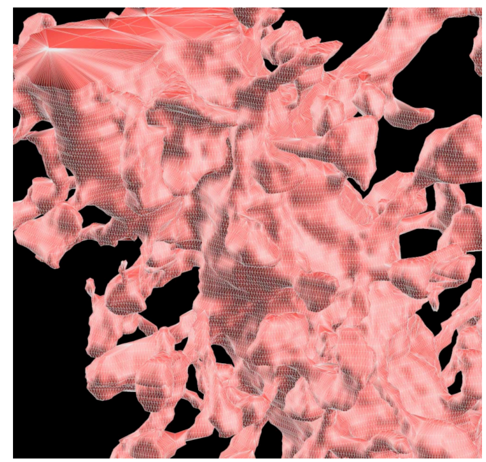










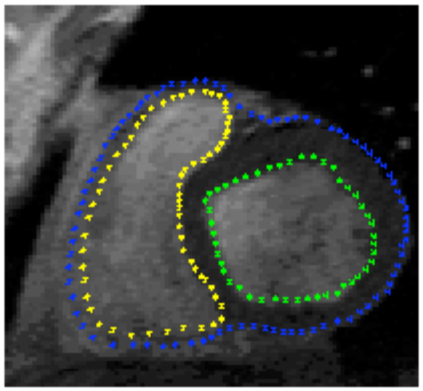


C.Bajaj, K. Lin, E. Coyle**: Arbitrary Topology Shape Reconstruction from Planar Cross-Sections,** Graphical Models and Image Processing, 58:6, 1996,

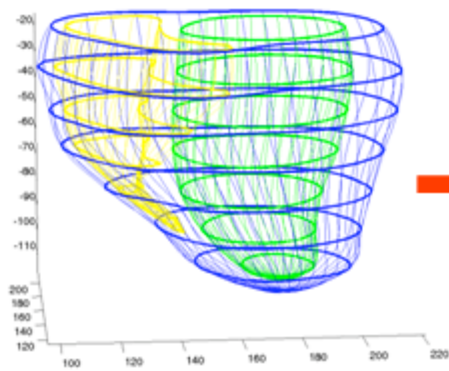
Heart Model via X-section Contour Lofting

First segment the heart into four independent planar contour stacks from MRI data: background (0), heart muscle (81), left ventricle (162), right ventricle (243) and then loft (skin) the planar contour stacks

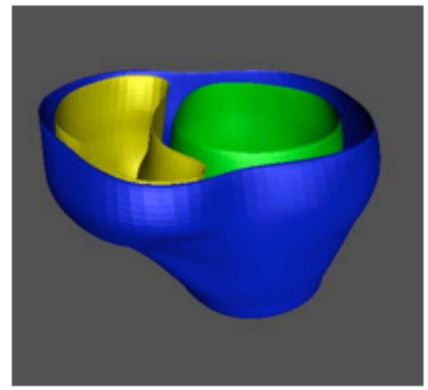
simulation of the electronic activity of the heart.



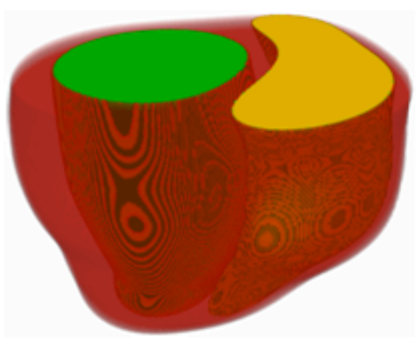
Raw MRI data



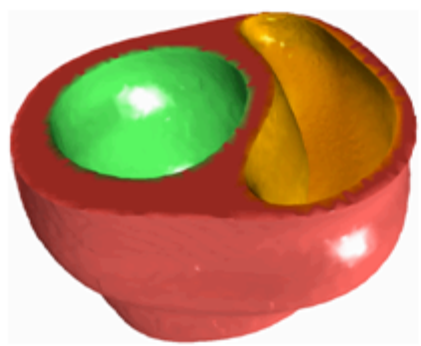
Manually digitized slices

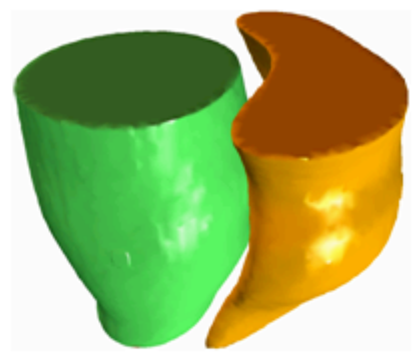


Continuous model

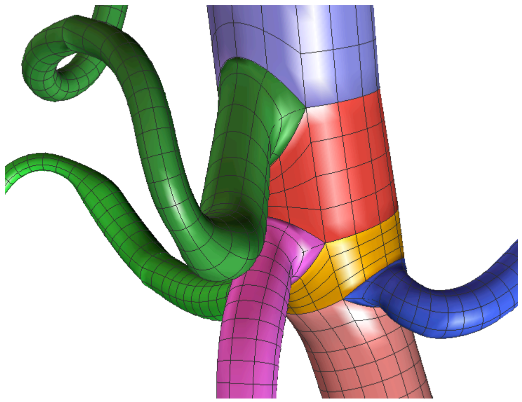


Volume rendering



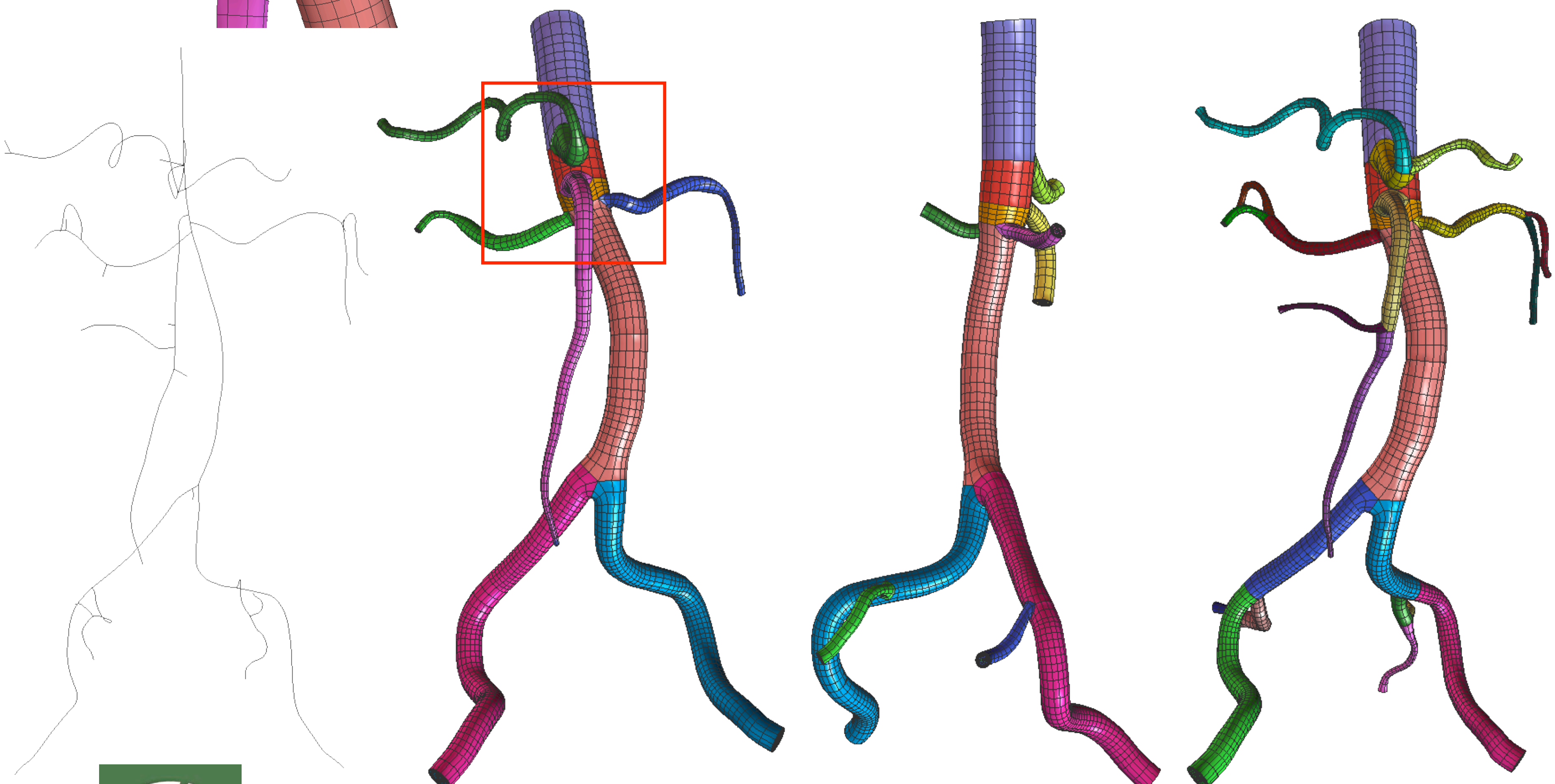


Smooth shading



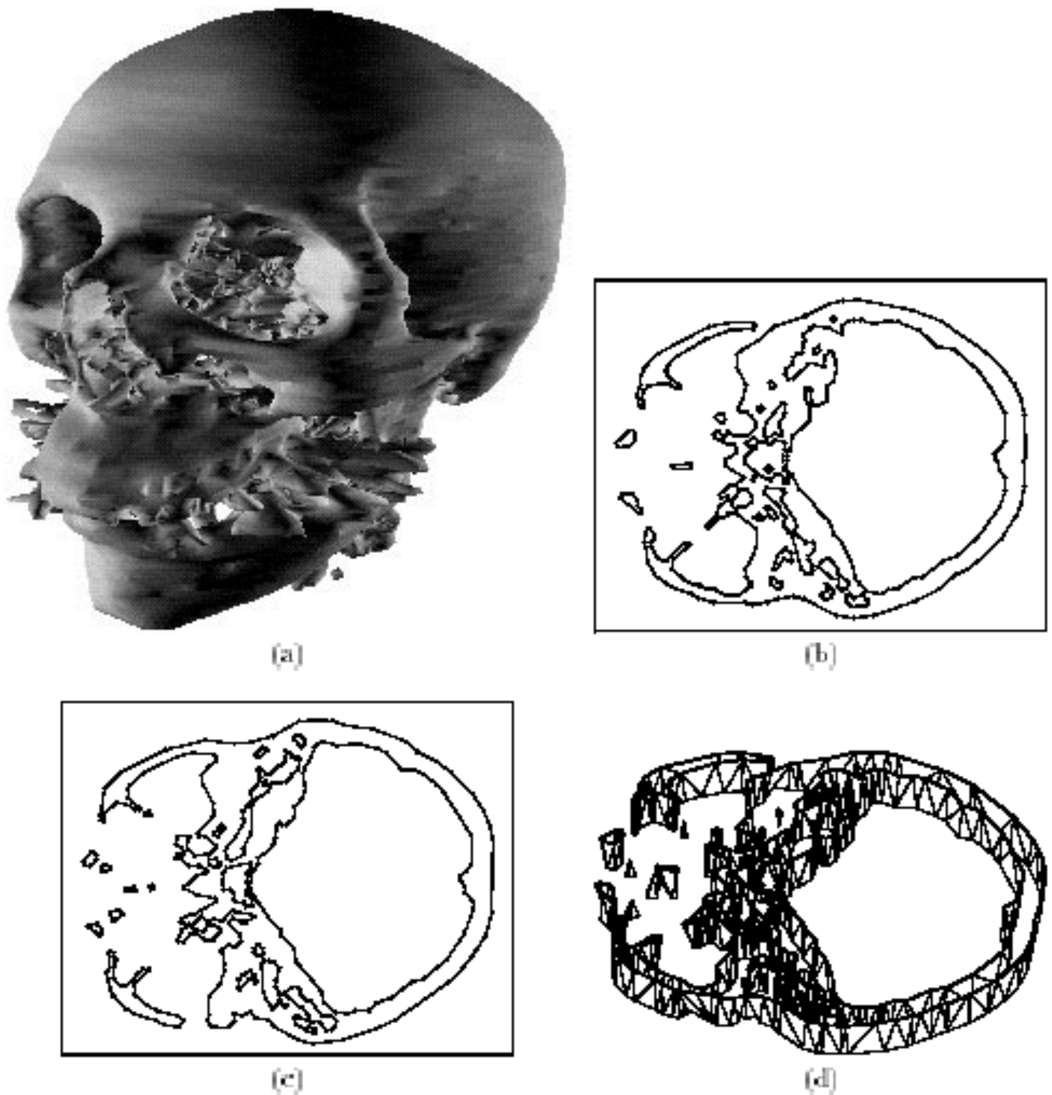
Abdominal Aorta

(Analysis Suitable Models)



Solid NURBS 1 Solid NURBS 2 (truncated) Solid NURBS 3

Triangular Meshing

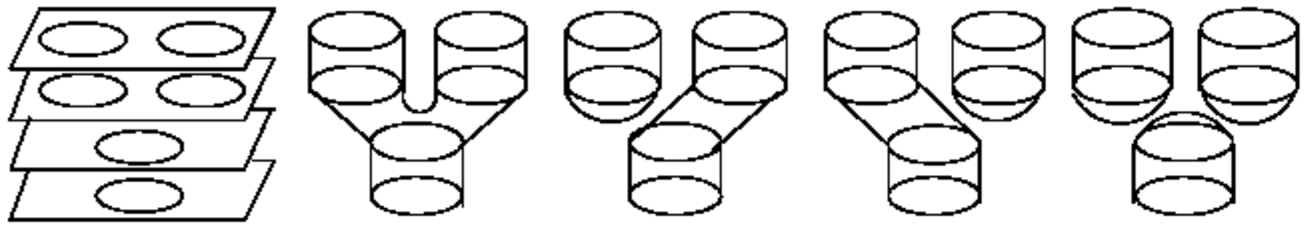


- To generate a boundary element triangular mesh from a stack of cross-sectional polygonal data.

