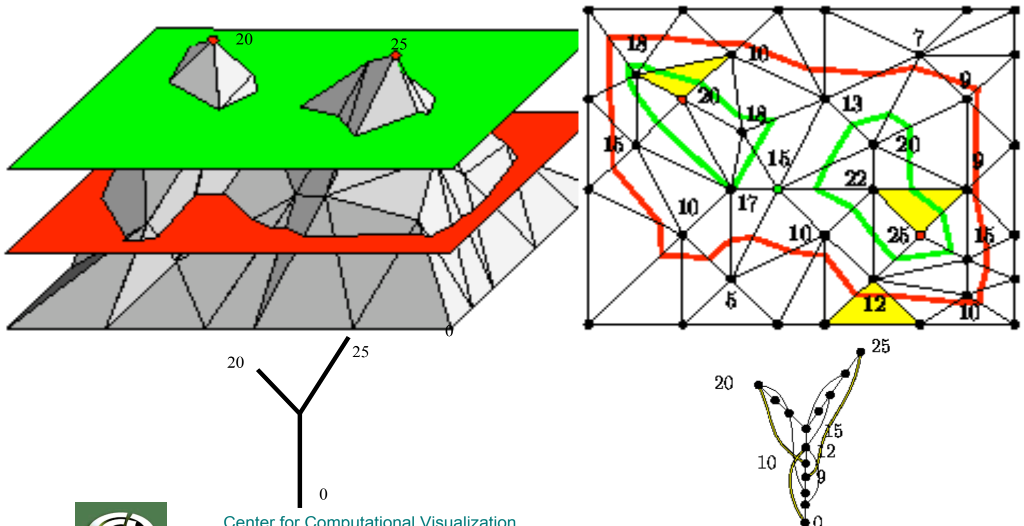
Geometric Modeling and Visualization

http://www.cs.utexas.edu/~bajaj/cs384R08/

Contouring: Capturing the Topology and Geometry of Zero Sets



Center for Computational Visualization
Institute of Computational and Engineering Sciences
Department of Computer Sciences
University of Texas at Austin
October 2007

Lecture 7

Surface Splines from Volumes III: Contouring Scalar Functions

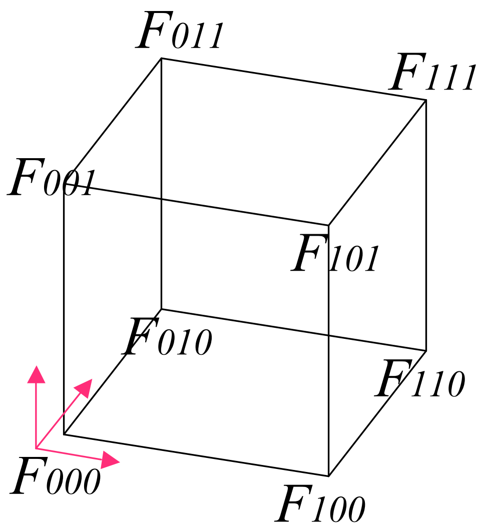
Computational Visualization Center
Institute of Computational and Engineering Sciences
Department of Computer Sciences
University of Texas at Austin

September 2008

Isosurface of Trilinear Function

- Trilinear Function

$$
\begin{aligned}
F(x,y,z) &= F_{000}(1-x)(1-y)(1-z) \\
&+ F_{001}(1-x)(1-y)z \\
&+ F_{010}(1-x)y(1-z) \\
&+ F_{011}(1-x)yz \\
&+ F_{100}x(1-y)(1-z) \\
&+ F_{101}x(1-y)z \\
&+ F_{110}xy(1-z) \\
&+ F_{111}xyz
\end{aligned}
$$

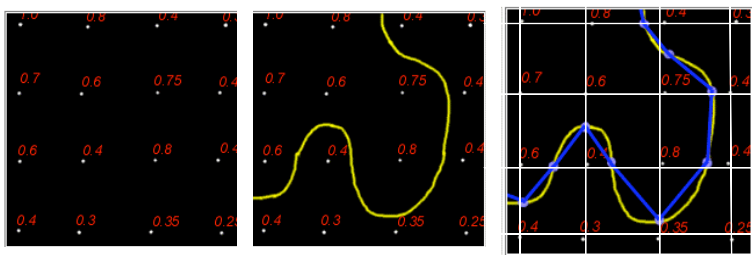


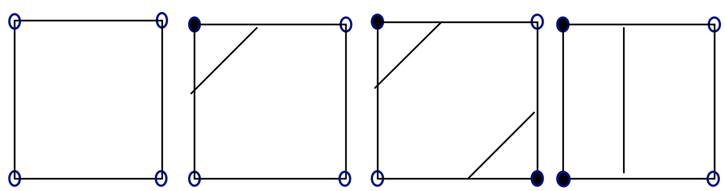
- Bilinear Function

$$
\begin{aligned}
F^f(x,y) &= F_{00}(1-x)(1-y) + F_{01}(1-x)y \\
&+ F_{10}x(1-y) + F_{11}xy
\end{aligned}
$$

Marching Cubes (MC) : Triangular Approximation

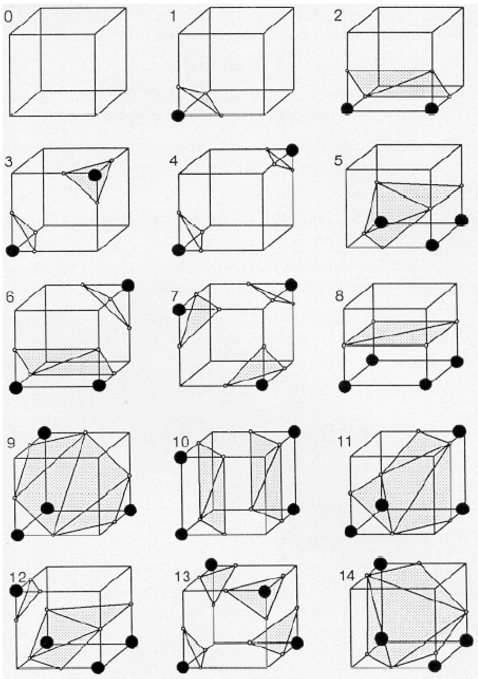
- 2D rectangle





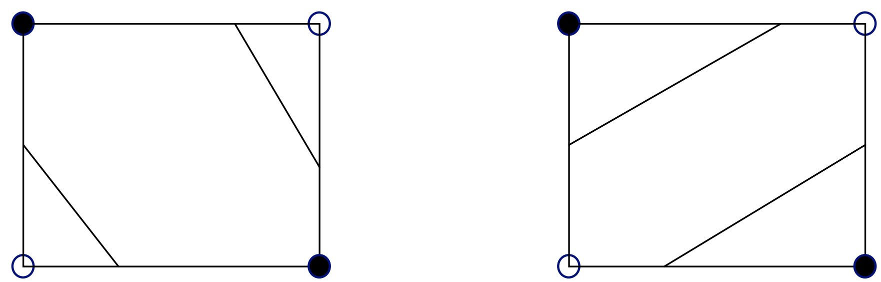
- 3D cube :

  15 Cases for Triangulation

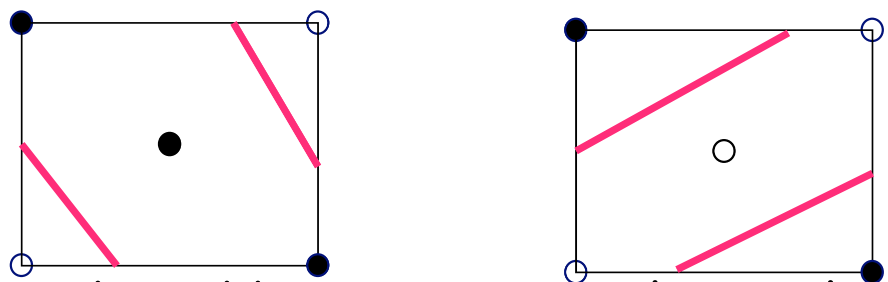


Decision on Contour Topology
(Nielson 92 : Asymptotic Decider)

- Resolving Face Ambiguity
  - Ambiguity (face saddle)



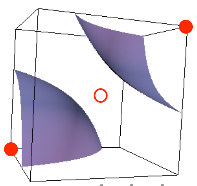
  - Decision based on the value *s* of saddle point



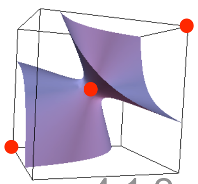
s is positive *s is negative*

Decision on Contour Topology (Natarajan 94)

- Resolving Internal Ambiguity
  - Ambiguity (body saddle)



4.1.1



4.1.2

- Decision based on the value *s* of saddle point

(i) s is positive → tunnel
(ii) s is negative → two pieces

Saddle Points Computation

- Face Saddle Point

  $F(x,y) = ax + by + cxy + d$ (bilinear interpolant)

  First derivatives : $Fx = a + cy = 0$, $Fy = b + cx = 0$

  Saddle point $S = (-b/c , -a/c)$

- Body Saddle Point

$$F(x,y,z) = a + ex + cy + bz + gxy + fxz + dyz + hxyz$$

First derivatives = 0 :

$$F_x = e + gy + fz + hyz = 0$$

$$F_y = c + gx + dz + hxz = 0$$

$$F_z = b + fx + dy + hxy = 0$$

Face and Body Saddle Points

- We obtain saddle points :

$$x = -\frac{c+dz}{g+hz}$$

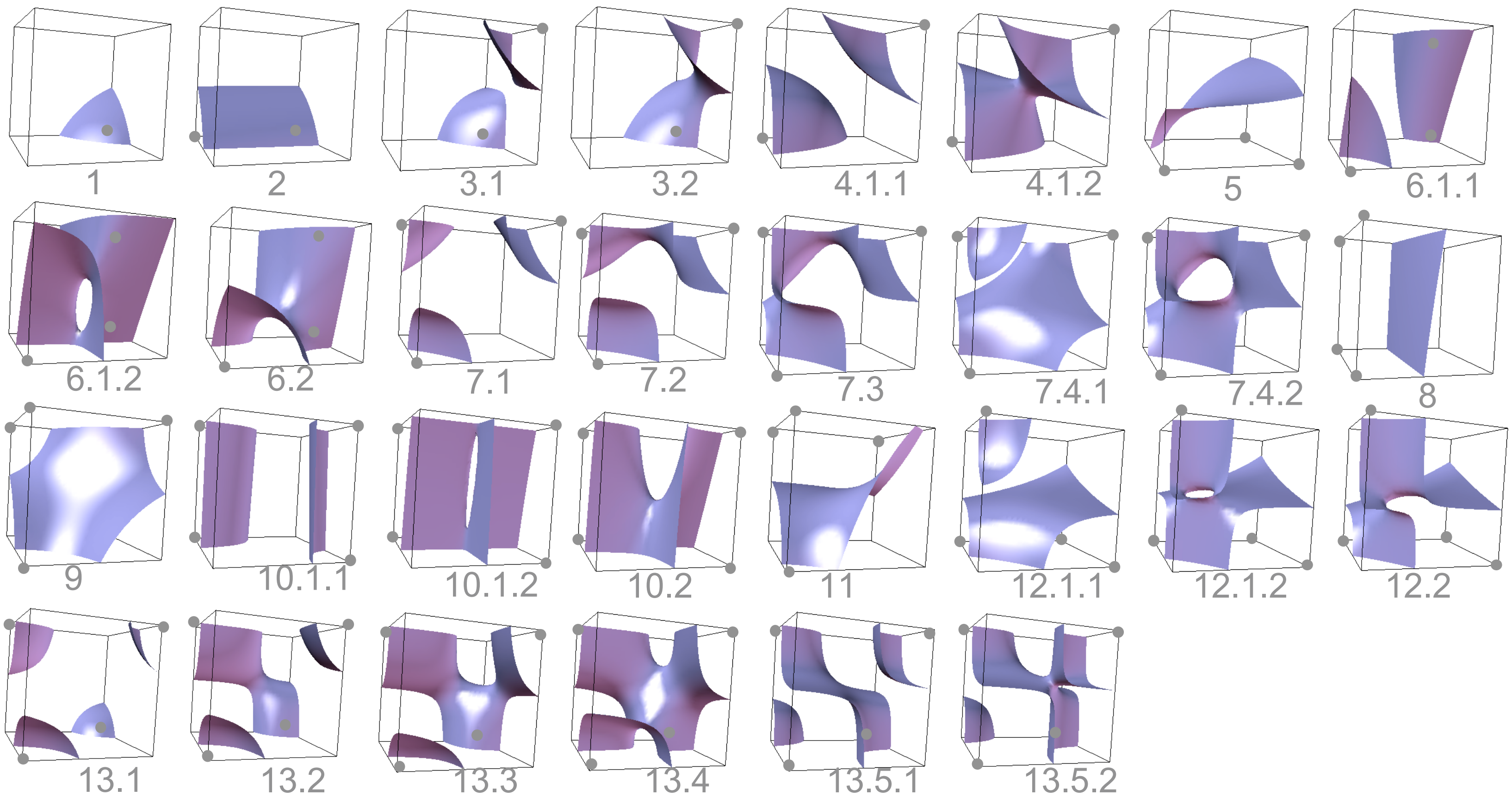
$$y = \frac{k_0+k_1 z}{k_2}$$

$$z = -\frac{g}{h} \pm \frac{\sqrt{g^2 k_1^2 - h k_1^{1/2} (e k_2 + g k_0)}}{h}$$

$$k_0 = cf - bg, k_1 = df - bh, k_2 = dg - ch$$

- saddle point outside the cube → discard
  (only case 13.5 has more than one valid body saddle point.)