- Subproblems
  - The correspondence problem
  - The tiling problem
  - The branching problem

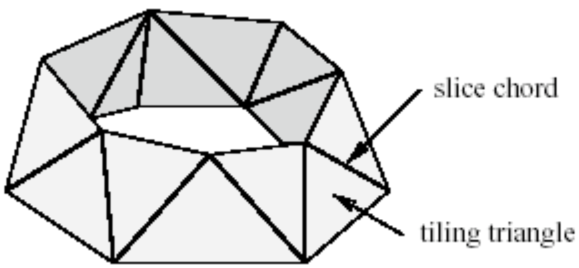
Sub-problems

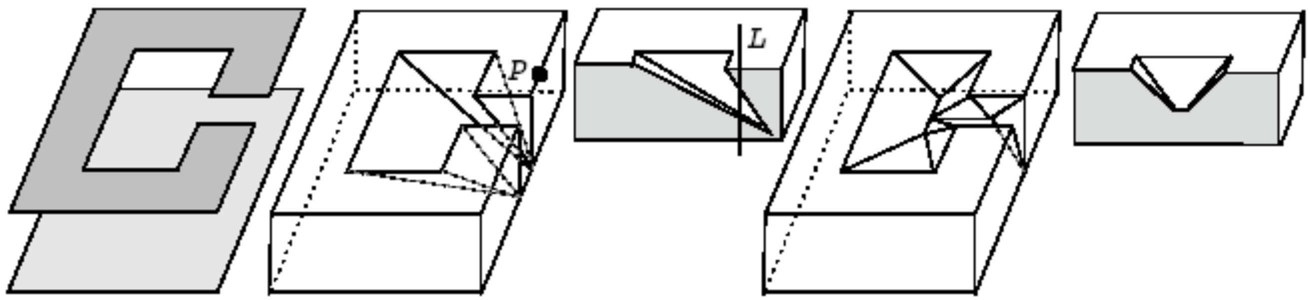
- Correspondence



(a) (b) (c) (d) (e)

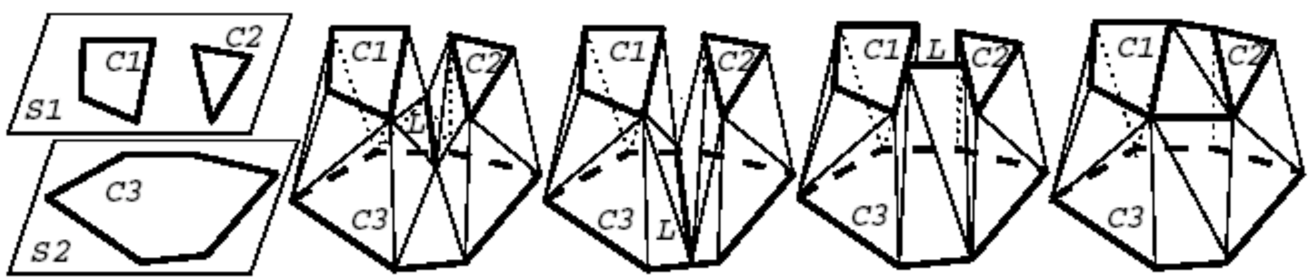
- Tiling





(a) (b) (c) (d) (e)

- Branching



(a) (b) (c) (d) (e)

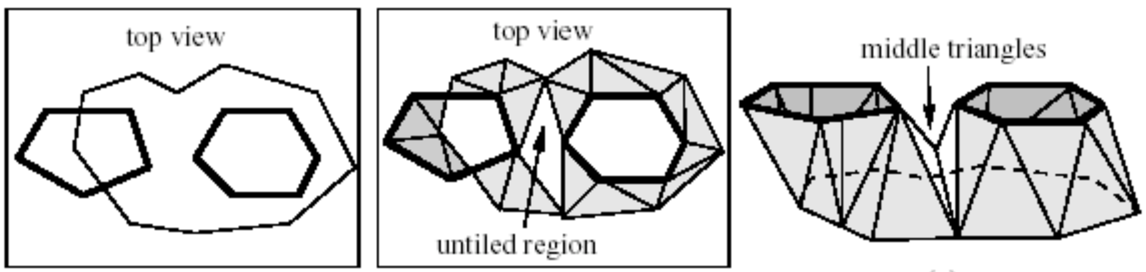
Incremental Construction

Algorithm Steps

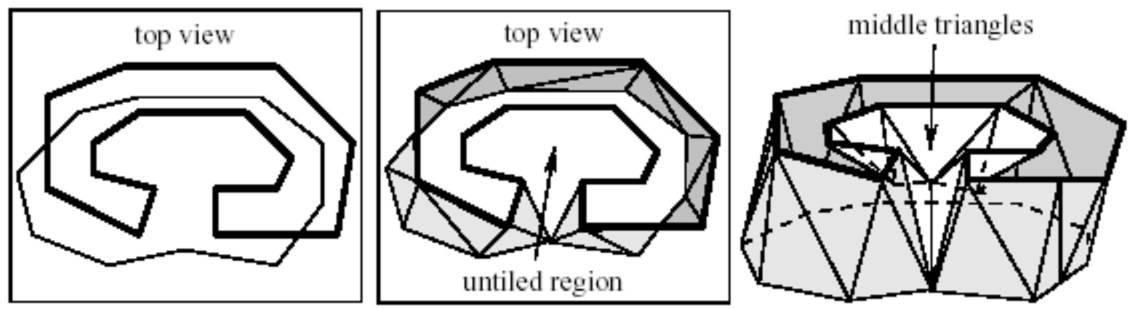
- Step a: Segment closed contours from 2D images
- Step b: Create any required augmented contours
- Step c: Find correspondences between contours
- Step d: Form the tiling region of each vertex
- Step e: Construct the tiling
- Step f: Collect the boundaries of untiled regions
- Step g: Form triangles to cover untiled regions based on their edge Voronoi diagram (EVD)

Algorithmic Subtleties

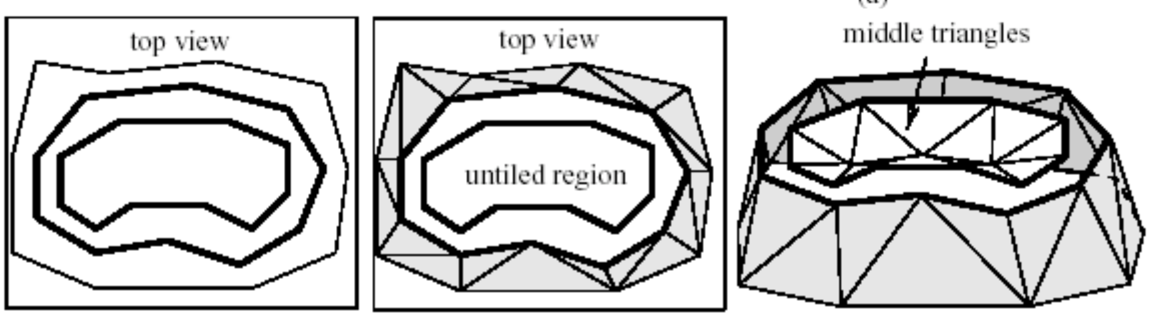
- A multi-pass tiling approach followed by the postprocessing of untiled regions



(a) (b) (c)

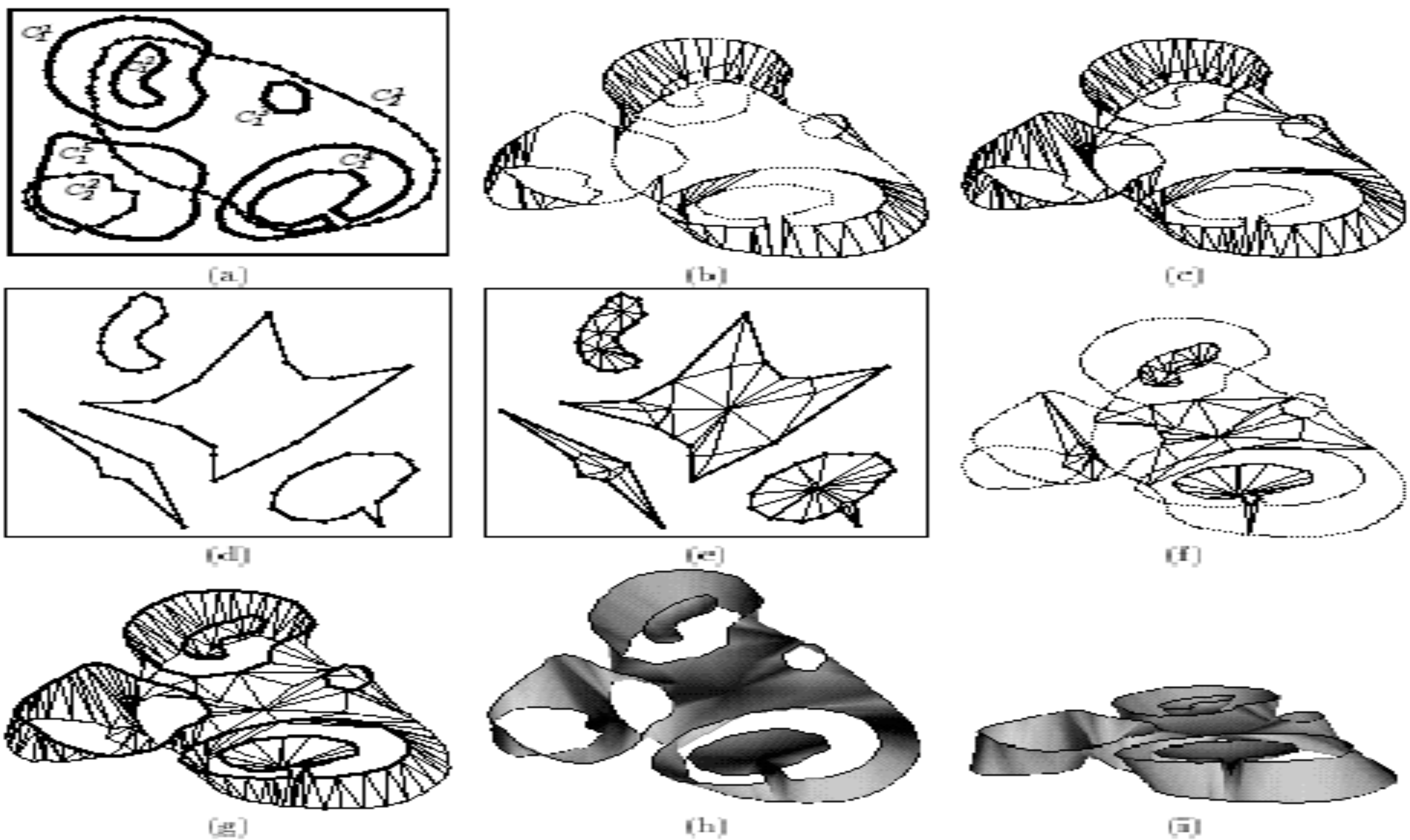


(a) (b) (c)

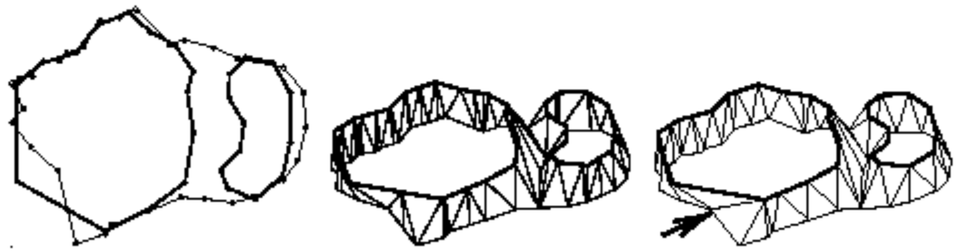


(a) (b) (c)

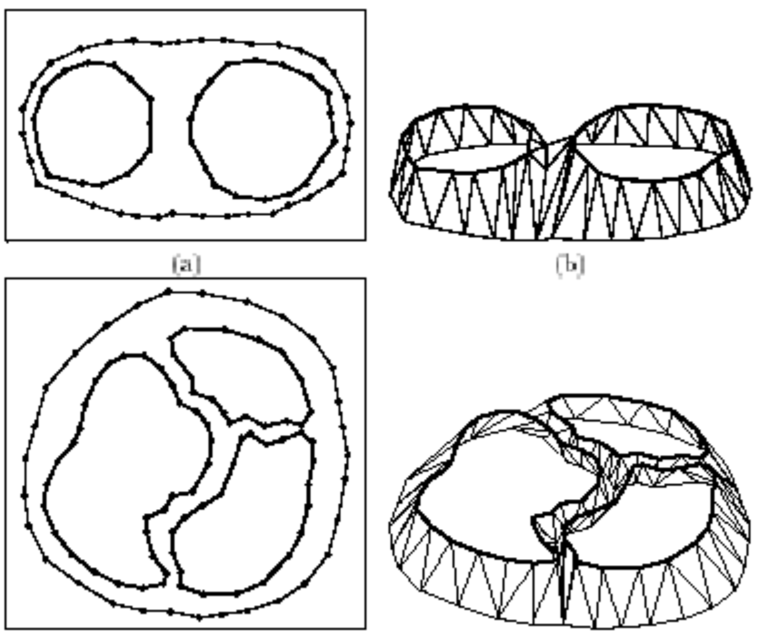
Algorithm Steps on actual data



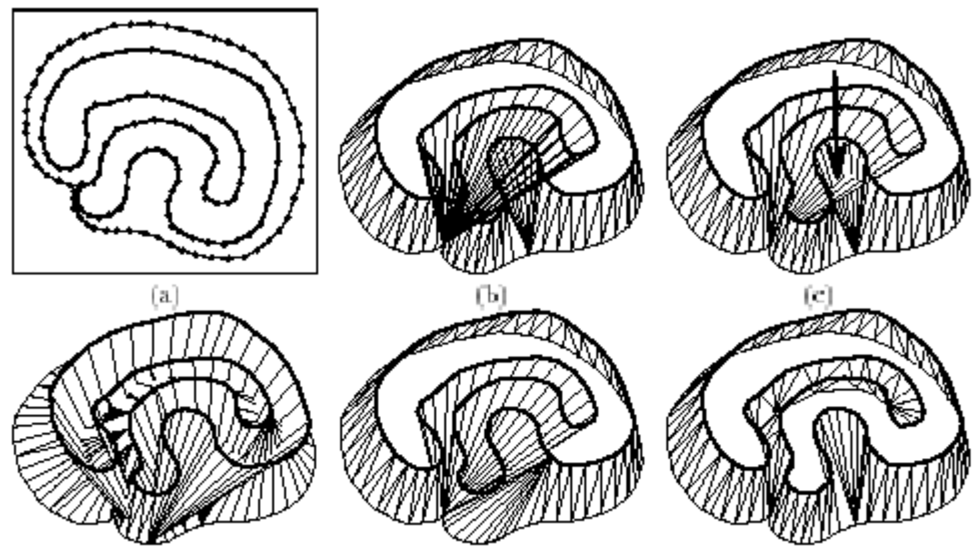
Using the Edge Voronoi Diagram as Ridges



(a) (b) (c)



(a) (b) (c) (d)



(a) (b) (c) (d) (e) (f)

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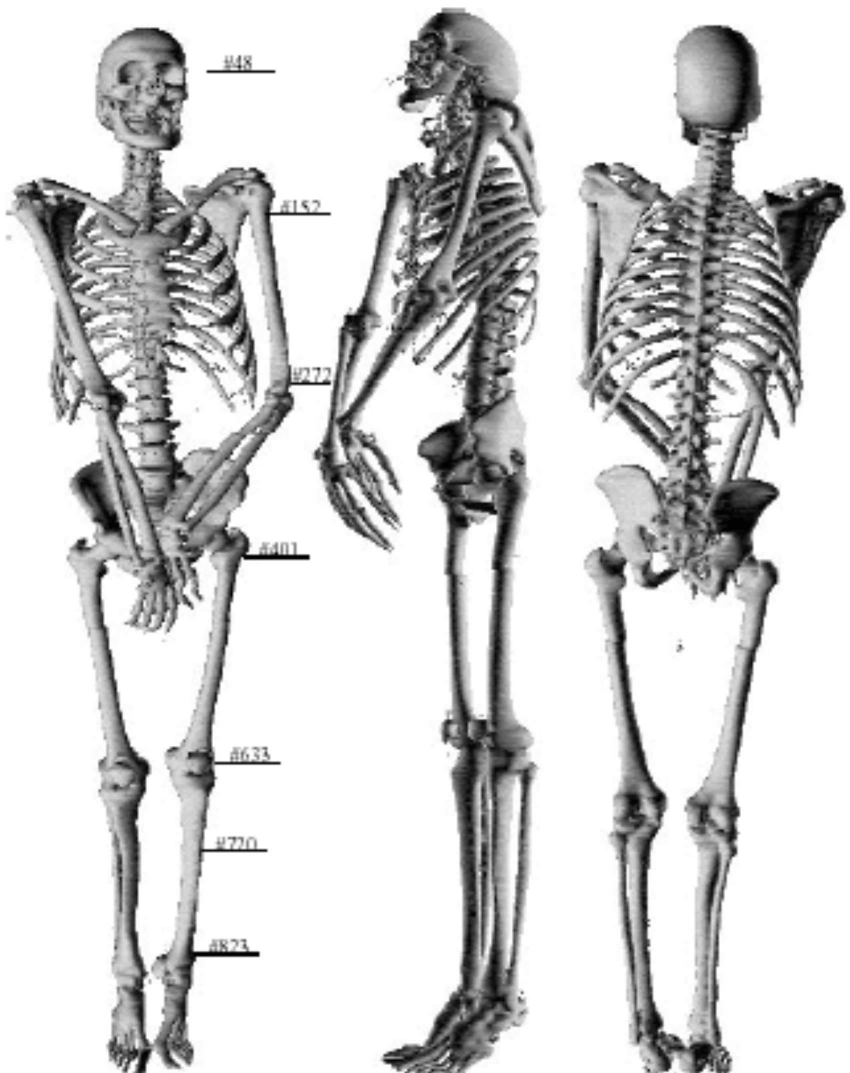
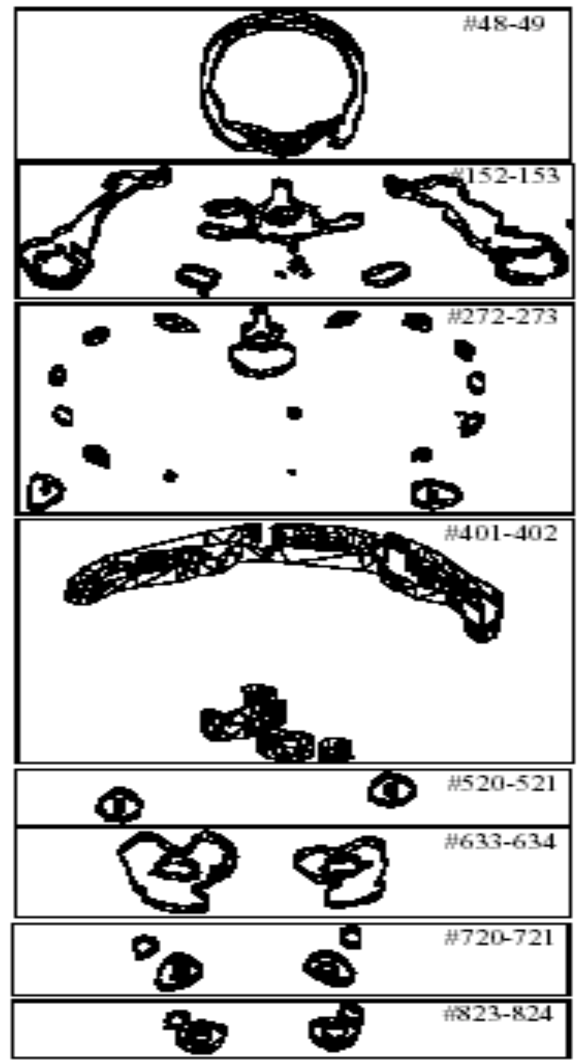
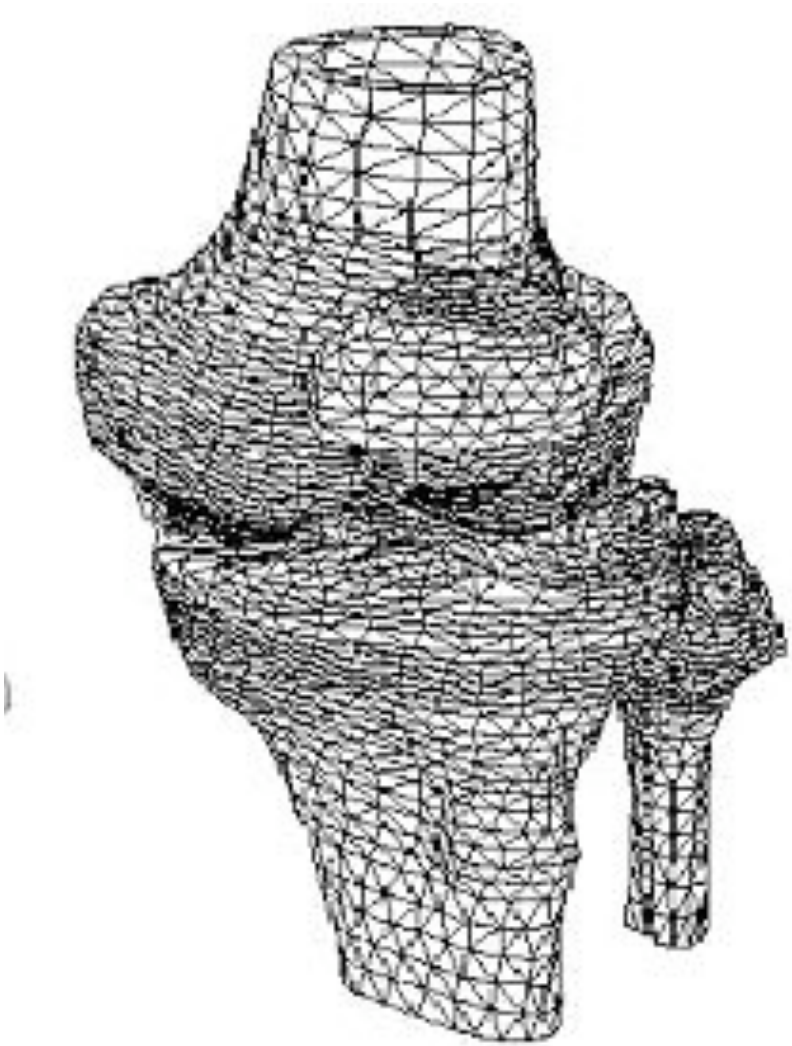
Boundary Element Triangular Mesh





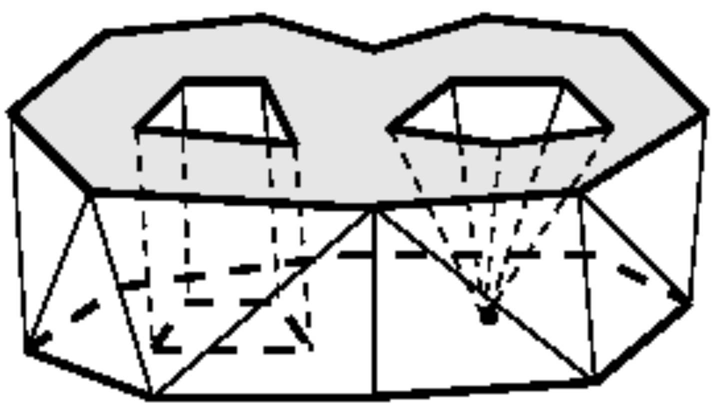
Tetrahedral Meshing

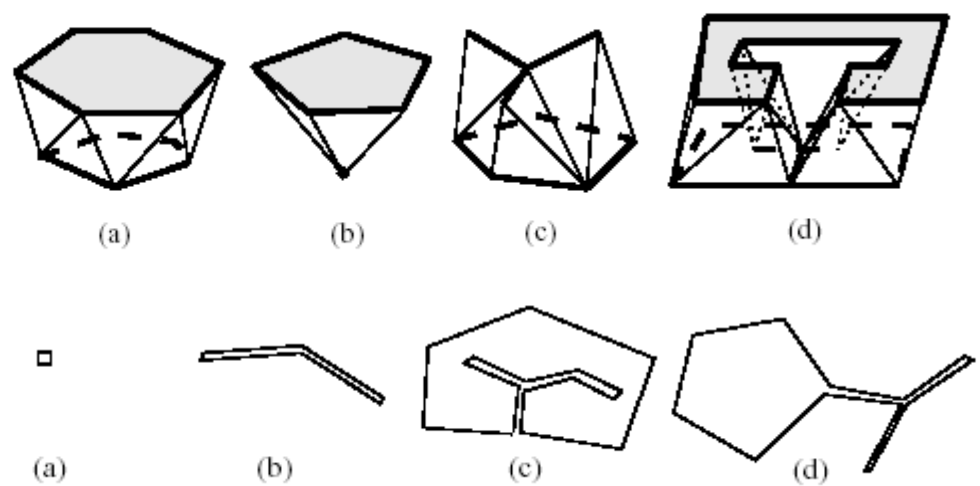


- To generate a 3D finite element tetrahedral mesh of the simplicial polyhedron obtained via the BEM construction of cross-section polygonal slice data.
- Subproblems
  - The shelling of tetrahedra to reduce polyhedron to prismatoids
  - The tetrahedralization of prismatoids

What is prismatoid?

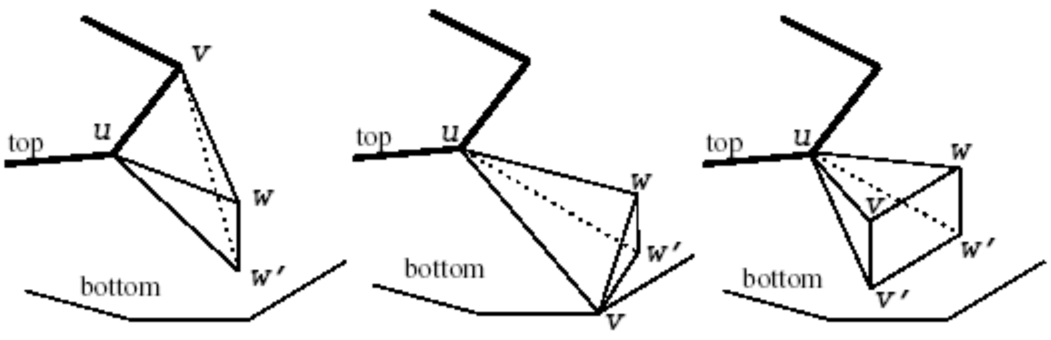
A prismatoid is a polyhedron having for bases two simple polygons (possibly degenerate) in parallel planes, and for lateral faces triangles or trapezoids having one vertex or side lying in one base (or plane), and the opposite vertex or side lying in the other base (or plane).





The Shelling Step

- Shell tetrahedra from the polyhedron, so the remaining part is a prismatoid or can be divided into prismatoids.



(a) (b) (c)

(a) (b) (c)

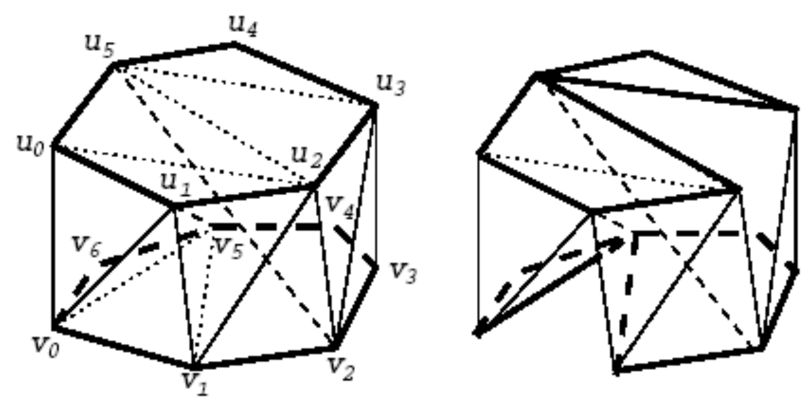
Prismatoid → Tetrahedra

- To tetrahedralize a non-nested prismatoid without Steiner points.
  1. For each boundary triangle on both slices, calculate its metric.
  2. Pick up the boundary triangle with the best metric and form one set of tetrahedra.
  3. Update the advancing front and go to Step 1.
  4. If the remaining part is non-tetrahedralizable, postprocess it.