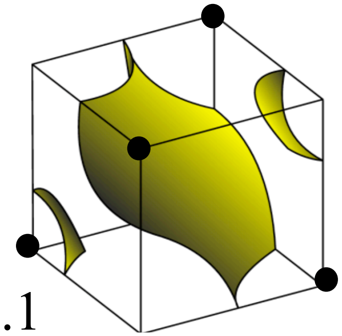
Trilinear Isosurface Topology 31 cases



Lopes, Brodlie 2003

31 Cases

- In the table,



13.5.1

MC case number

Face ambiguity

Interior ambiguity

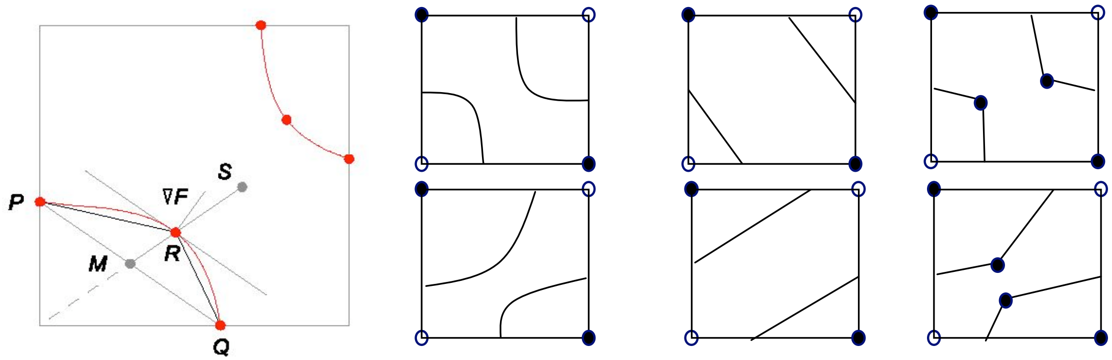
- MC : 15(further reduced to 14) cases based on vertex coloring (symm).

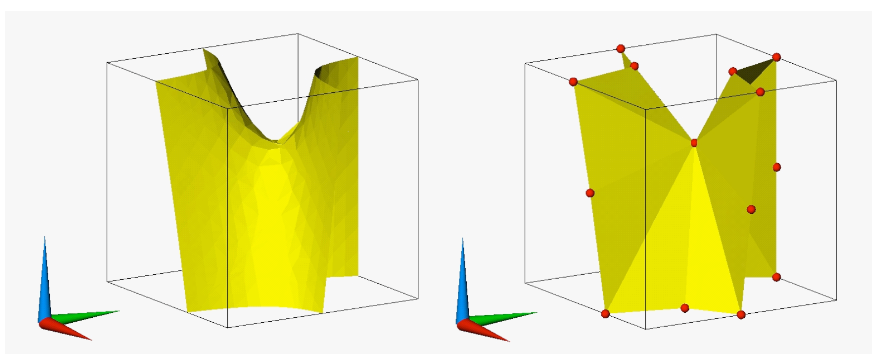
- 31 cases ⟵ (vertex coloring , face ambiguity , internal ambiguity)

Symmetry of different configurations are used to reduce the cases.

Geometric Approximations

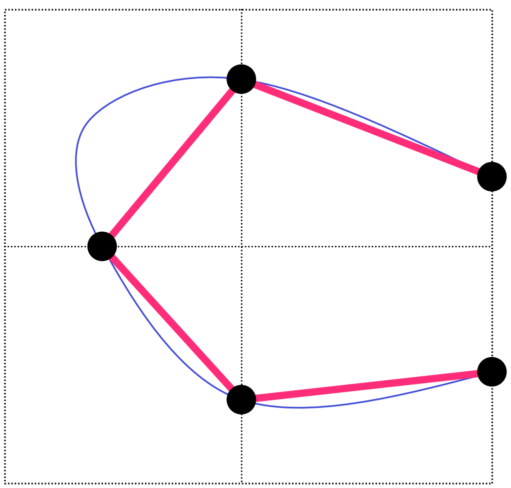
- Better appoximation of trilinear interpolant
  - Adding a shoulder and inflection points



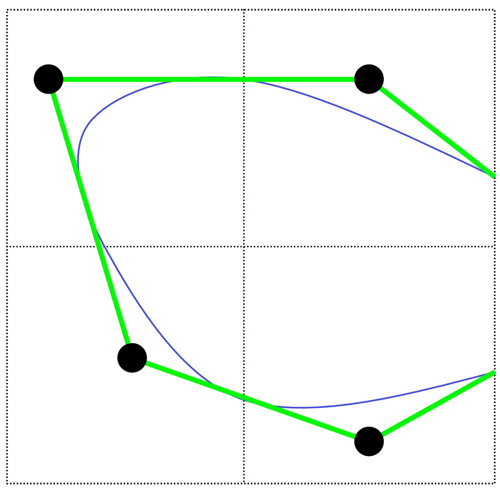


Dual Contouring

- Primal Contouring vs Dual Contouring



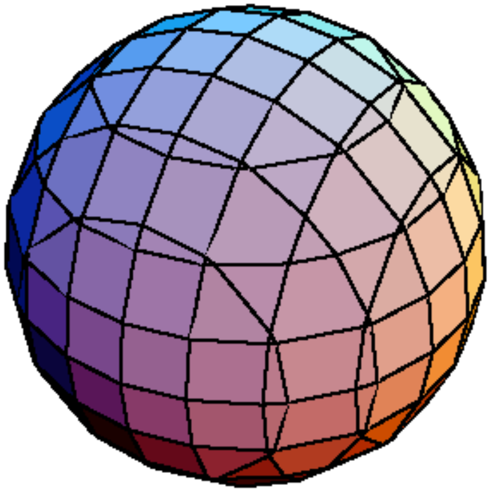
Primal contour

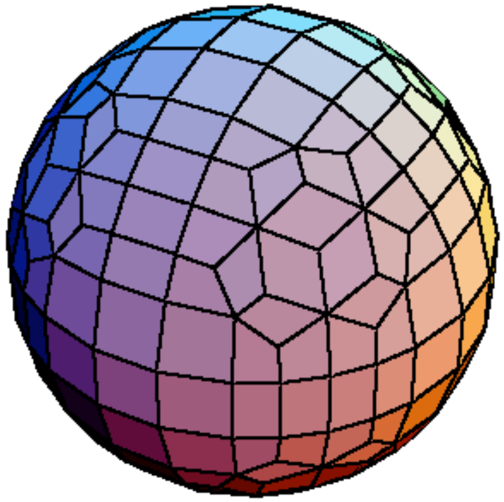


Dual Contour

Dual Contouring

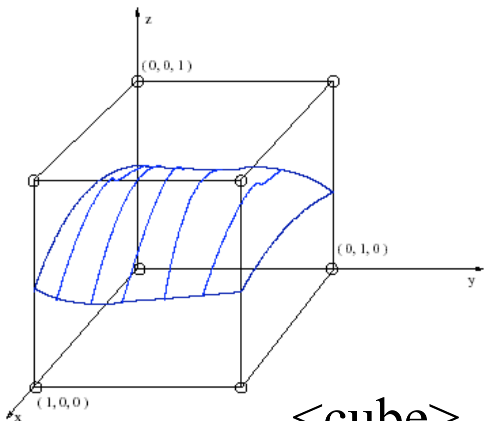
- Polygons with better aspect ratio



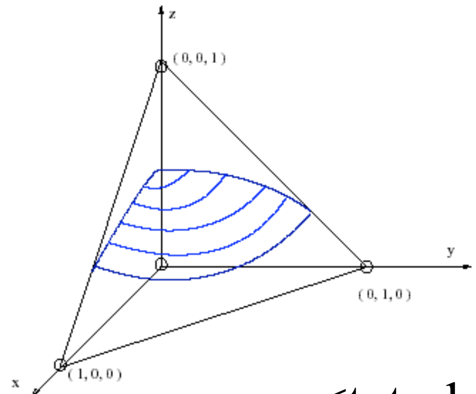


Algebraic Patches: Smooth Boundary Elements

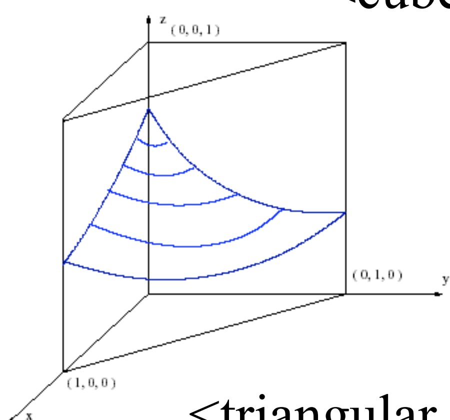
- Implicit form of Isocontour : $f(x,y,z) = w$



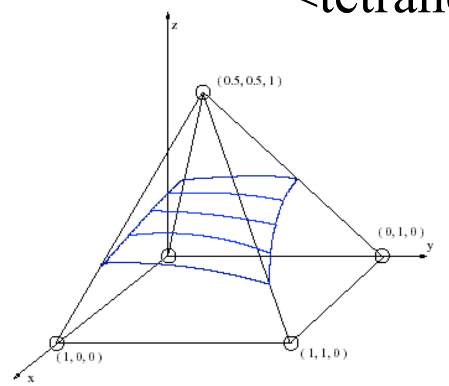
<cube>



<tetrahedron>



<triangular prism>



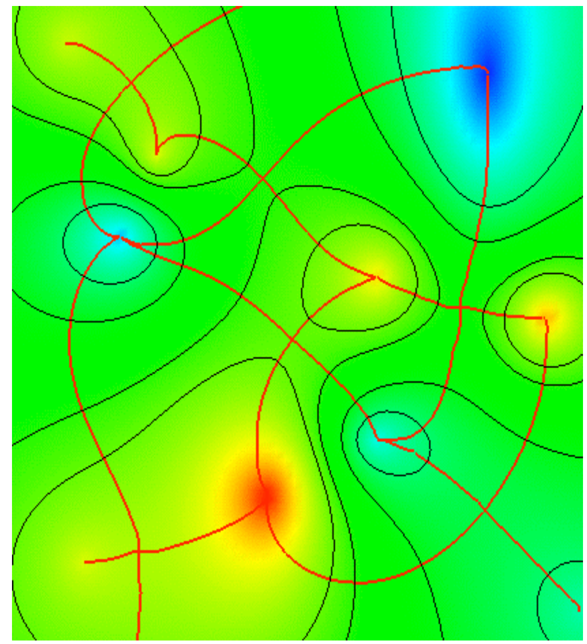
<square pyramid>

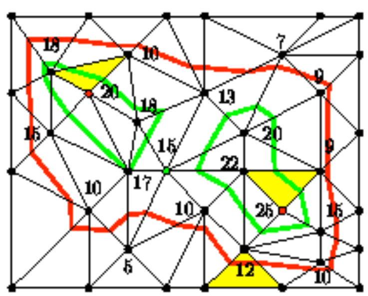
Interactive Isocontour Queries

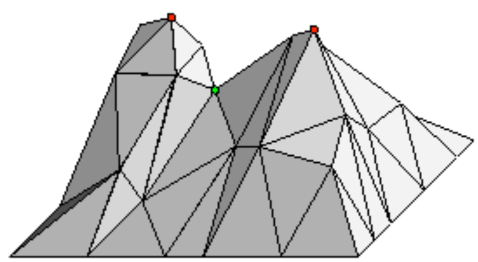
- Input:
  - Scalar Field *F* defined on a mesh
  - Multiple Isovalues *w* in unpredictable order