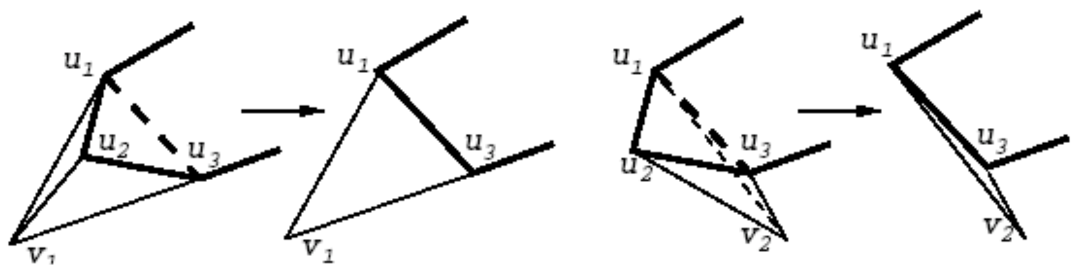
Metric, Weight Factor, Grouping

- Metric = volume/(edge)^3
- Weight factor

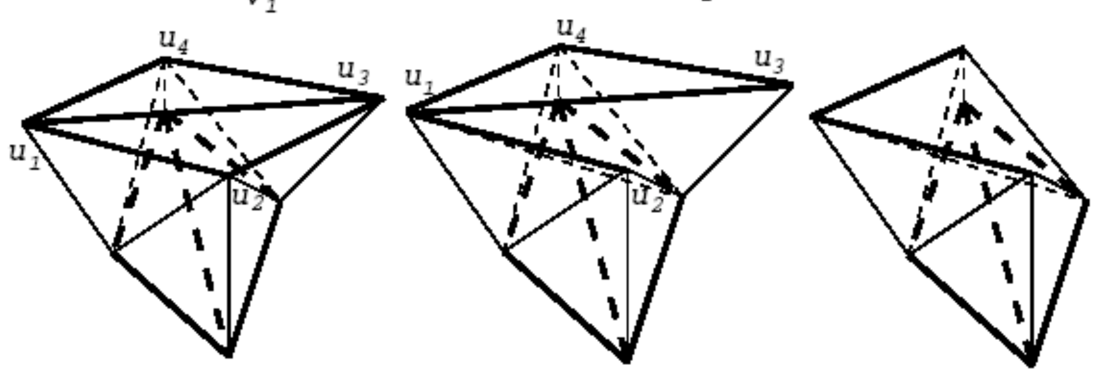
$$w = \begin{cases} 2(1-\frac{d}{h}) & \text{if } d \le 0.5h \\ 1 & \text{if } 0.5h < d < h \\ \frac{h}{d} & \text{if } d \ge h \end{cases}$$



(a) (b)



- Grouping can avoid irregular remaining part

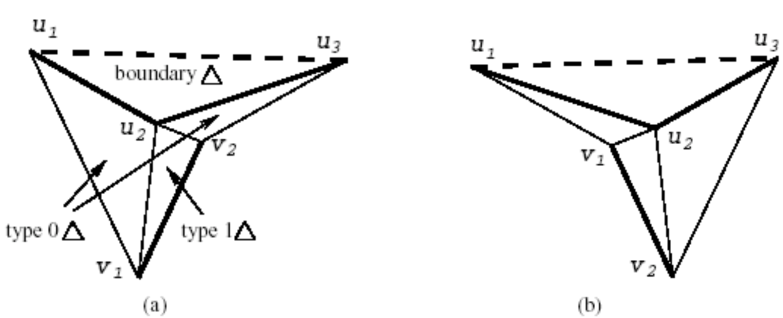


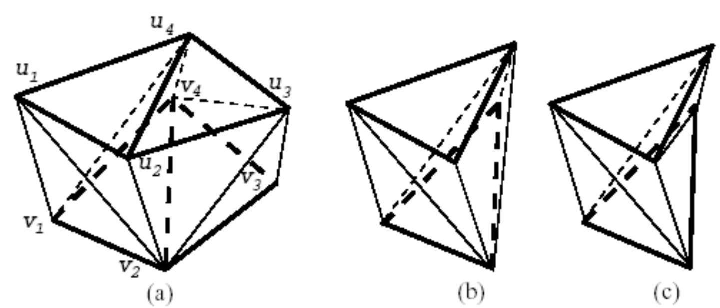
(a) (b) (c)

Protection Rule

Lemma 1: Suppose a top boundary triangle $\Delta u_1u_2u_3$ is under the constraint that no more than one type 1 triangle is between the two type 0 triangles containing the contour segments u_1u_2 and u_2 u_3. Furthermore, let the bottom vertices of the two type 0 triangles be v_1 and v_2. Our grouping operation cannot apply to $\Delta u_1u_2u_3$ to form a set of tetrahedra, if and only if all the following conditions are satisfied.

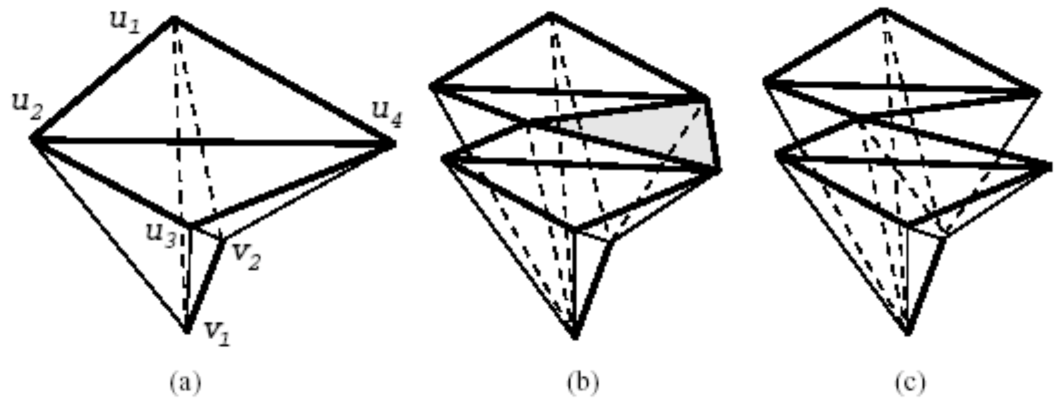
1. v_1v_2 is exactly one contour segment.
2. One of the slice chords u_2v_1 and u_2v_2 is reflex and the other is convex.
3. Both u_1v_2 and u_3v_1 are not inside the prismatoid.



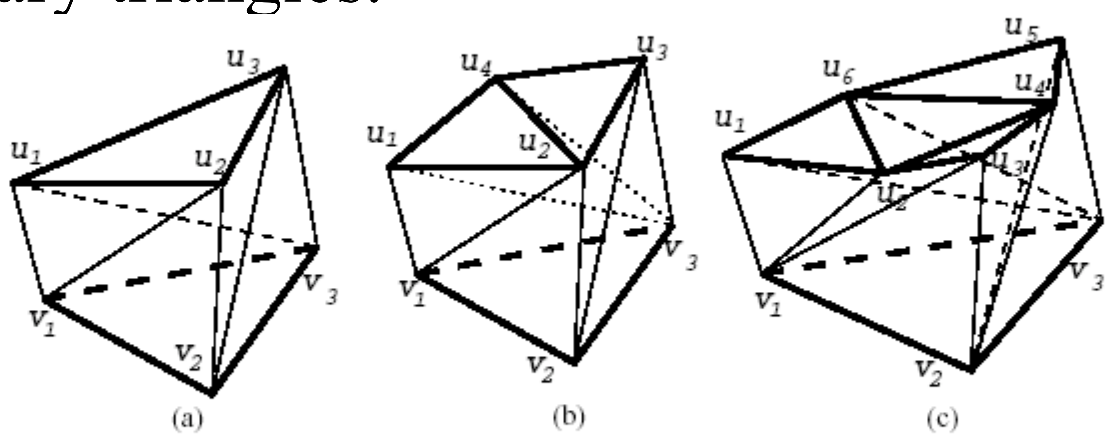


Classification of Untetrahedralizable Prismatoids

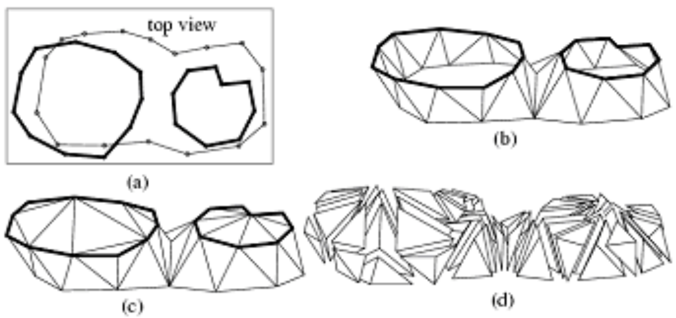
1. Has two boundary triangles on the top face and one line segment on the bottom face.



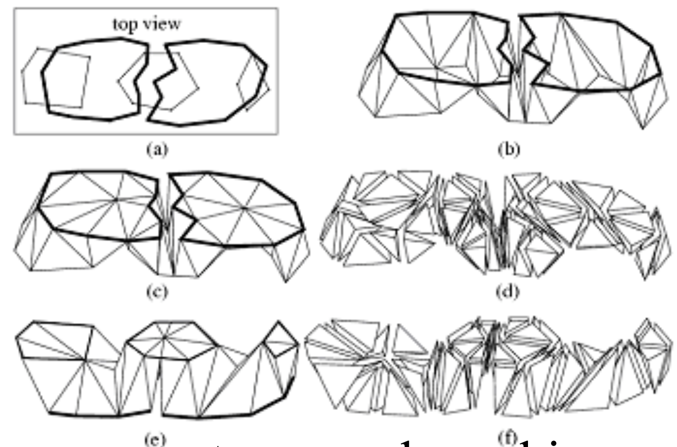
2. Has one bottom triangle which is treated as three boundary triangles.



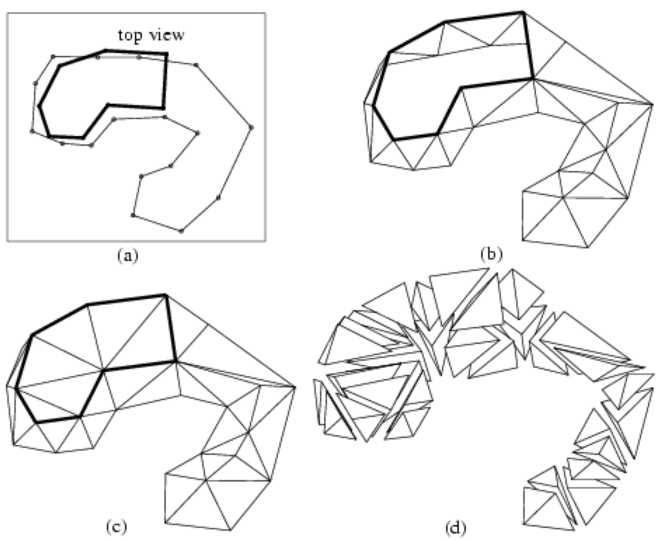
Multiple Tetrahedralizable Cases



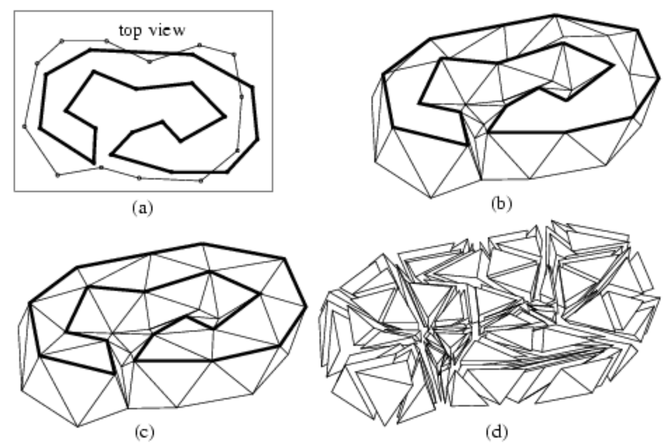
One-to-many branching



many-to-many branching

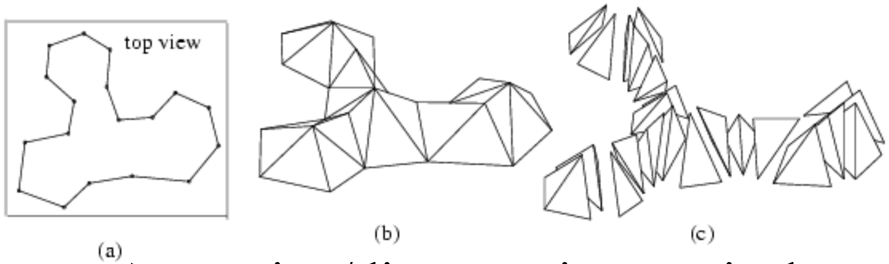


Dissimilar region (the right bottom portion of the bottom contour)

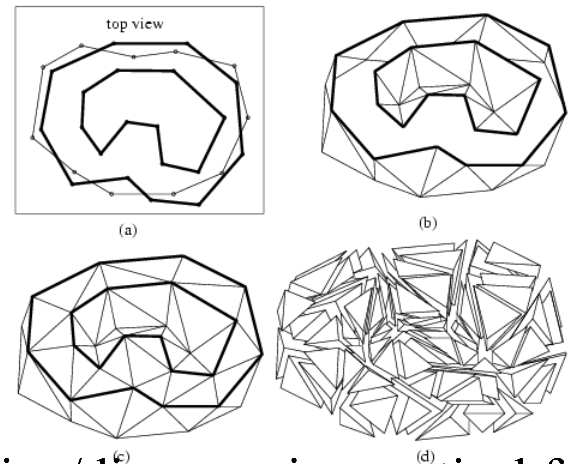


Dissimilar region (the inner portion of the top contour)

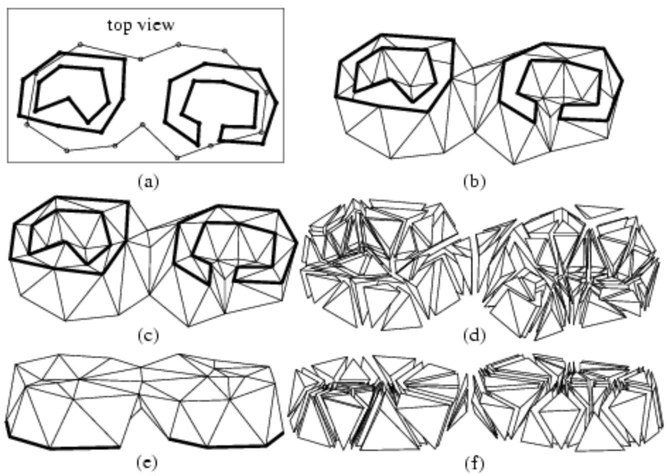
Multiple Tetrahedralizable Cases



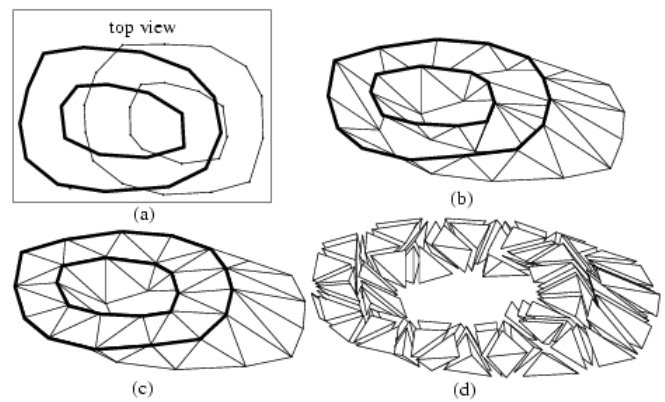
Appearing/disappearing vertical feature of a solid interior



Appearing/disappearing vertical feature (the top inner contour) of a void interior

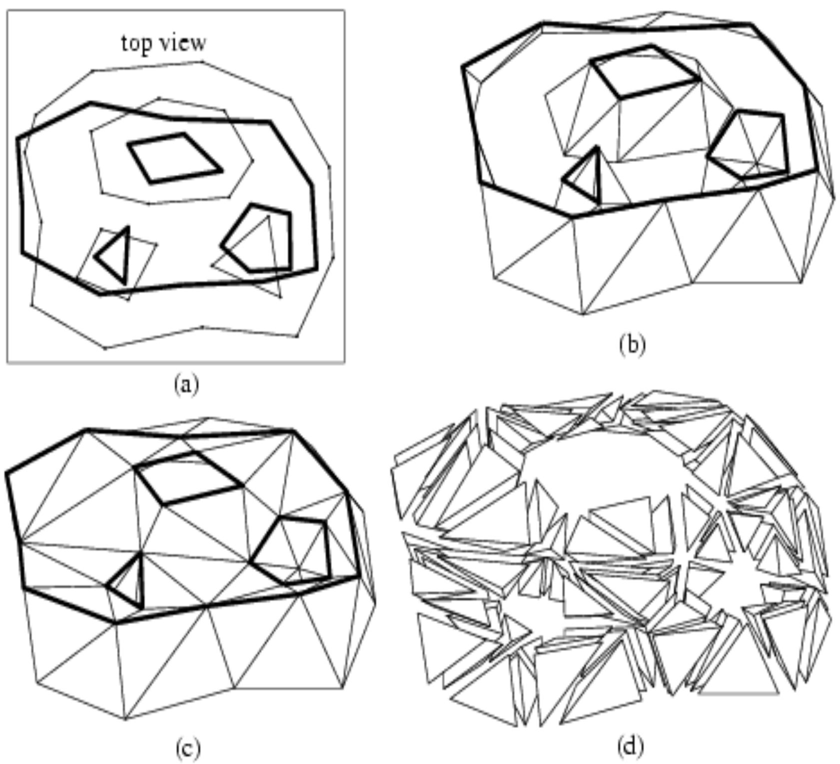


A branching, a dissimilar portion (the inner portion of the top right contour), and an appearing/disappearing vertical feature (the inner contour at the left of the top slice)

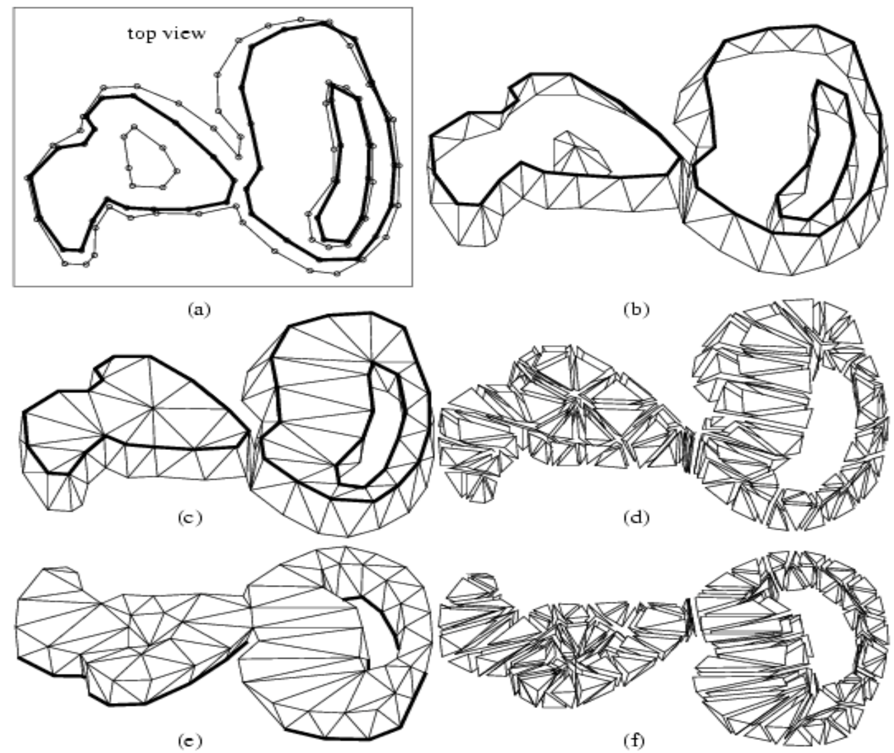


Nested prismatoids

Multiple Tetrahedralizable Cases

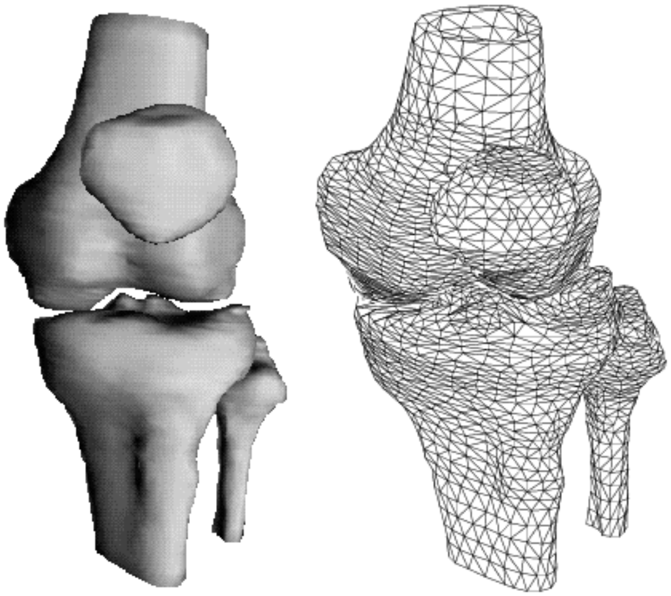


Multiply-nested prismatoid

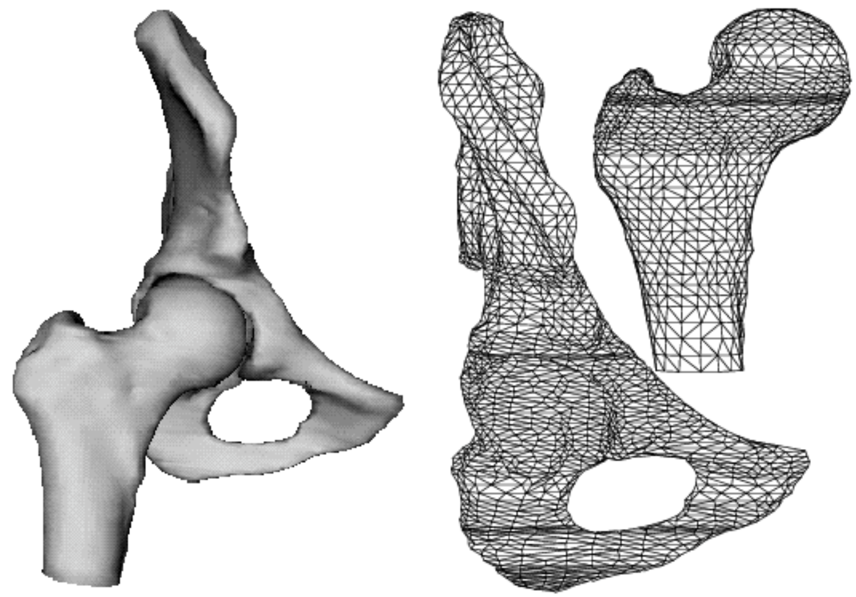


Solid region between two slices of a human tibia

Examples



(a) (b)



(a) (b)

Knee joint (the lower femur, the pper tibia and fibula and the patella)

(a) Gouraud shaded

(b) The tetrahedralization

Hip joint (the upper femur and the pelvic joint)

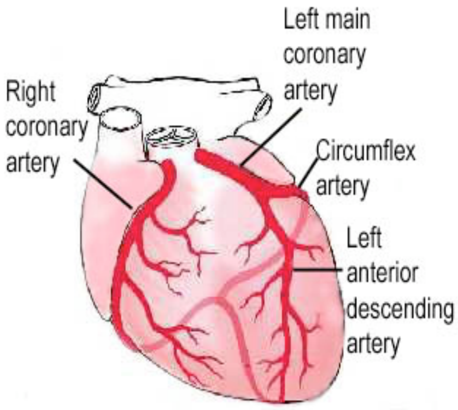
(a) Gouraud shaded

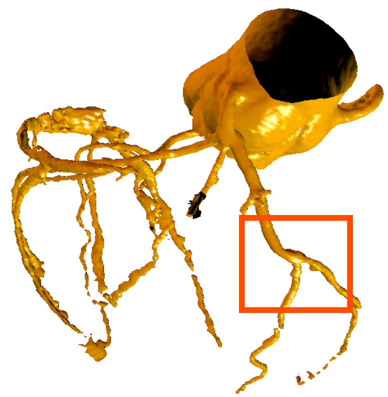
(b) The tetrahedralization

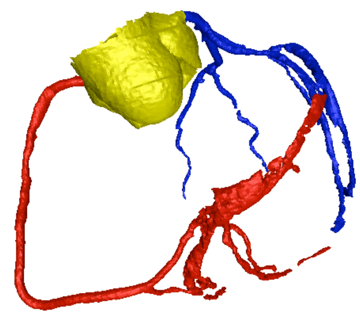
Mini-summary

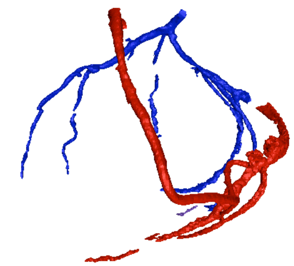
- The characterization, avoidance of non-tetrahedralizable polyhedra is one of the main challenges
- The mix of numerical precision and topological decision making needs precise rules so errors don't propagate.