- Output (for each isovalue *w*):

  Contour *C(w)* = {*x* | **F**(*x*) = *w*}



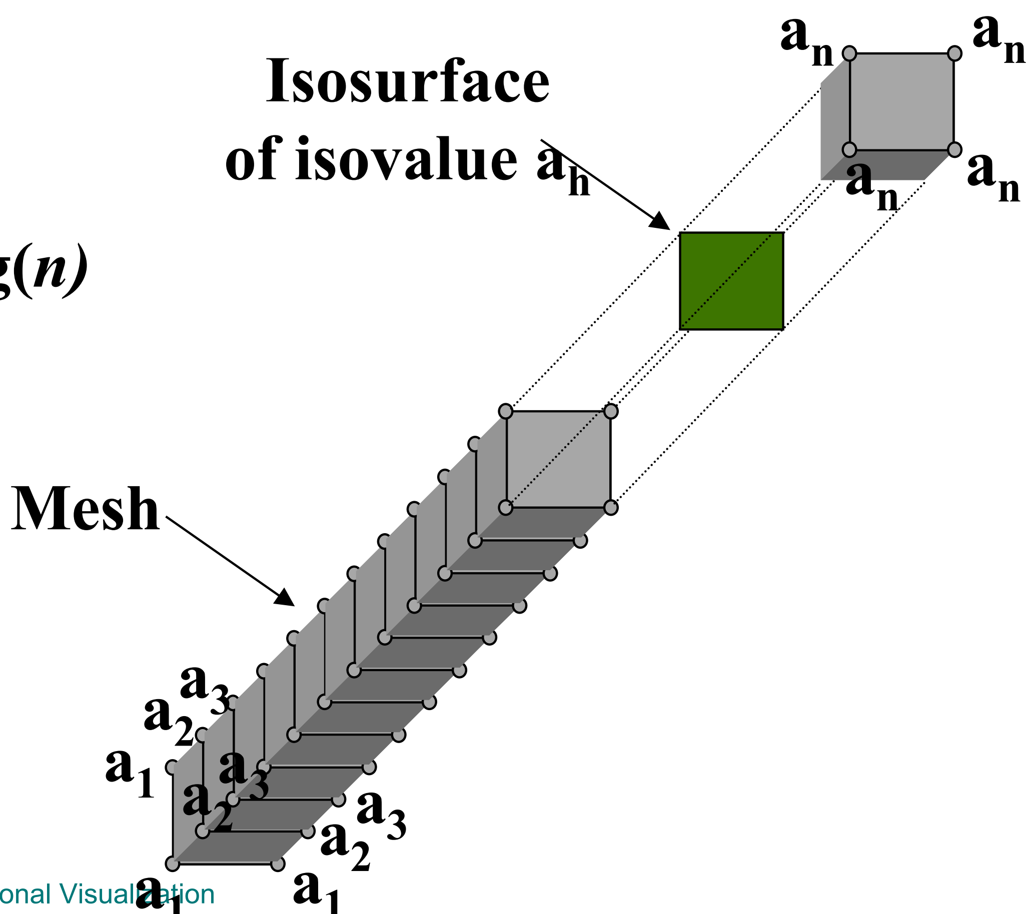




Isocontour Query Problem

Lower Bound

Input size n } $m + \log(n)$
Output size m

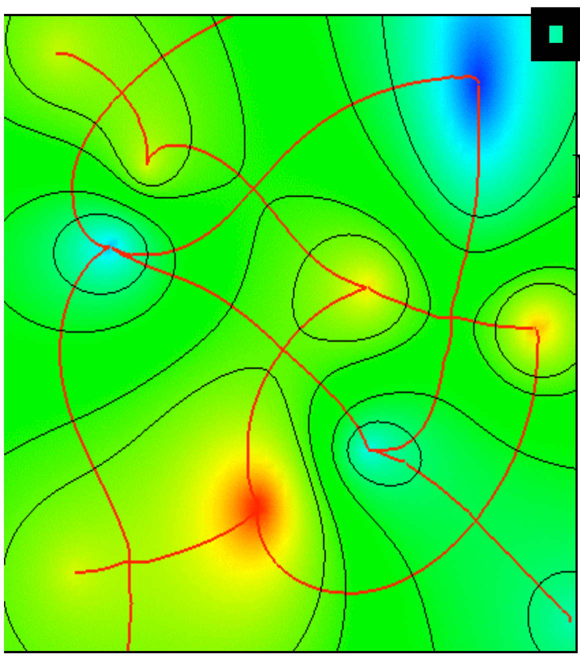


The search for a_h takes at least $\log(n)$

$a_1, a_2, a_3, a_4, \ldots, a_n$

Optimal Single-Resolution Isocontouring

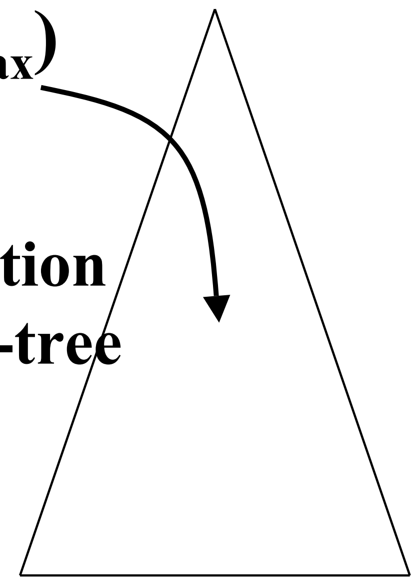
The basic scheme



Preprocessing: (f_{min}, f_{max})

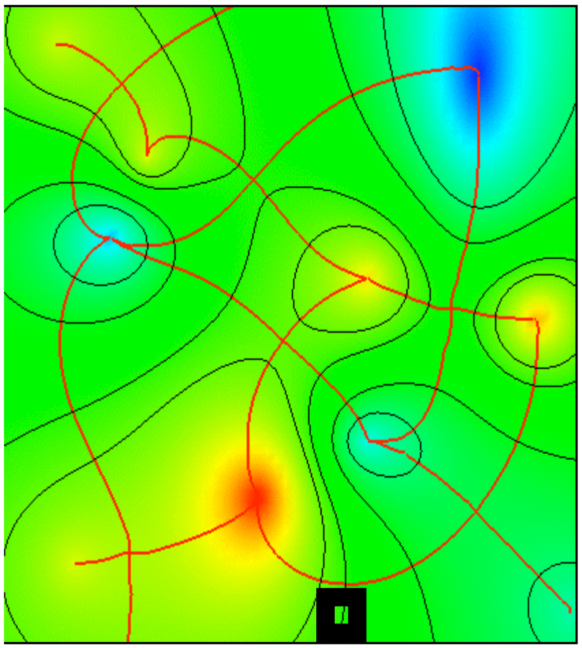
For each cell c in *M*

Enter its range of function values into an interval-tree



Optimal Single-Resolution Isocontouring

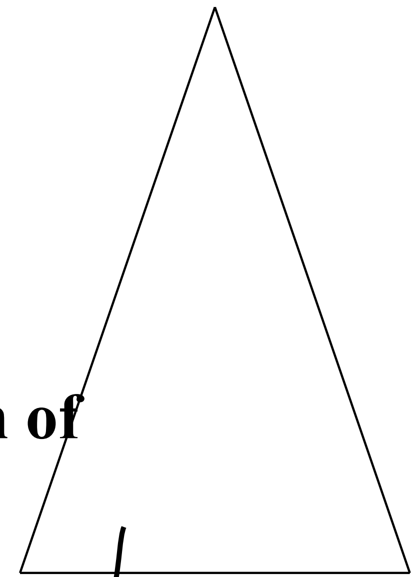
The basic scheme



Isocontour query *W*

For each interval containing *W*

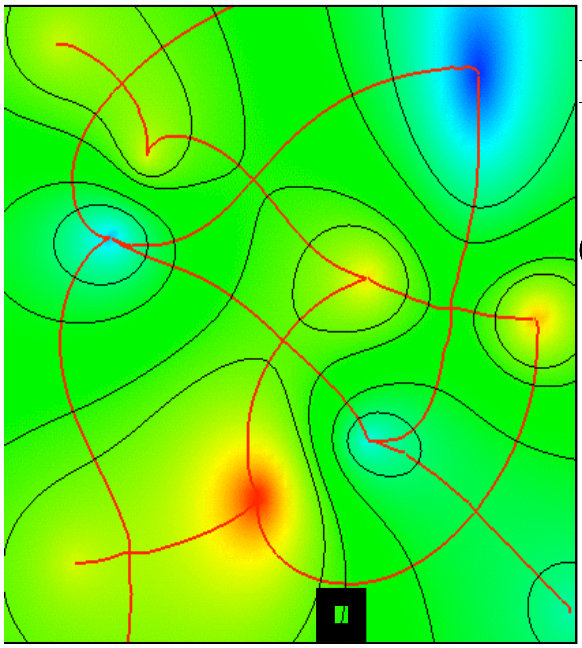
Compute the portion of isocontour in the corresponding cell



(f_{min}, f_{max})

Optimal Single-Resolution Isocontouring

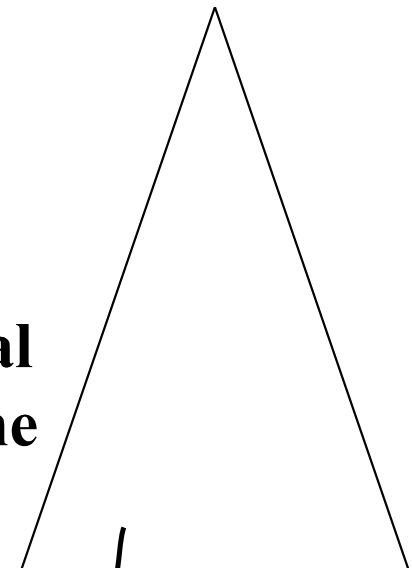
The basic scheme



Isocontour query *W*

Complexity: $m + \log(n)$

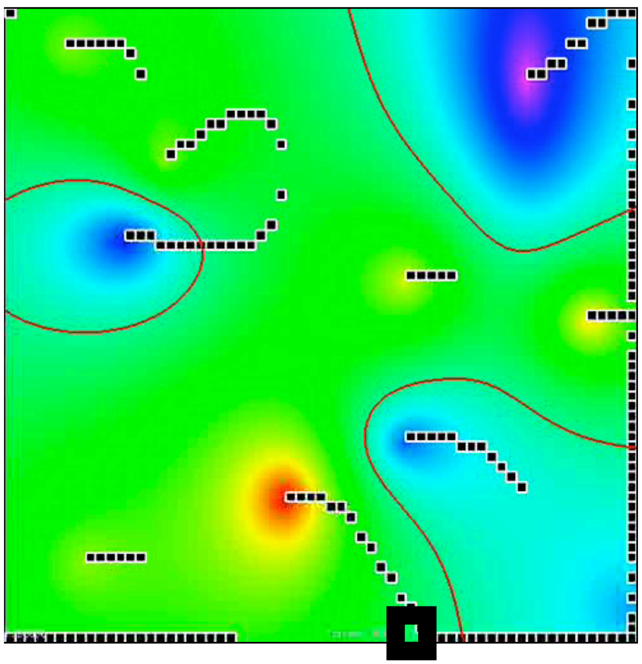
Optimal but impractical because of the size of the interval-tree



(f_{min}, f_{max})