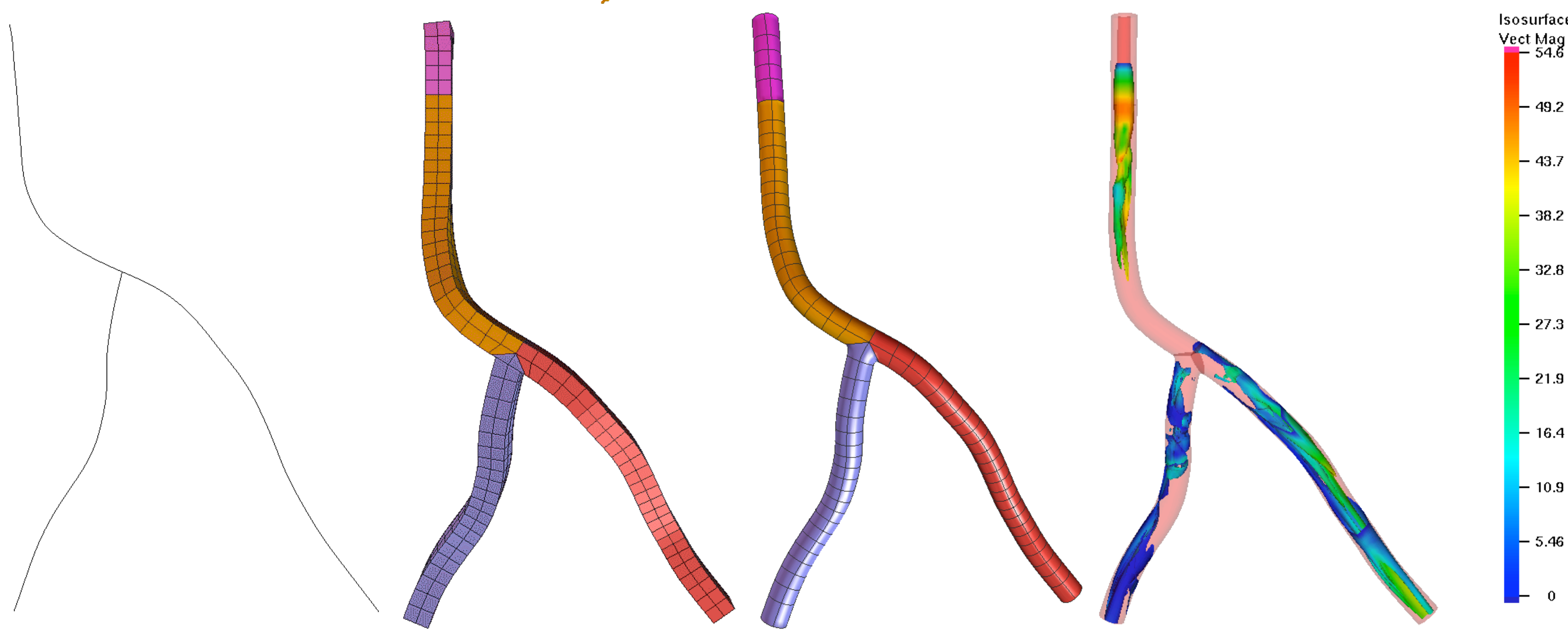
Coronary Arteries







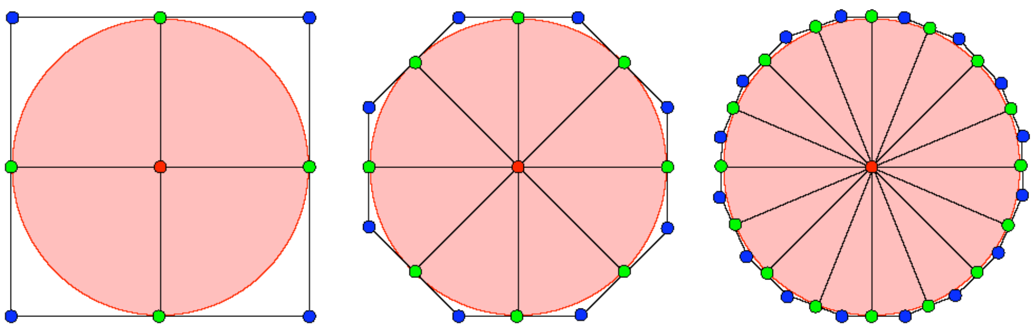




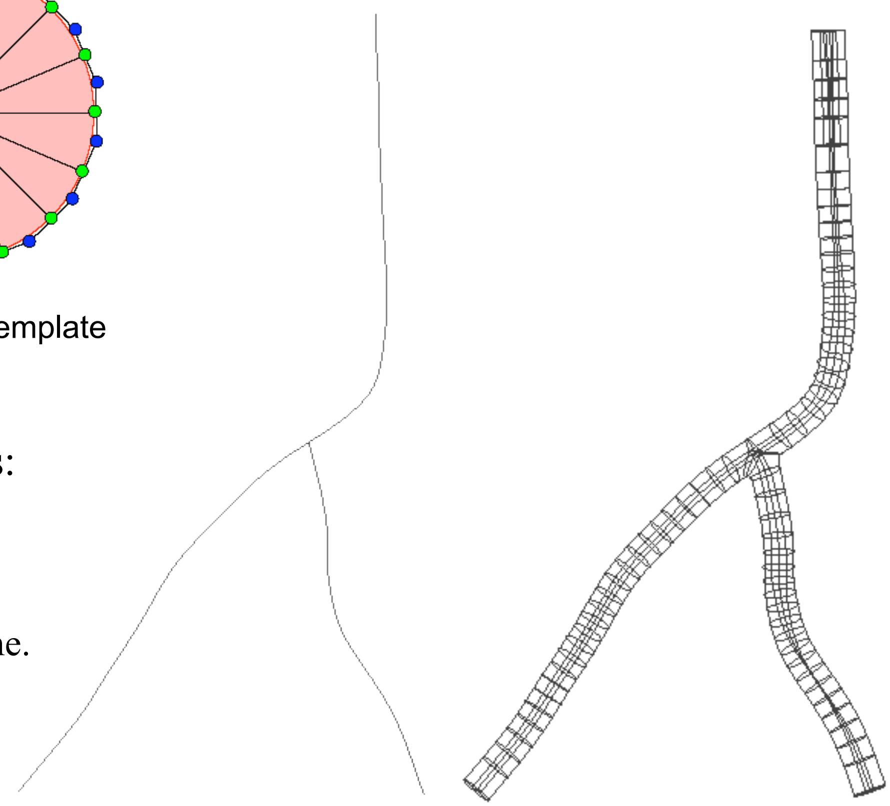
(b) Path (c) Control mesh (d) Solid NURBS (e) Simulation results

Sweep based Hexahedral Mesh

- To project a templated quad mesh of a circle onto each cross-section of the tube, then connect corresponding vertices in adjacent cross-sections to form a hex mesh.



Level-1-template Level-2-template Level-3-template



Control meshes satisfy the following four requirements:

1. Any two cross-sections can not intersect with each other.

2. Each cross-section should be perpendicular to the path line.

3. In the n-furcation region of several branches, each cross-section should remain perpendicular to the vessel surface.

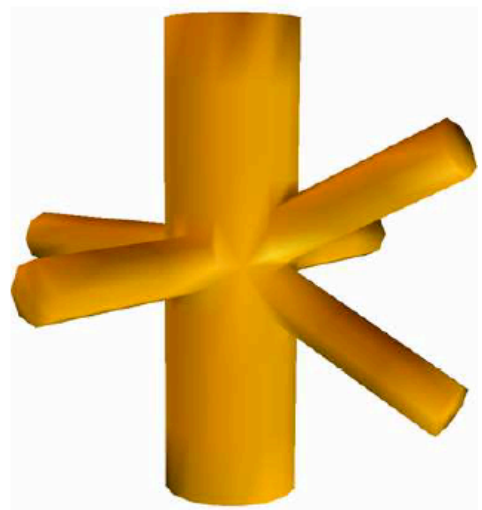
Vasculature Branchings

One-to-one sweeping requires the source and the target surfaces have similar topology. Various templates are designed to decompose arteries into mapped meshable regions for different branching configurations.

- *n*-**Branching:** A *n*-branching is a situation when *n* branches join together, where $n \geq 3$.
- **Bifurcation:** A bifurcation is a situation when three branches join together. A bifurcation is also a 3-branching.
- **Trifurcation:** A trifurcation is a situation when four branches join together. A trifurcation is also a 4-branching.

Bifurcation

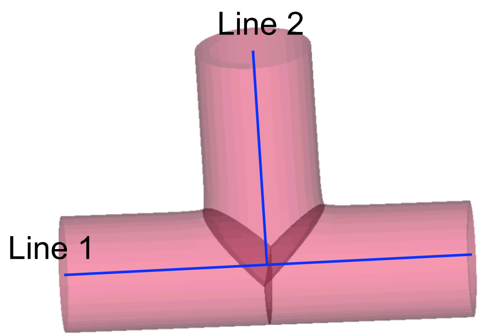
Trifurcation



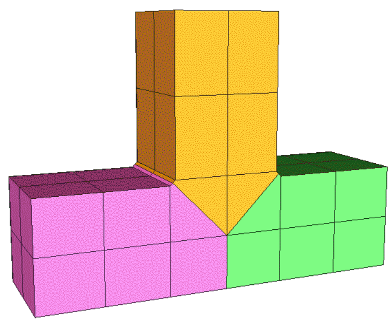
7-branching

Bifurcation Templates

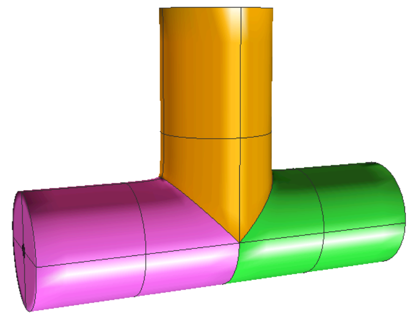
The bifurcation geometry is decomposed into three patches: the master branches contain two branches and the slave branch has one branch.



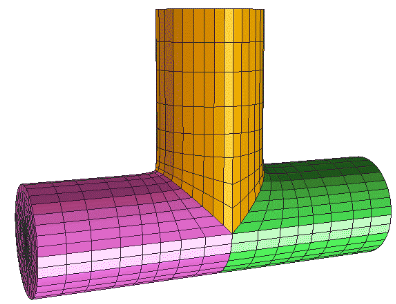
Path



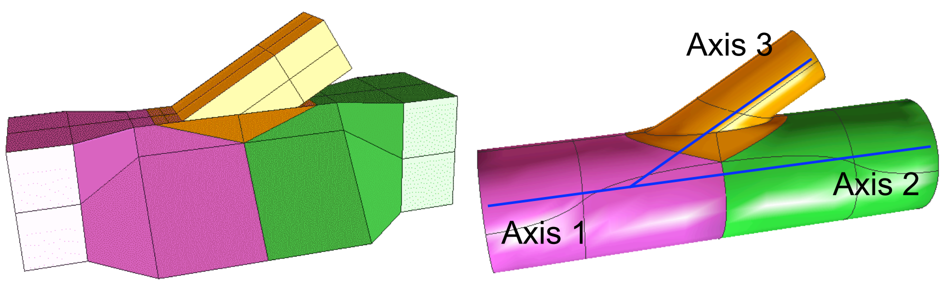
Control mesh



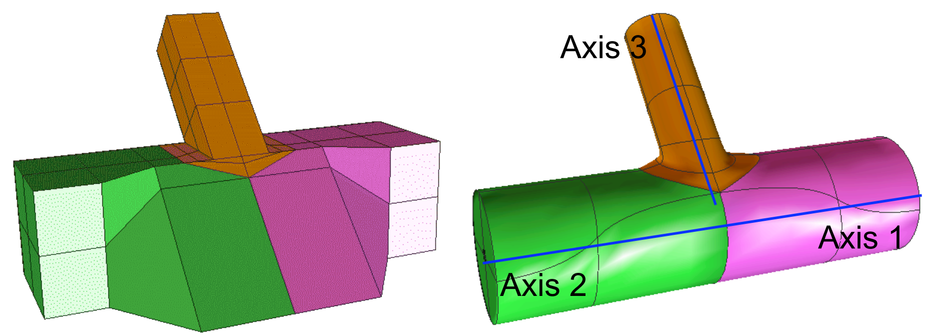
Solid NURBS



Hex meshes



The master and slave branch axes are non-orthogonal.



The master and slave branch axes are not coplanar.

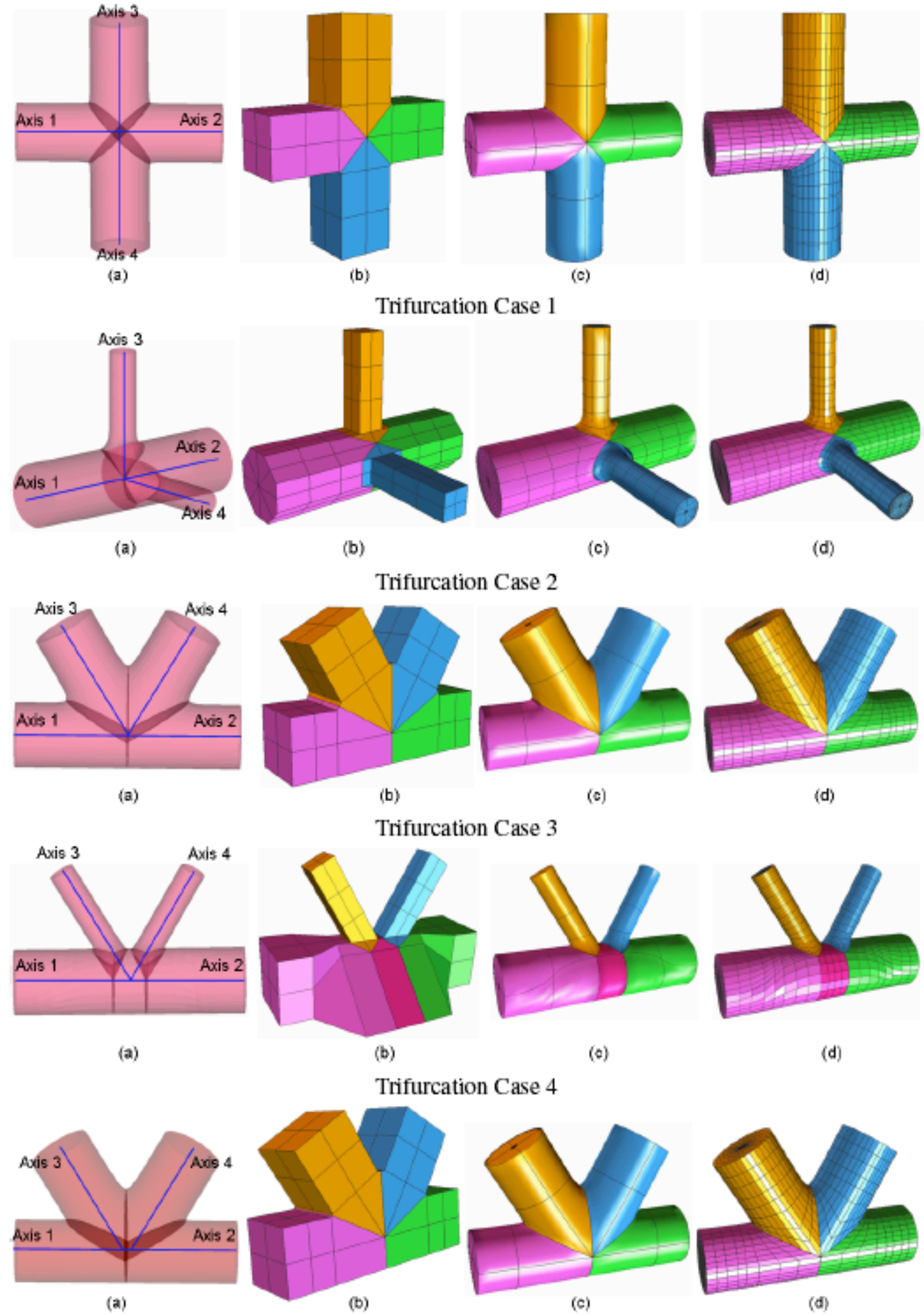
Center for Computational Visualization
Institute of Computational and Engineering Sciences
Department of Computer Sciences
University of Texas at Austin

October 2007

Trifurcation Templates

- Trifurcation has one master branch and two slave branches (4/5 patches).
- All possible trifurcations are classified into five cases according to the position of slave branches relative to the master branch (peripheral/axial).

1. Level-1-template for the master branch, at most two slave branches.
2. Level-2-template for the master branch and Level-1-template for the slaves.
3. Axial direction, two slave branches intersect with each other.
4. Axial direction, two slave branches do not intersect. One trifurcation degenerates into two bifurcations.
5. Two bifurcations merge into one trifurcation.