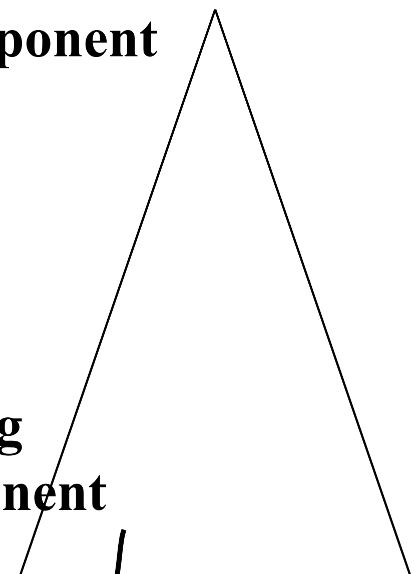
Optimal Single-Resolution Isocontouring

Seed Set Optimization



For each connected component we need only one cell (and then propagate by adjacency in the mesh)

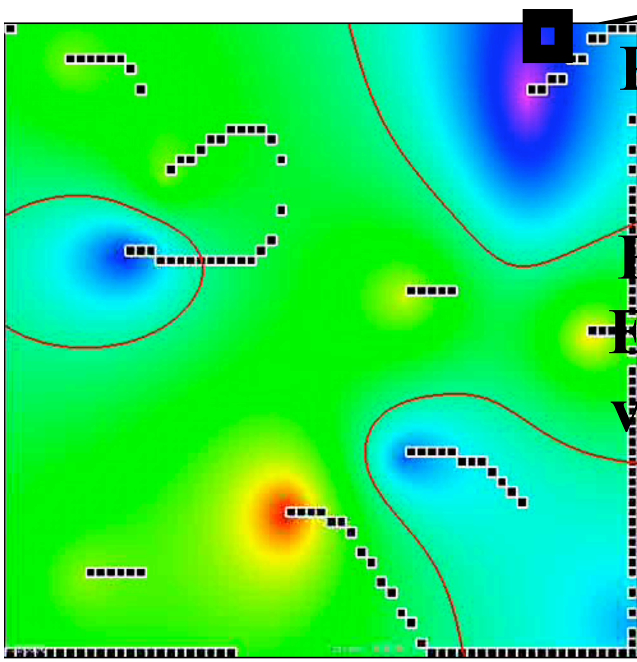
Seed Set: a set of cells intersecting every connected component of every isocontour



(f_{min}, f_{max})

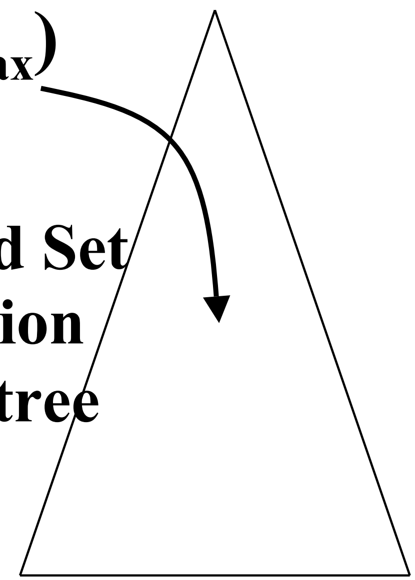
Optimal Single-Resolution Isocontouring

The basic scheme



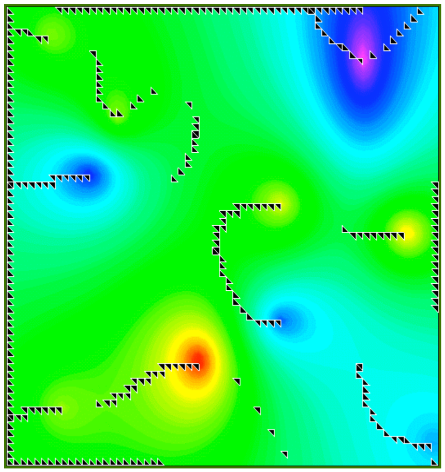
Preprocessing (f_{min}, f_{max})
(revised):

For each cell c in a Seed Set
Enter its range of function values into an interval-tree

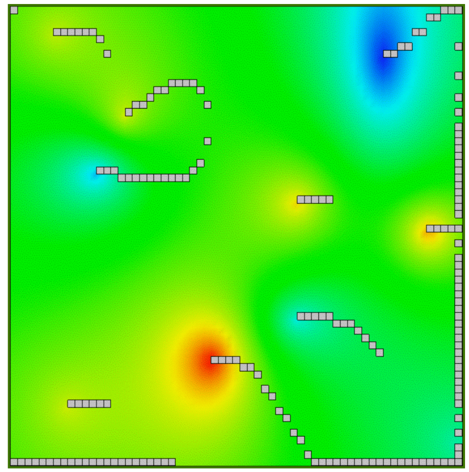


Optimal Single-Resolution Isocontouring

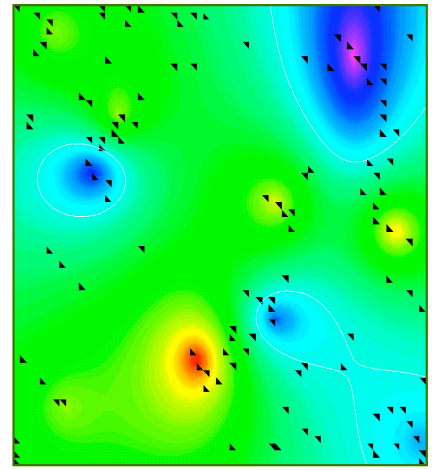
Seed Set Generation (*k* seeds from *n* cells)



Domain Sweep



Responsibility Propagation

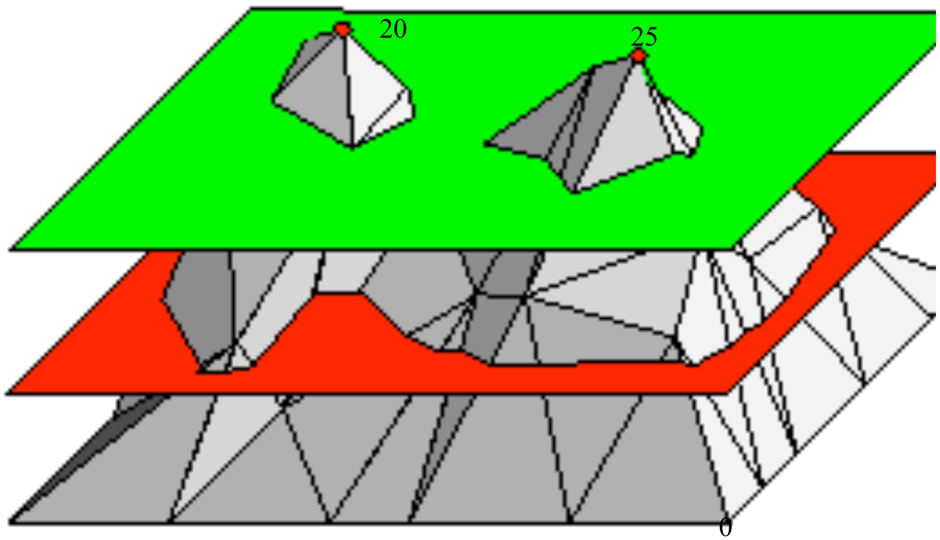


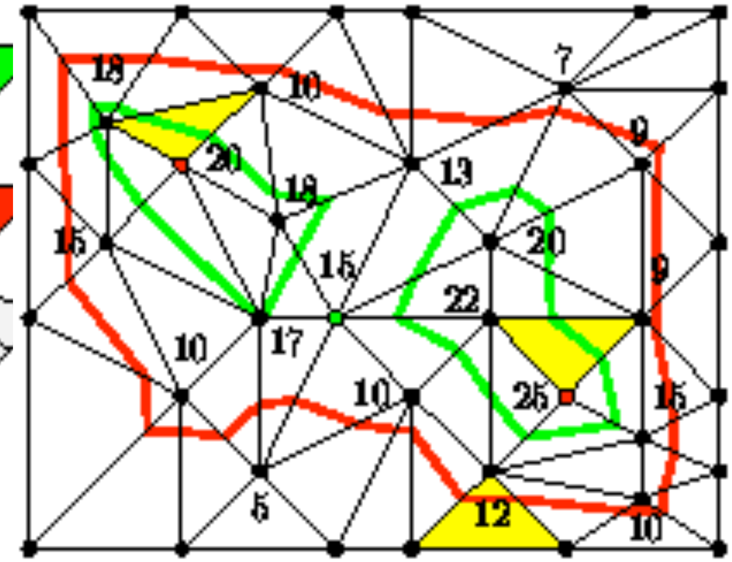
Range Sweep

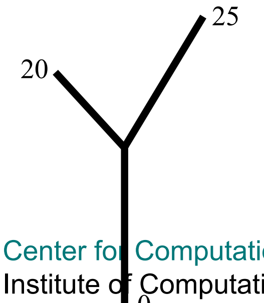
	Domain Sweep	Responsibility Propagation	Range Sweep
Time	**O(*n*)**	**O(*n*)**	**O(*n* log *n*)**
Space	**O(*k*)**	**O(*k*)**	**O(*n*)**
***k* =**	**?**	**?**	**2 k_{min}**
Test	238 seed cells 0.01 seconds	177 seed cells 0.05 seconds	59 seed cells 1.02 seconds