Trifurcation Case 1
Trifurcation Case 2
Trifurcation Case 3
Trifurcation Case 4
Trifurcation Case 5

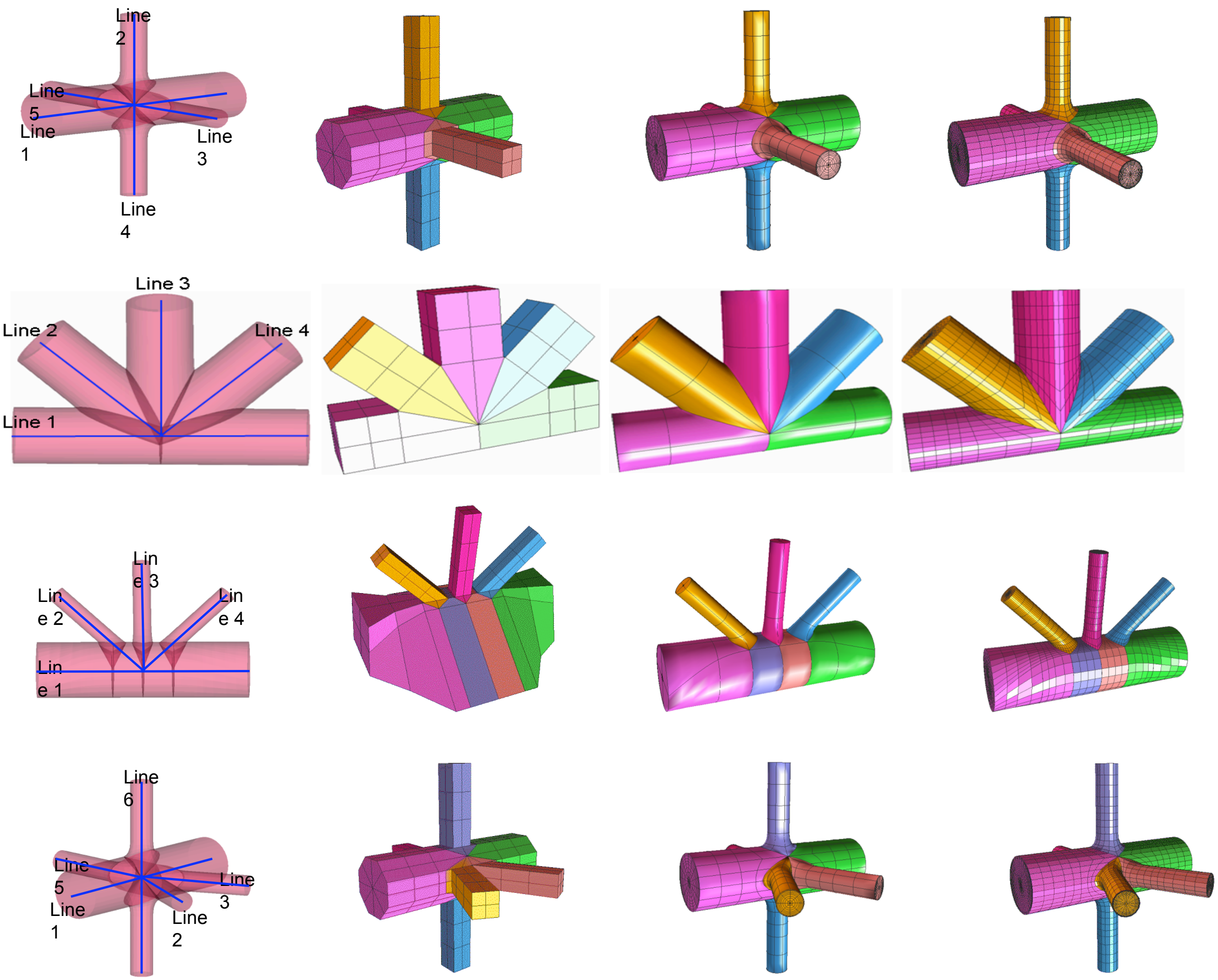
n-branching Templates (n>4)

Case 1:
Peripheral direction

Case 2
Axial direction

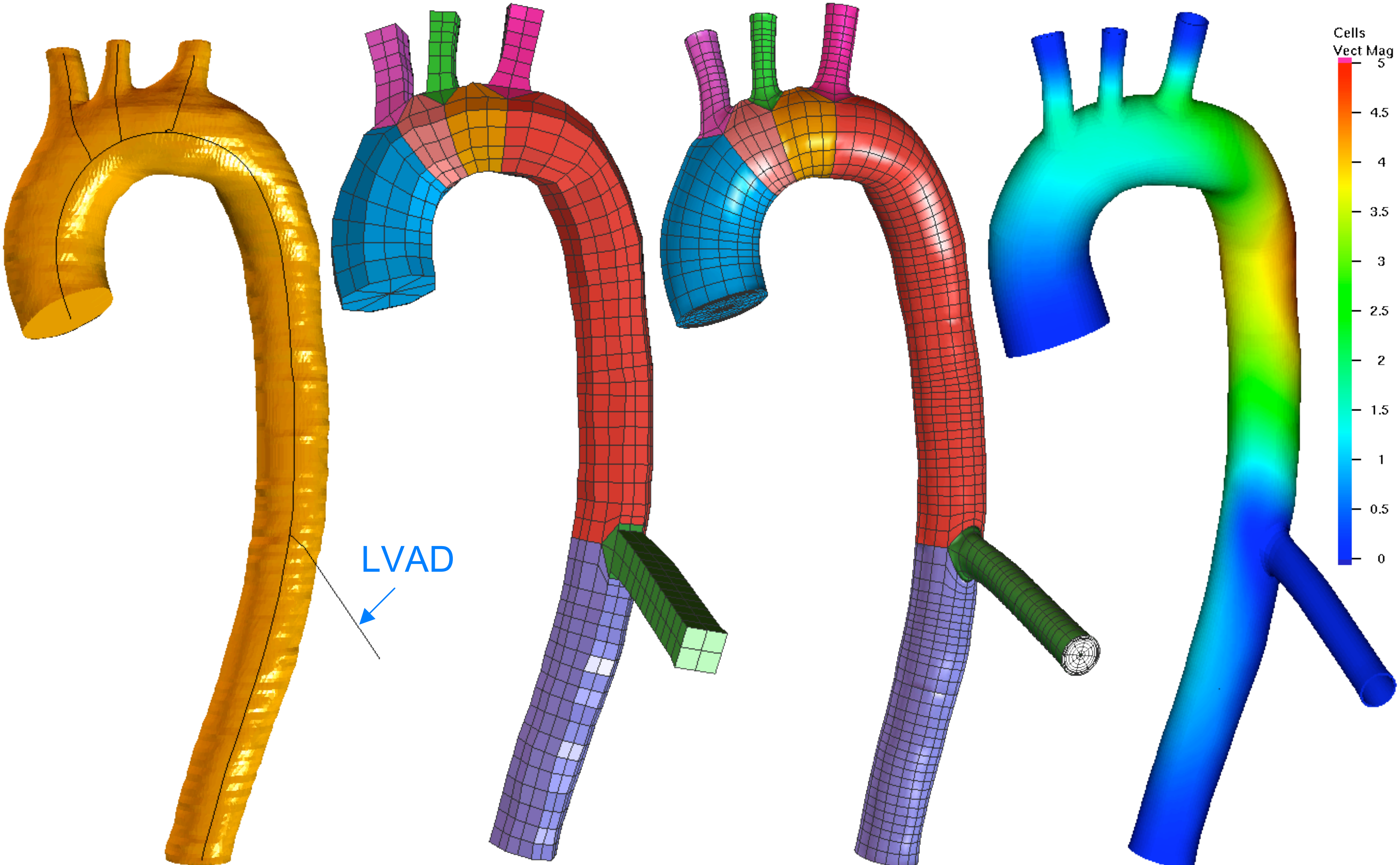
Case 3
Axial direction
n-braching degenerates into several *m*-branchings.

Case 1&2



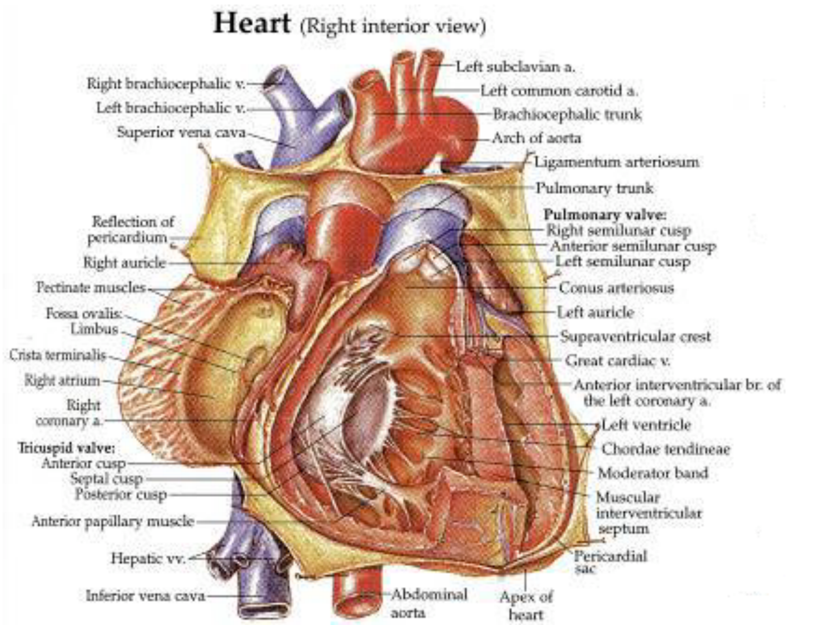
patch Control mesh Solid NURBS Hex meshes

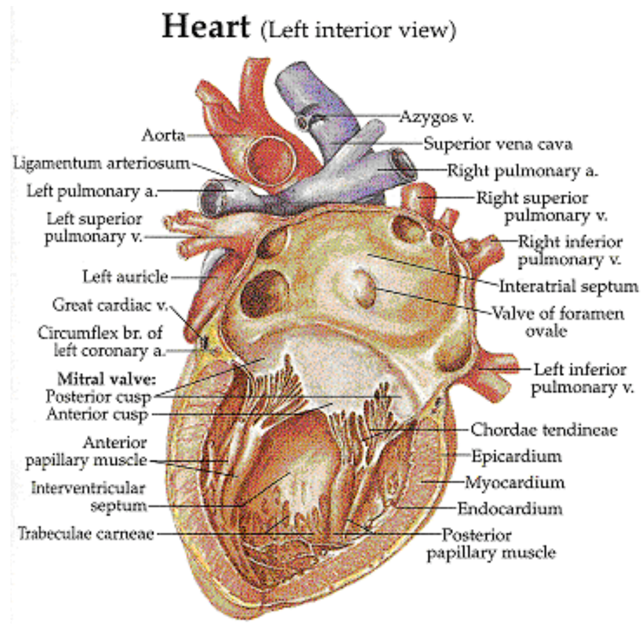
Thoracic Aorta

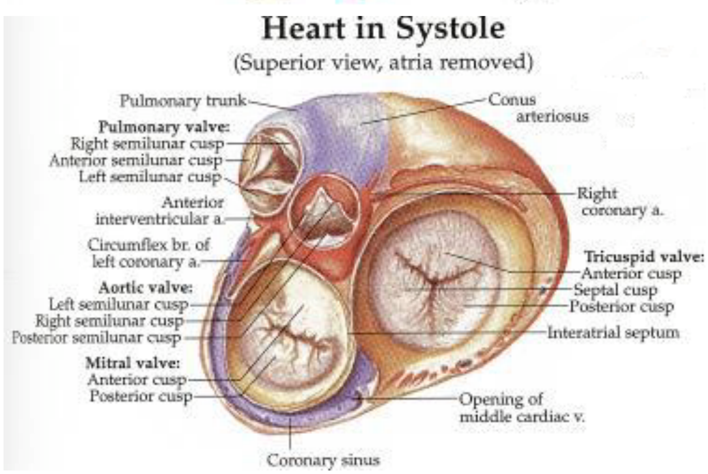


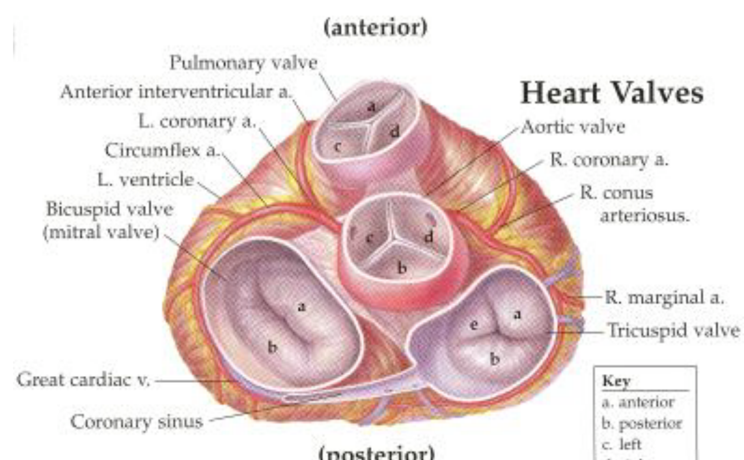
(a) Surface model and path (b) Control mesh (c) Solid NURBS (d) Simulation results