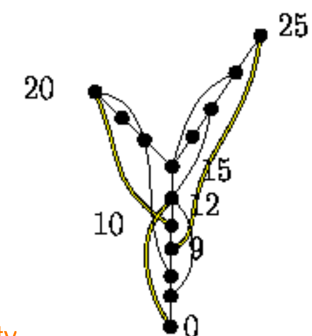
Optimal Single-Resolution Isocontouring

Contour tree

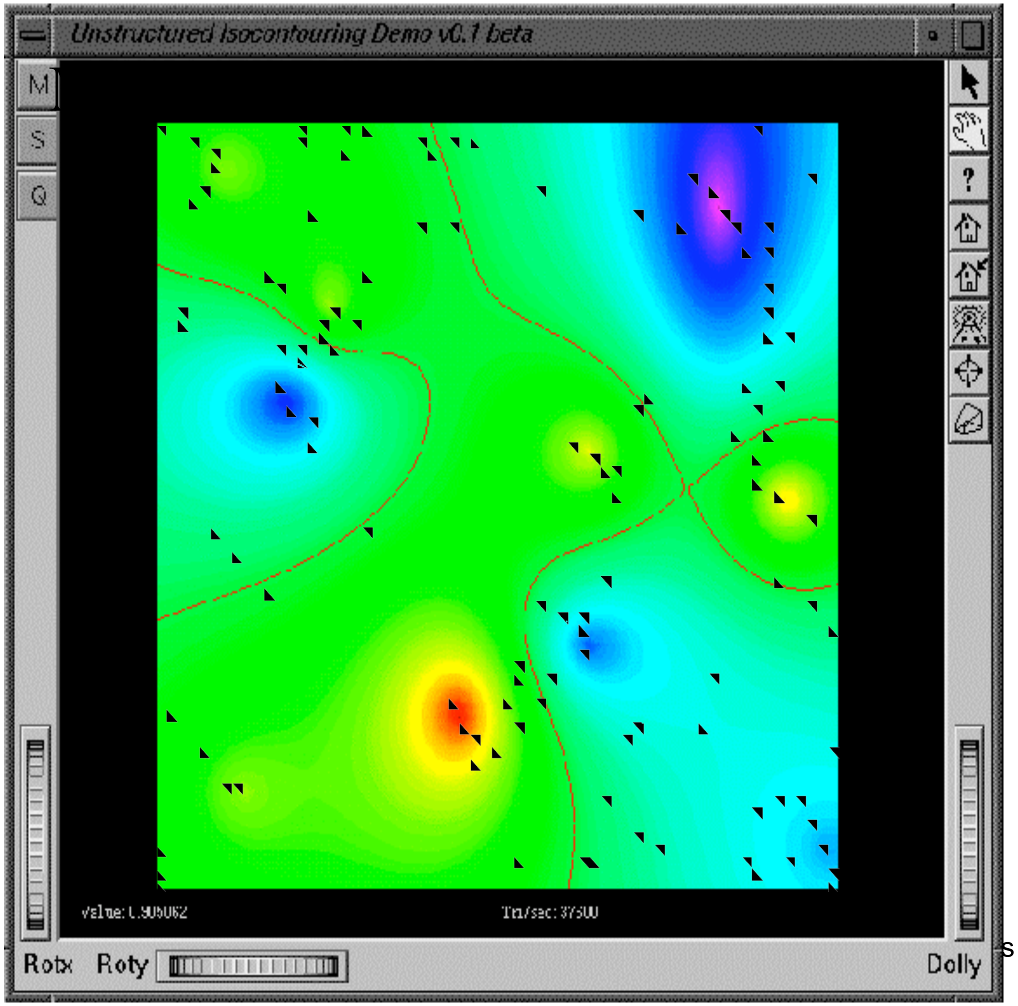


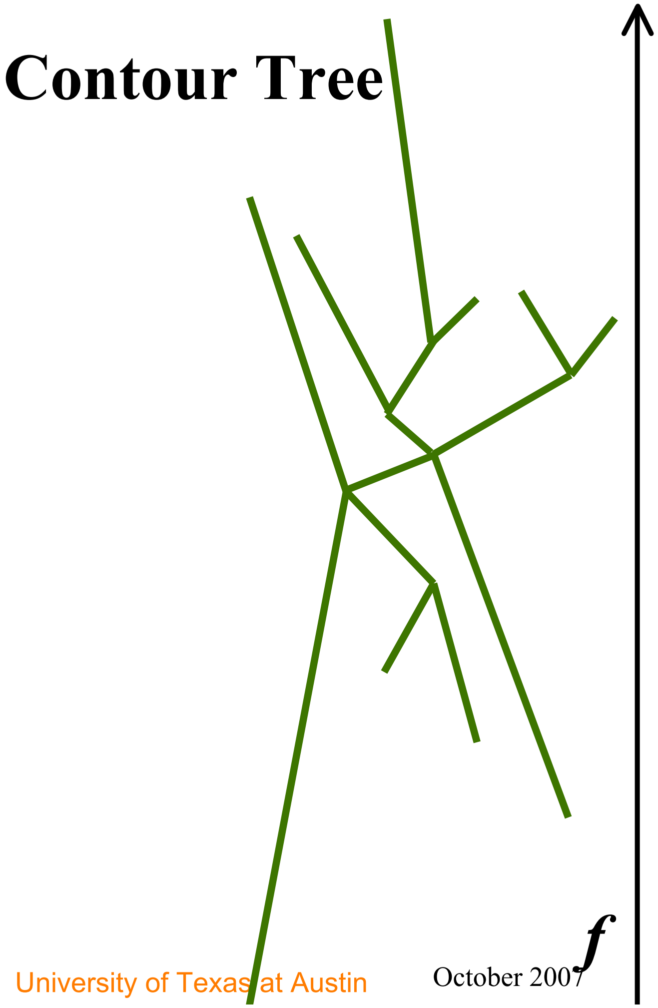




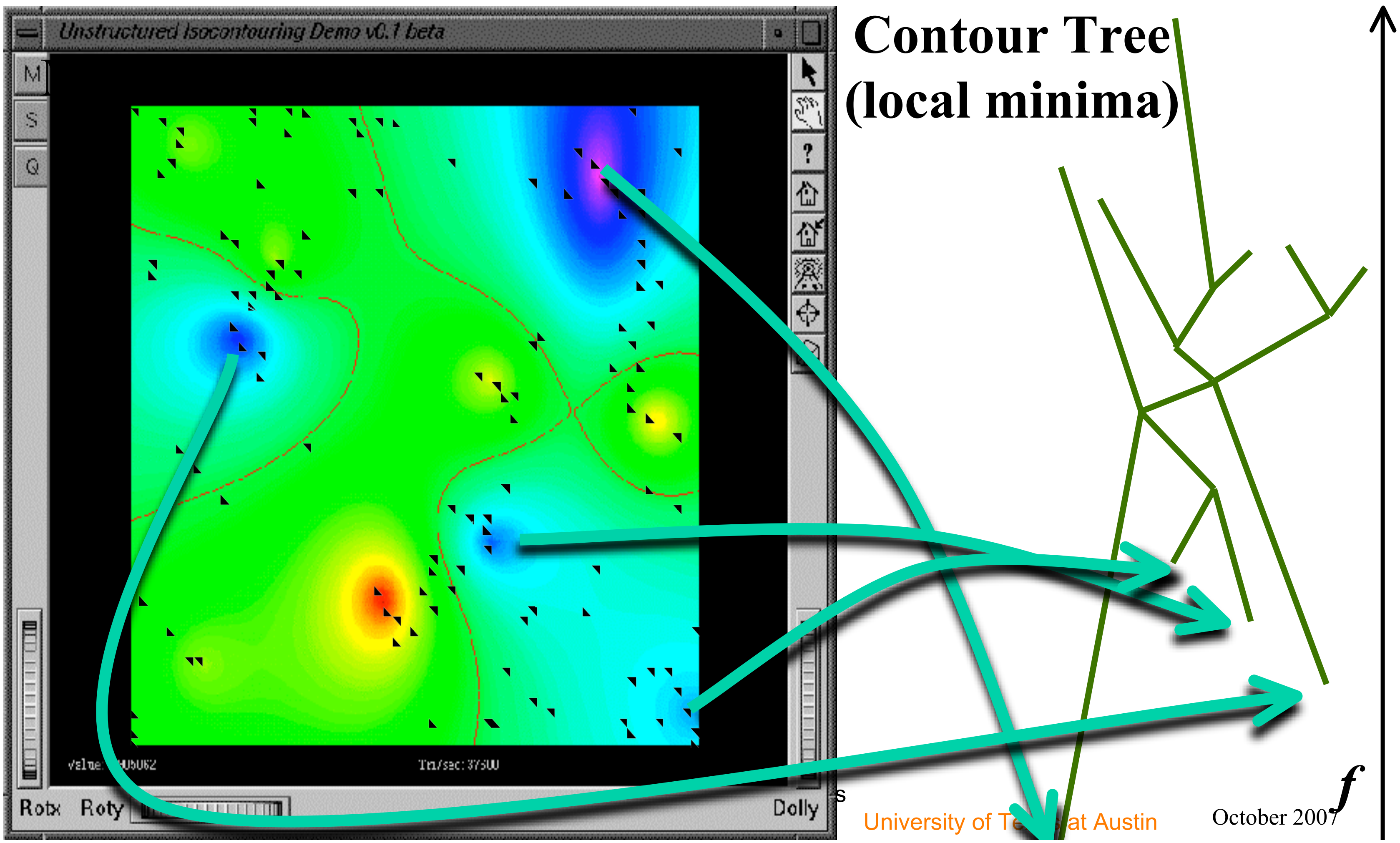


Optimal Single-Resolution Isocontouring

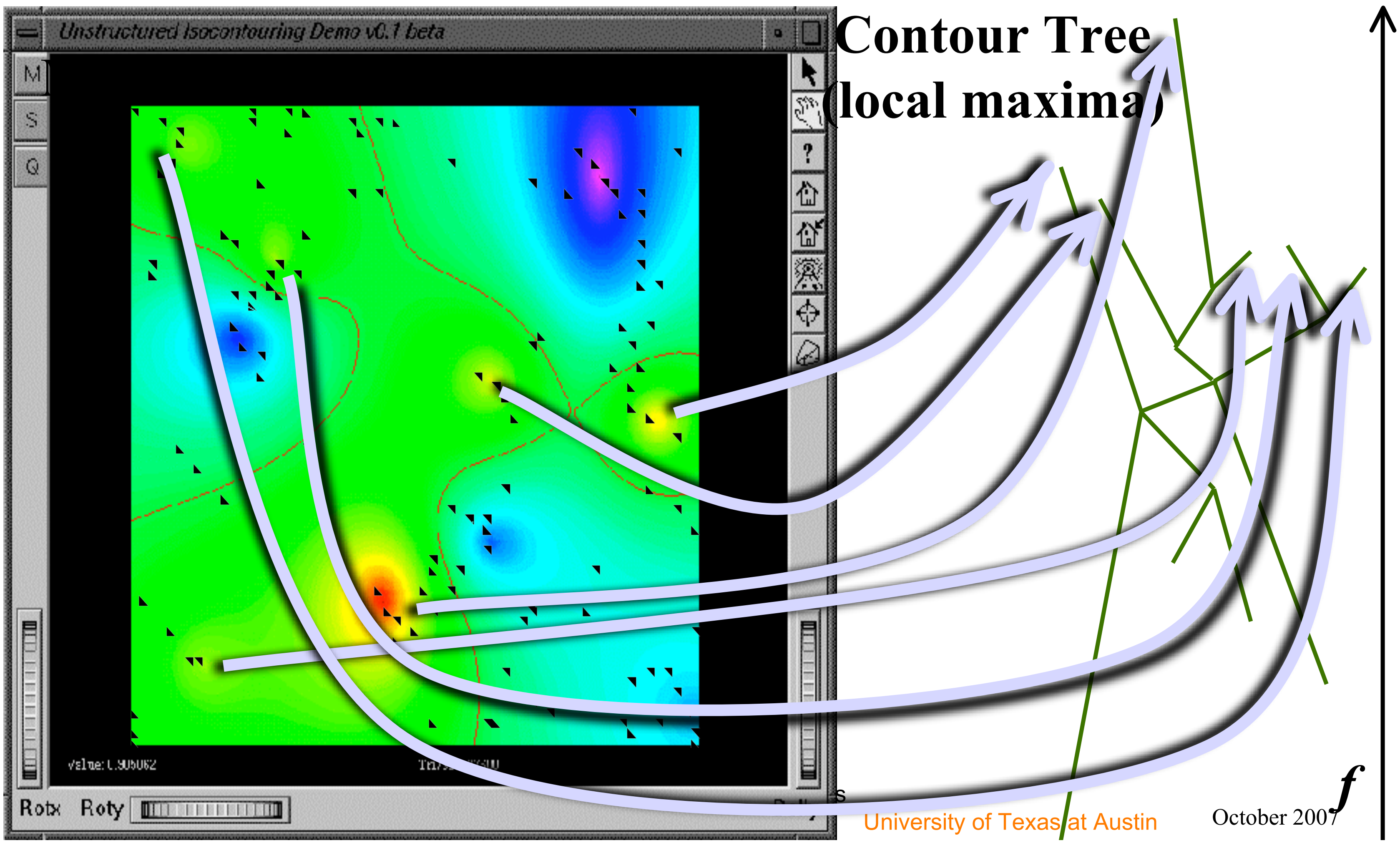




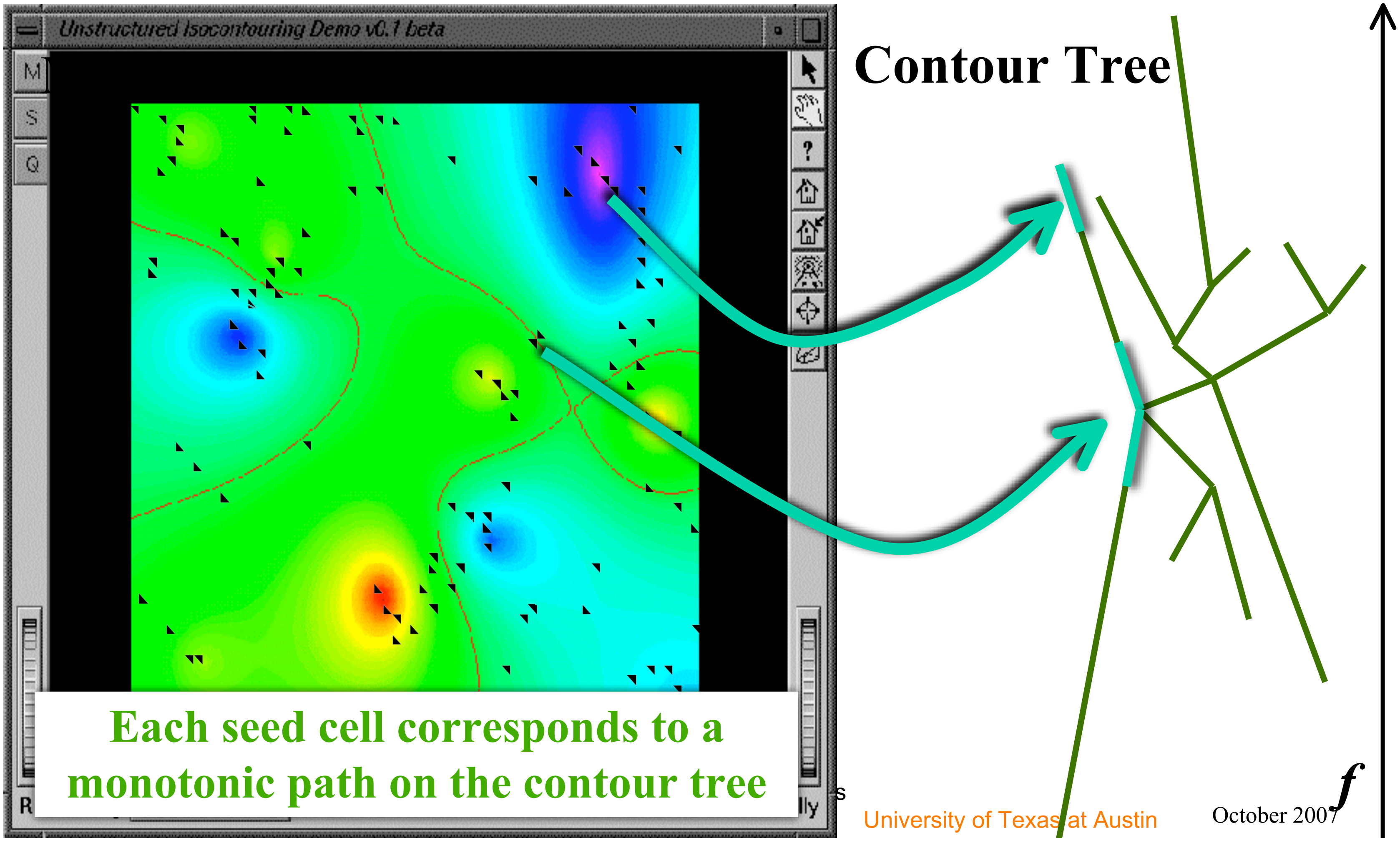
Optimal Single-Resolution Isocontouring



Optimal Single-Resolution Isocontouring

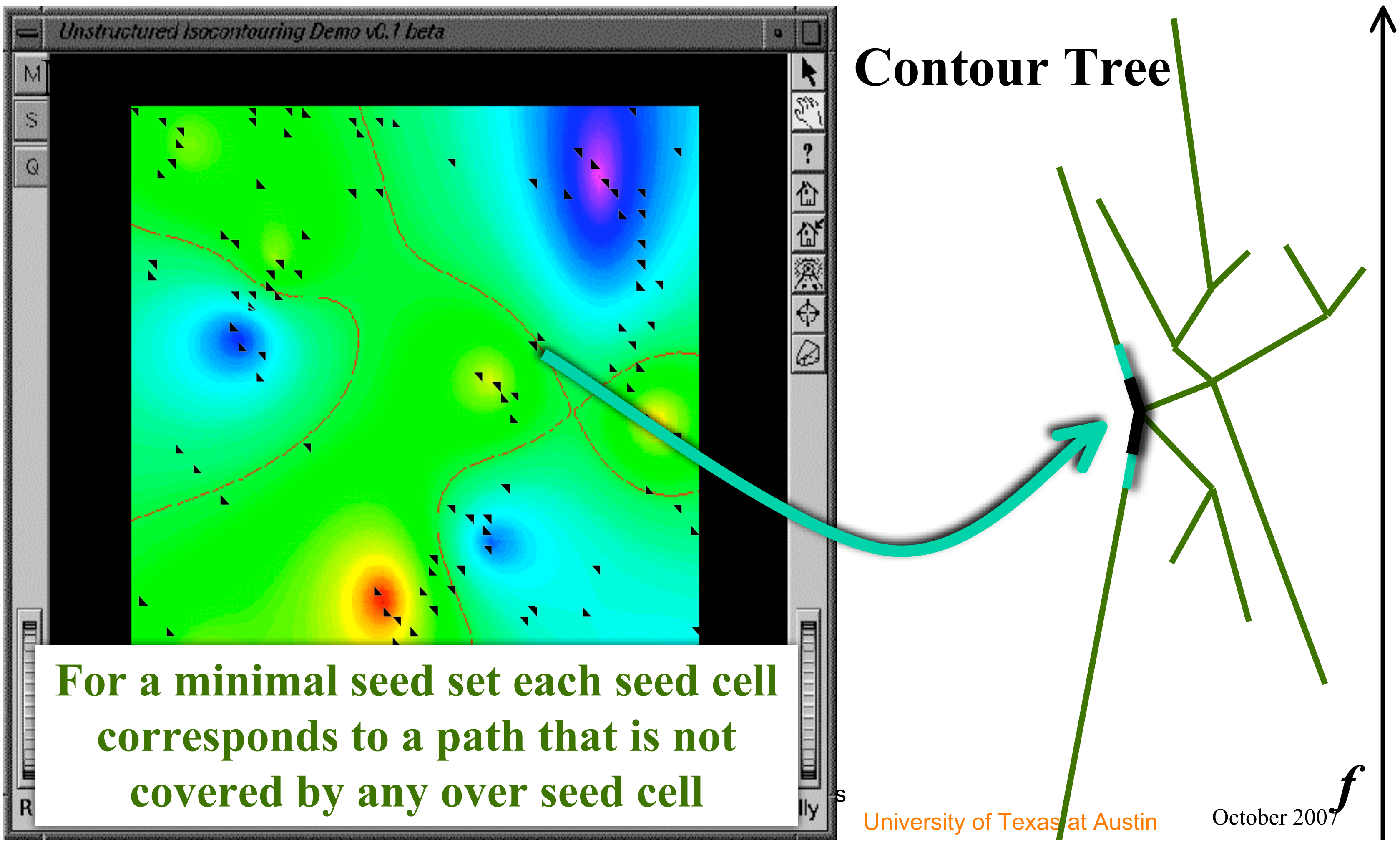


Optimal Single-Resolution Isocontouring



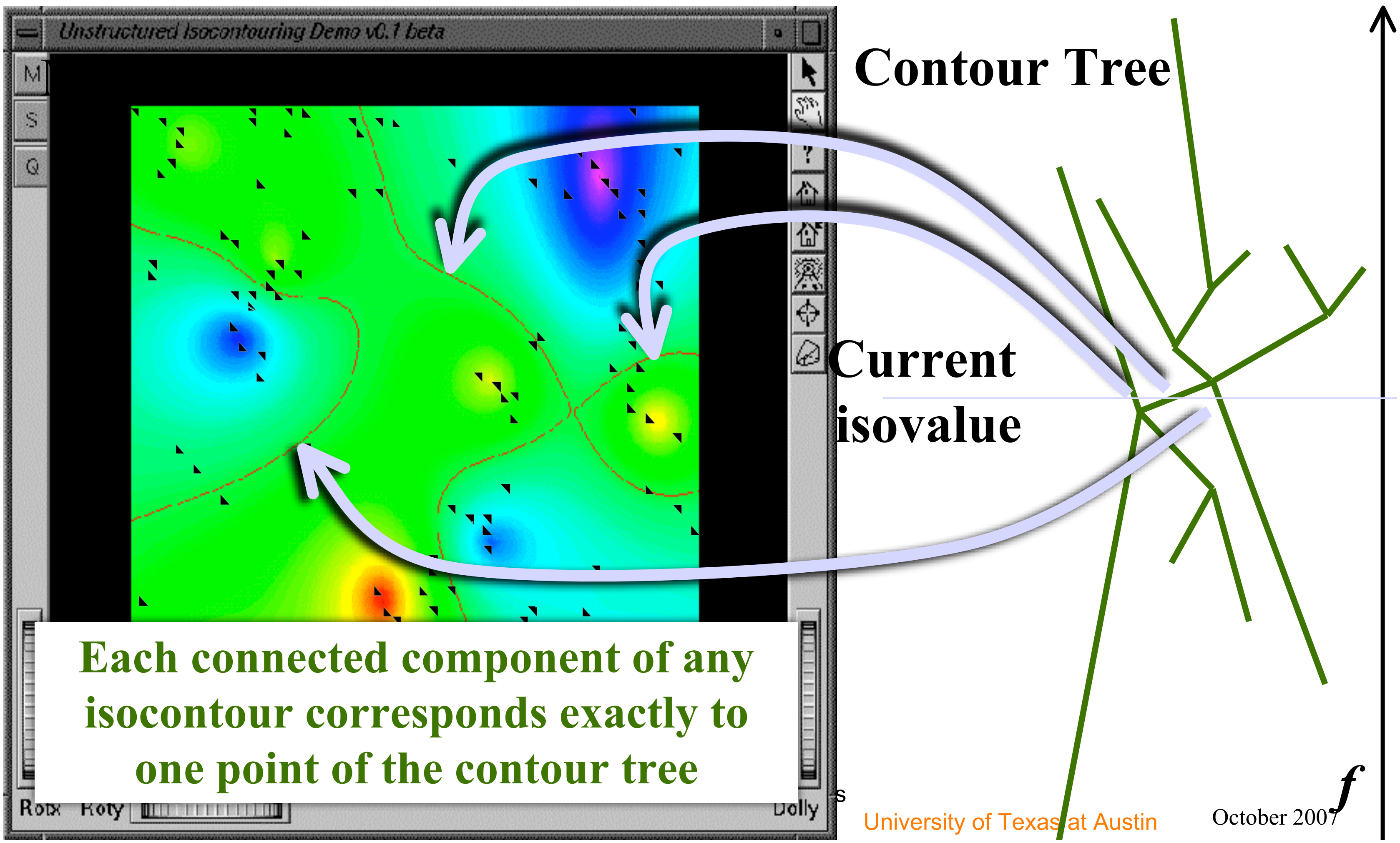
Each seed cell corresponds to a monotonic path on the contour tree

Optimal Single-Resolution Isocontouring



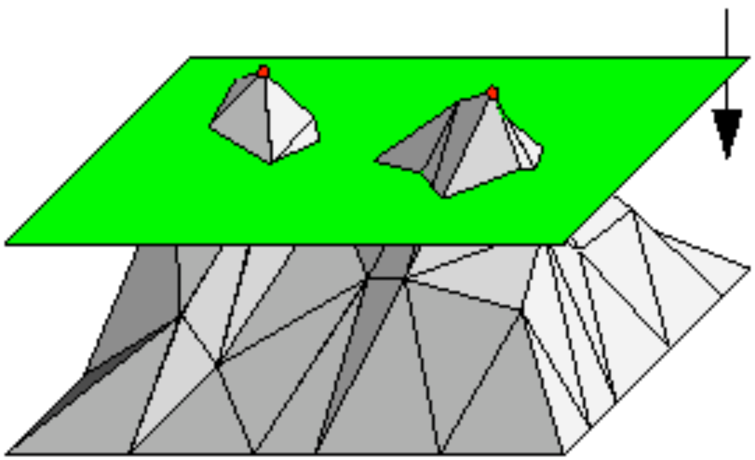
For a minimal seed set each seed cell corresponds to a path that is not covered by any over seed cell

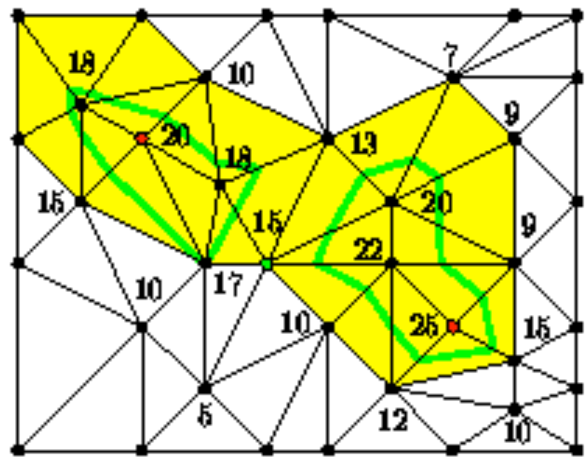
Optimal Single-Resolution Isocontouring

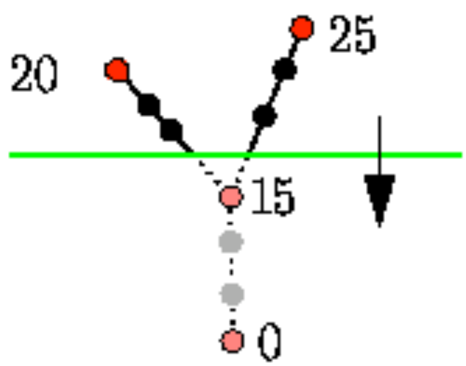


Each connected component of any isocontour corresponds exactly to one point of the contour tree