Human Heart Anatomy





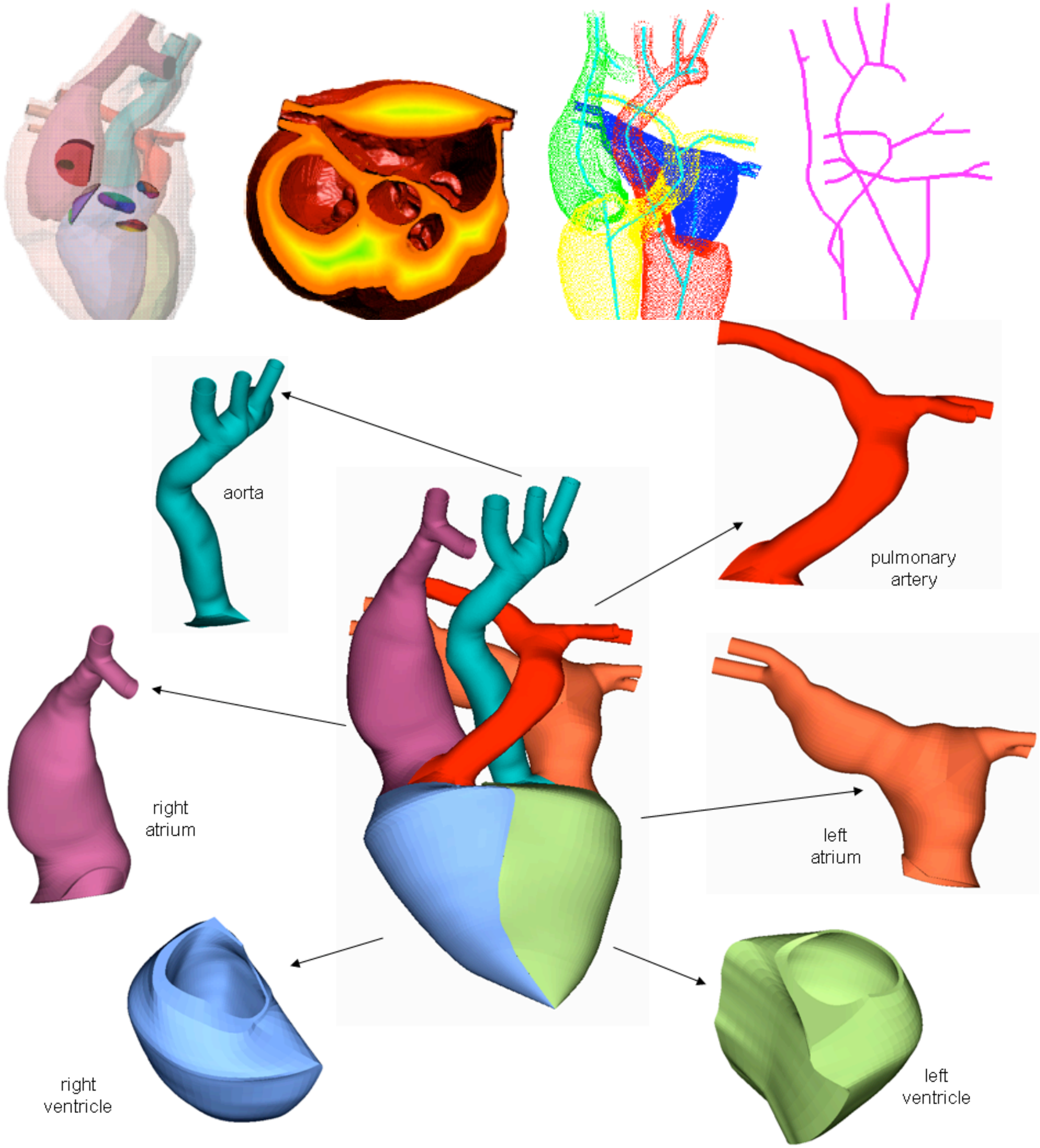




Anatomical Charts [1]

[1] The World's Best Anatomical Charts. Anatomical Chart Company Skokie, IL. ISBN 0-9603730-5-5

Swept Volume Model of the Heart



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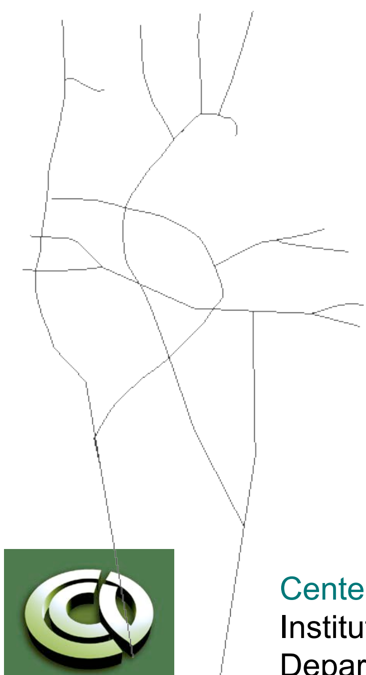
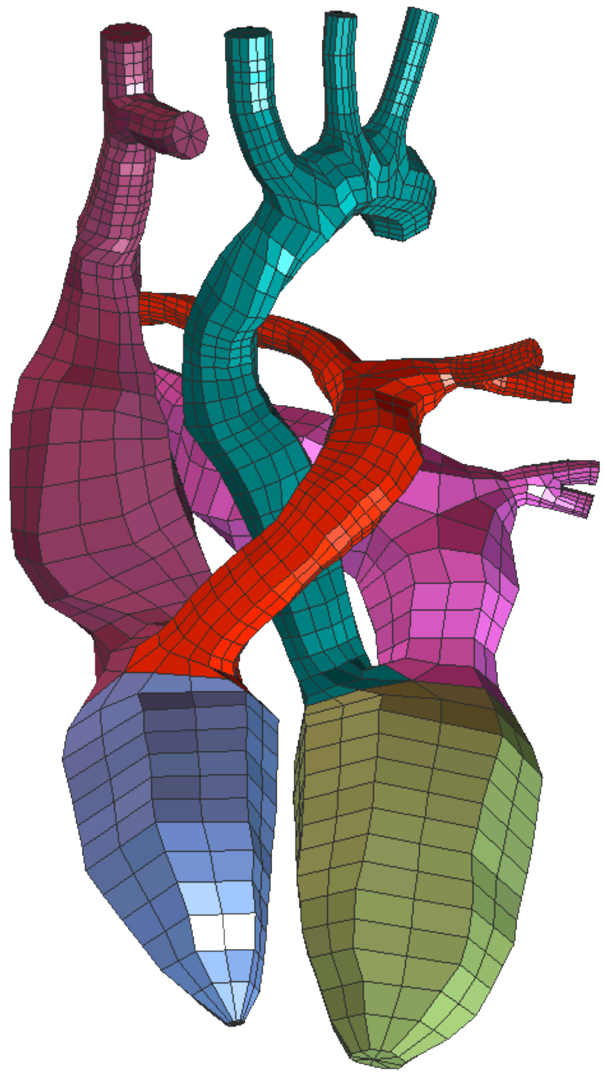
Fluid Volume Mesh

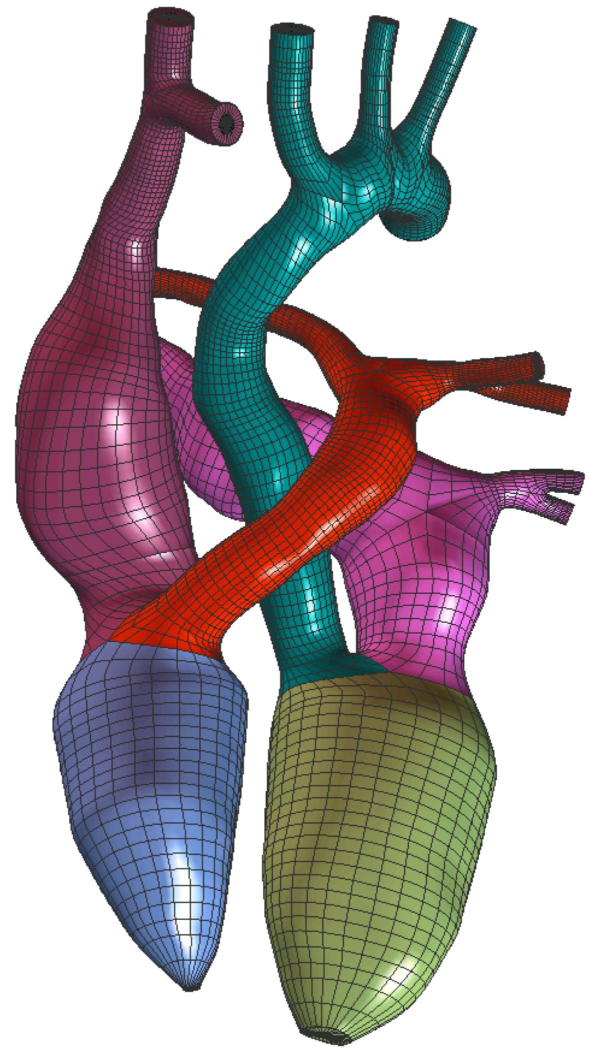
NYU heart model



path

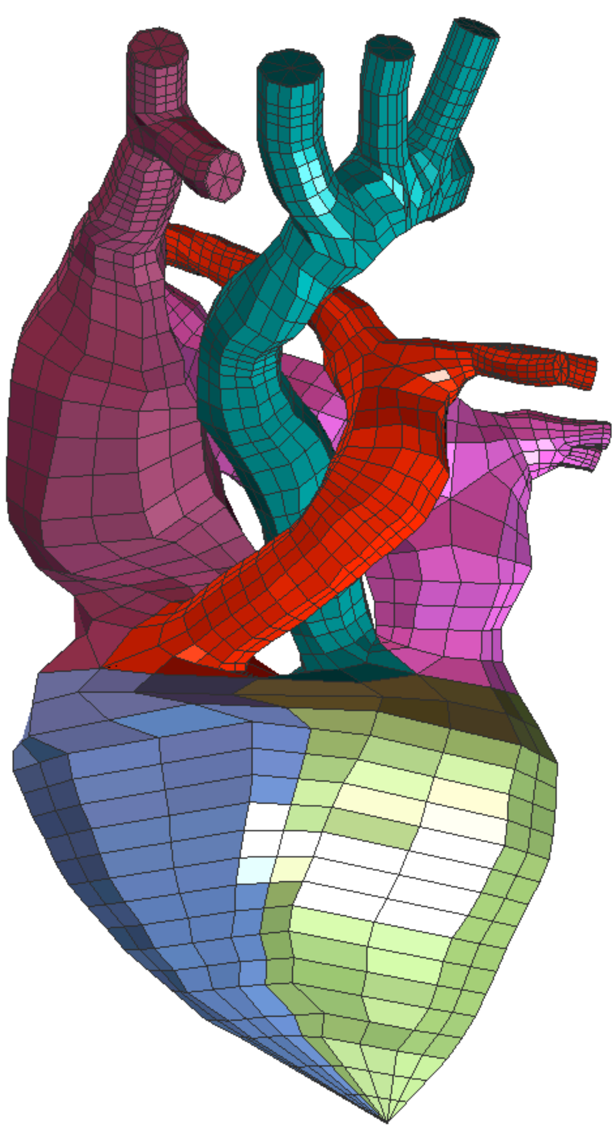


control mesh

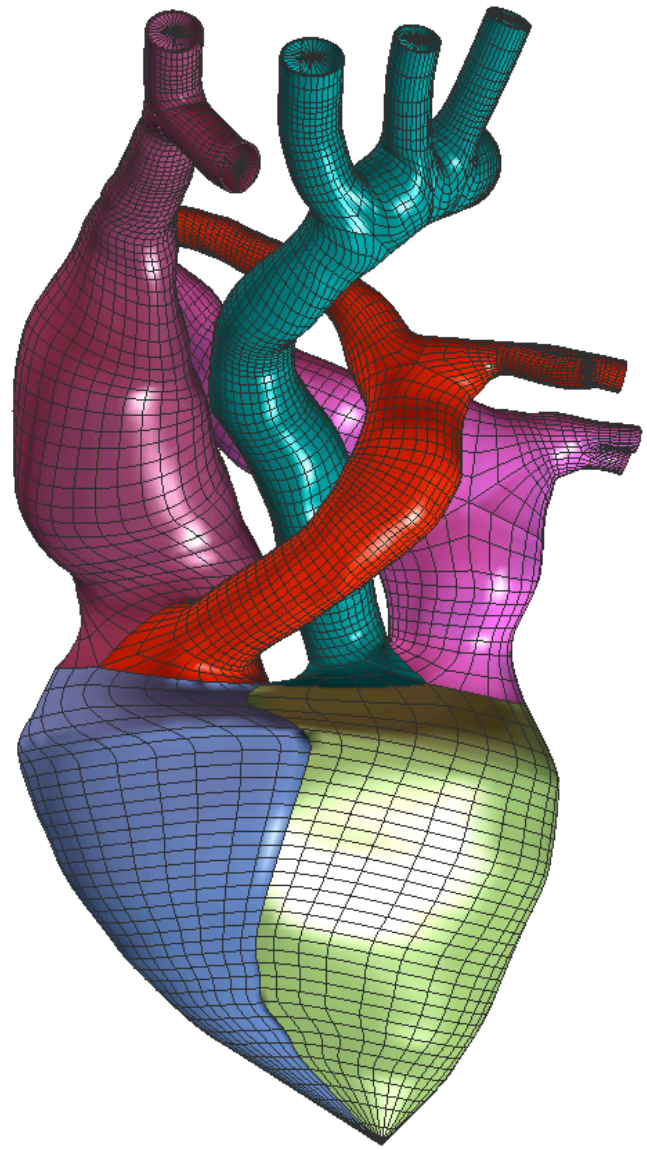


solid NURBS

Muscle Wall



control mesh



solid NURBS

Further Reading

[1] C. Bajaj, E. Coyle, K. Lin. Arbitrary topology shape reconstruction from planar cross sections. *Graphical Models and Image Processing*, 58(6):524-543, Nov.1996.

[2] C. Bajaj, T. Dey, Convex Decompositions of Polyhedra and Robustness. *Siam Journal on Computing*, 21, 2, (1992), 339-364.

[3] MEYERS, D., Multiresolution Tiling. *Computer Graphics Forum* 13, 5 (December 1994), 325--340.

[4] C. Bajaj, E. Coyle, K. Lin. Tetrahedral meshes from planar cross sections. *Computer Methods in Applied Mechanics and Engineering*, Vol. 179 (1999) 31-52

[5] S. Goswami, T. Dey, C. Bajaj **Identifying Flat and Tubular Regions of a Shape by Unstable Manifolds** *Proc. 11th ACM Sympos. Solid and Physical Modeling, pp. 27-37, 2006*

[6] Y. Zhang, Y. Bazilevs, S. Goswami, C. Bajaj, T. J.R. Hughes
Patient-Specific Vascular NURBS Modeling for Isogeometric Analysis of Blood Flow *Proceedings of 15th International Meshing Roundtable, 2006.*