Optimal Single-Resolution Isocontouring

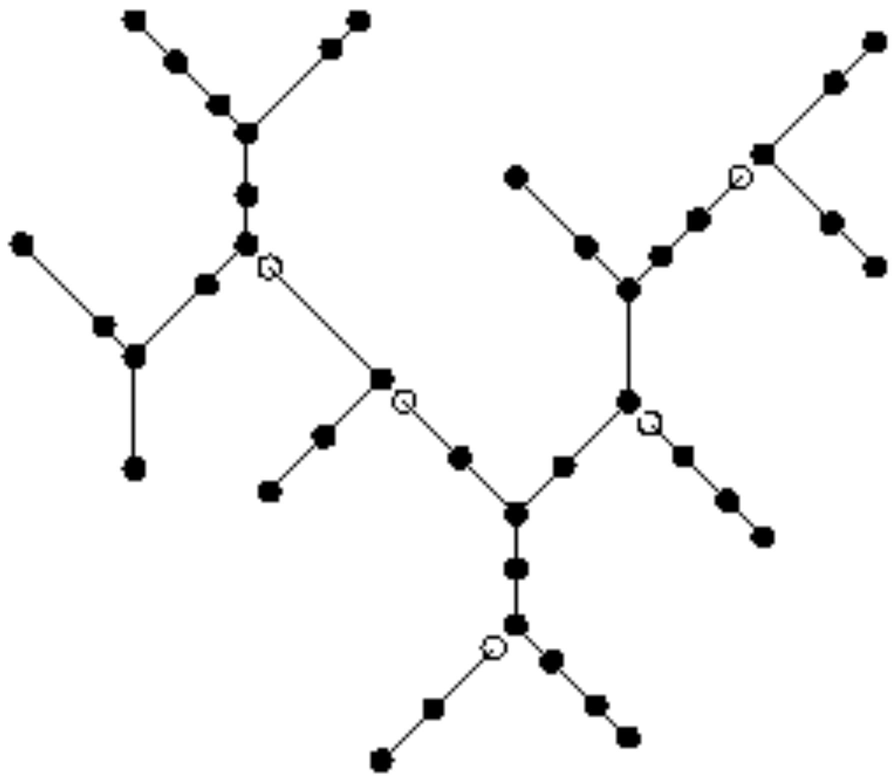






Optimal Single-Resolution Isocontouring

- The number of seeds selected is the minimum plus the number of local minima.



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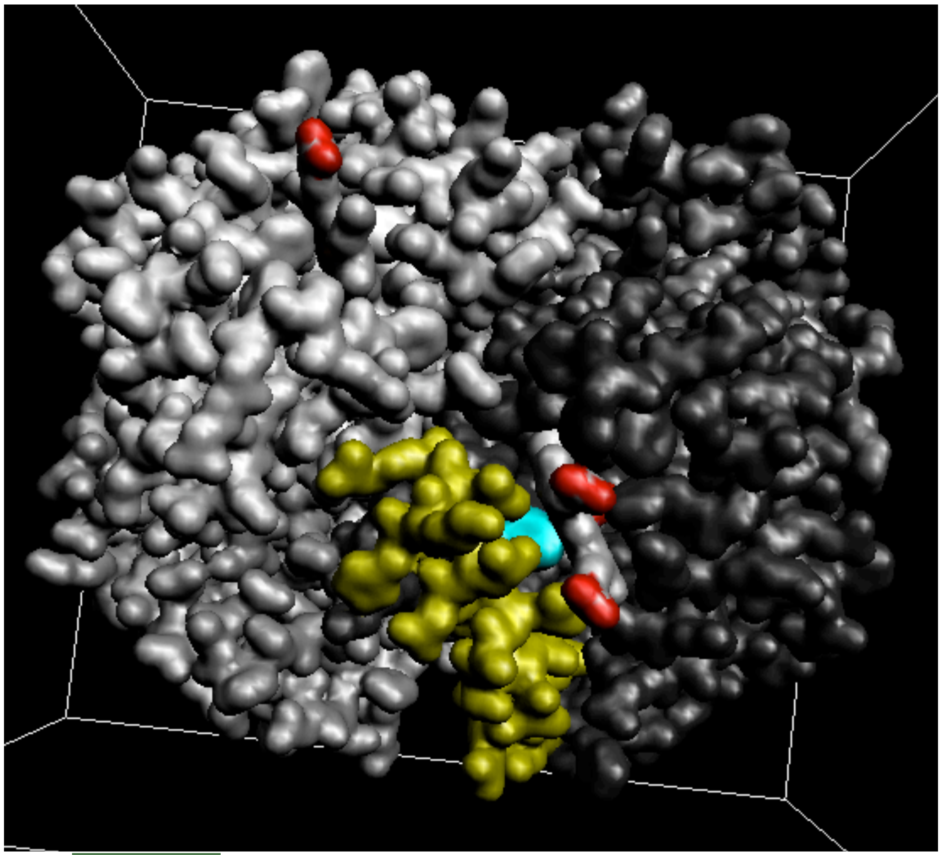
Institute of Computation
Department of Compute

Structural Analysis

Contour Spectrum and Contour Tree on Hemoglobin Dynamics



Subunit A
Subunit B
Subunit C
Subunit D

Within Subunit A

F helix
Histidine Ligand(HIS87)
O_2

- Oxy process : O2 binds to the Fe2+ ion on the opposite side of the histidine ligand. F helix shifts position through the oxy-deoxy cycle.

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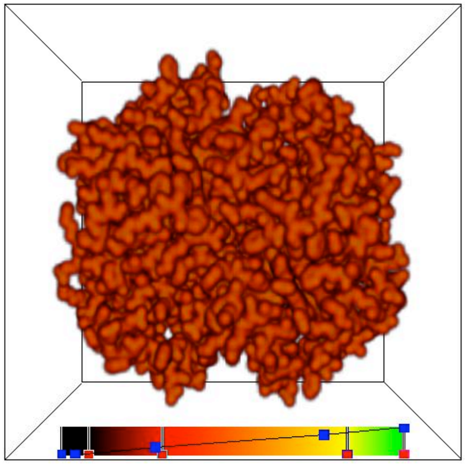
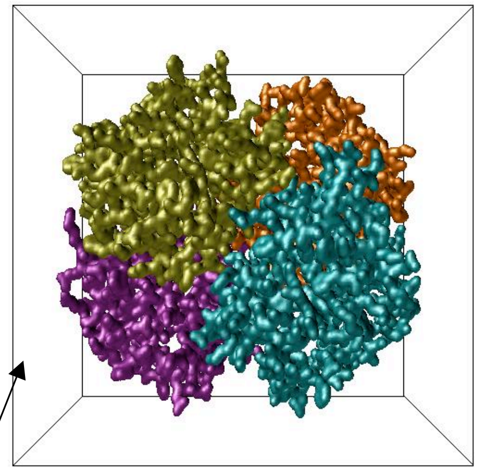
Topological Analysis & Visualization

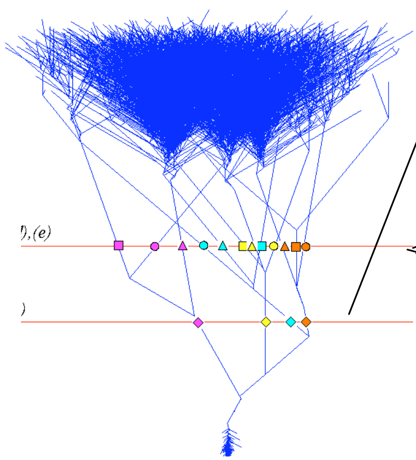


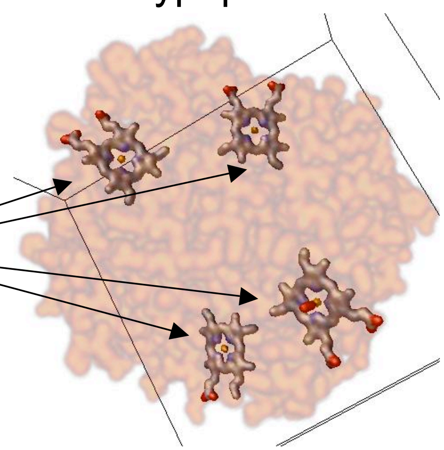


Four Polypeptide chains

Contour Tree
of
Electron Density Map

3D chemical bonding
structures
with different levels





Functional groups
Atoms belonging to the same contour have stronger linkage

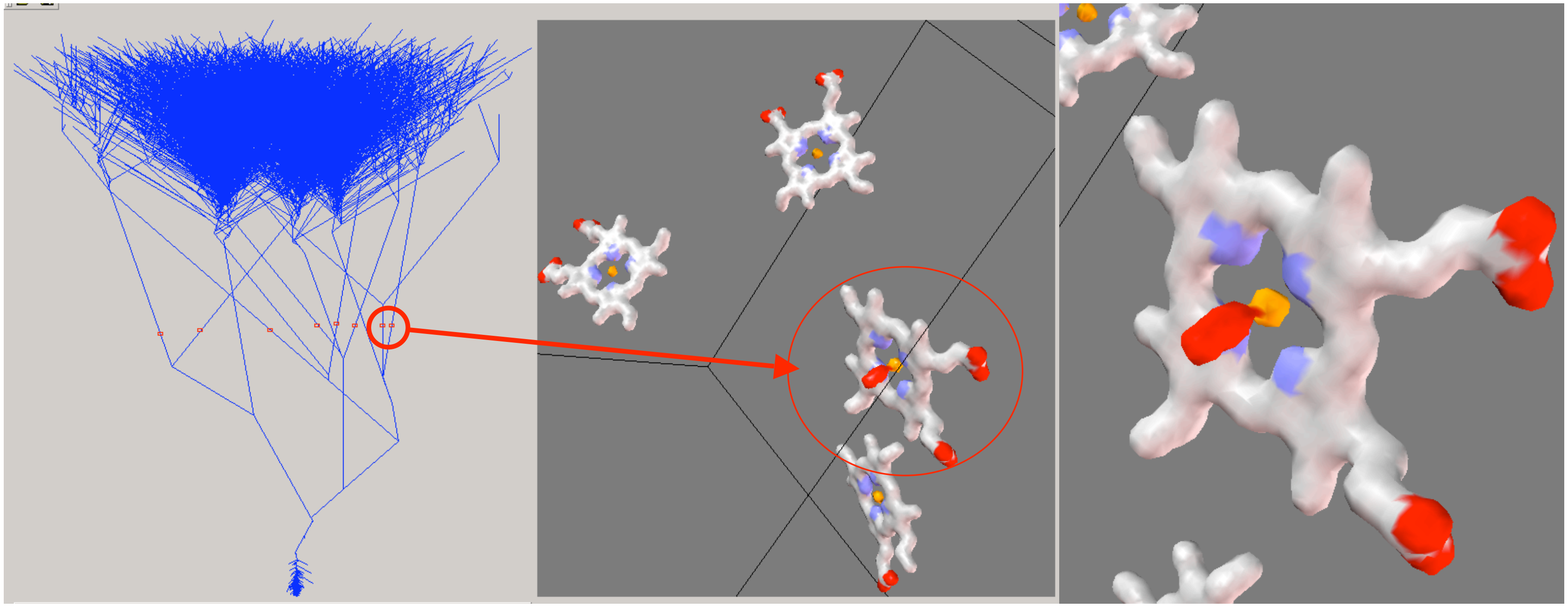
Each chain consists of heme, iron, and globin

•M. van Kreveld, R. van Oostrum, C. Bajaj, V. Pascucci, and D. Schikore, *Chap5, pg 71 - 86, 2004 ed. by S. Rana, John Wiley & Sons, Ltd, 2004*

•C. Bajaj, V.Pascucci, and D.Schikore, *Proceedings of the 1997 IEEE Visualization Conference,167-173, October 1997 Phoeniz, Arizona*

Topological Analysis using the CONTOUR TREE

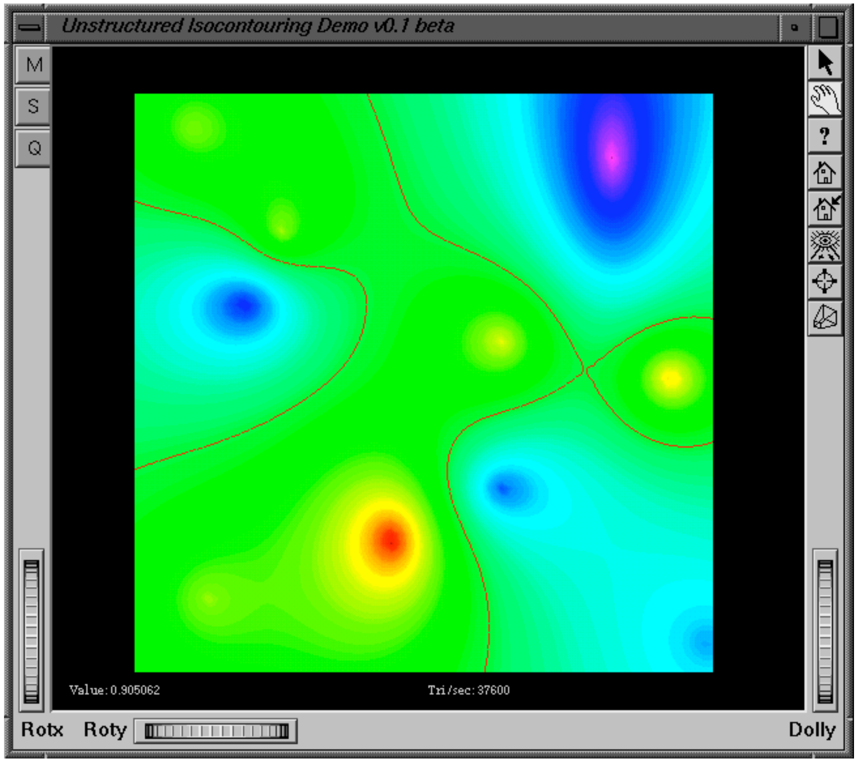
- Oxygenated Hemoglobin (T=1)



<isovalue = 31>

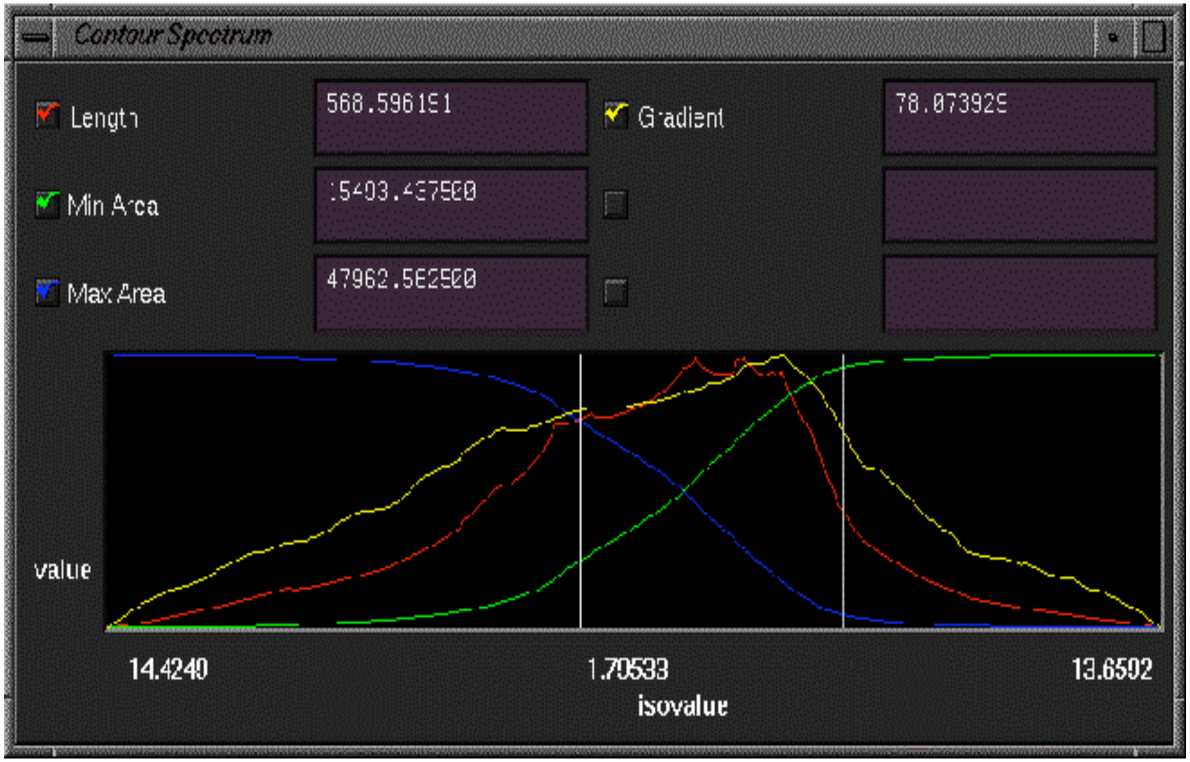
Spectral Analysis

- Consider a terrain of which you want to compute the length of each isocontour and the area contained inside each isocontour.



Spectral Analysis

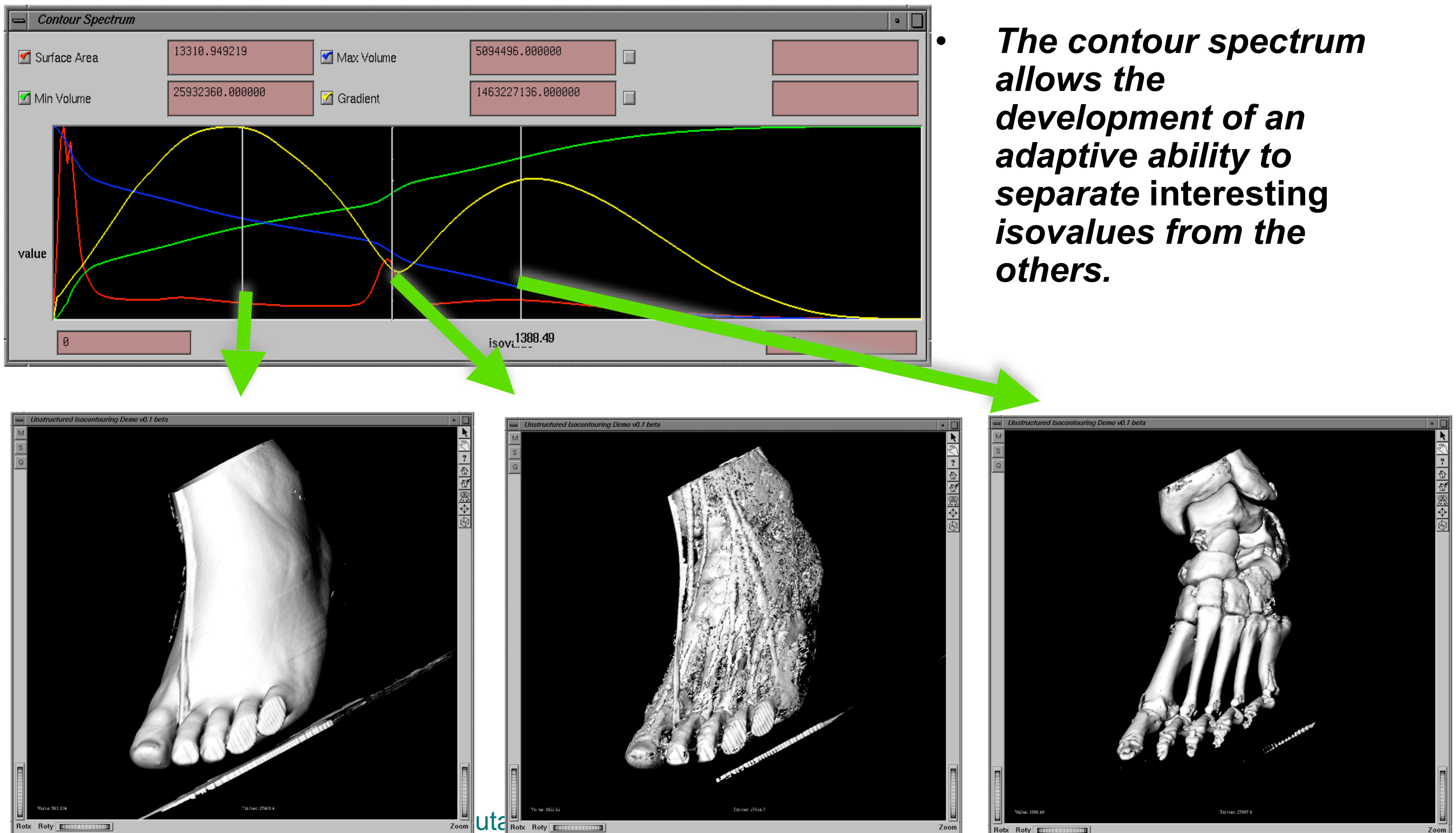
Graphical User Interface for Static Data



– The horizontal axis spans the scalar values α.
– Plot of a set of signatures (length, area, gradient ...) as functions of the scalar value α.

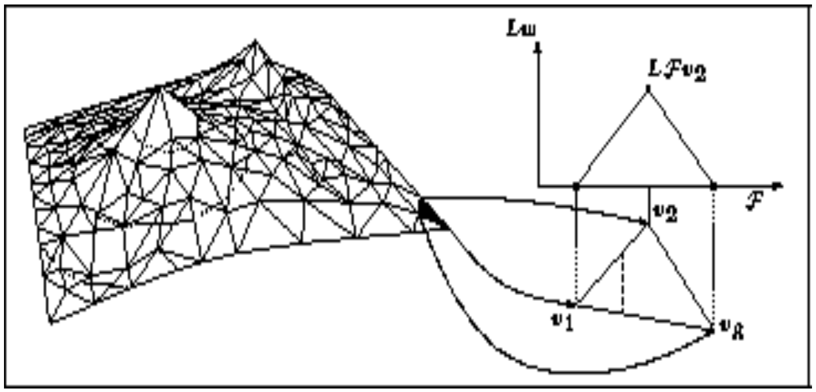
- Vertical axis spans normalized ranges of each signature.

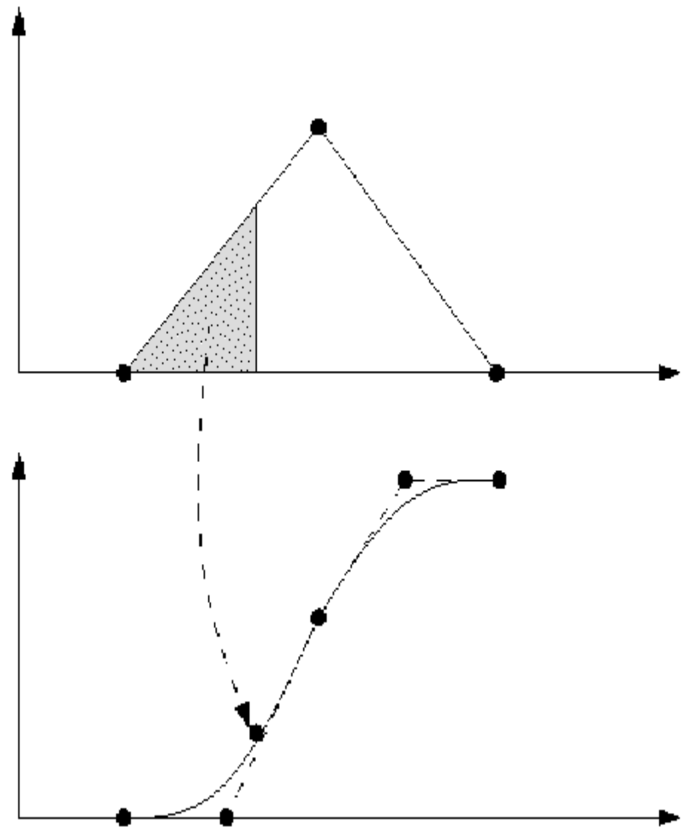
Contouring based Selection



- ***The contour spectrum allows the development of an adaptive ability to separate interesting isovalues from the others.***

Spectral Analysis
(signature computation)



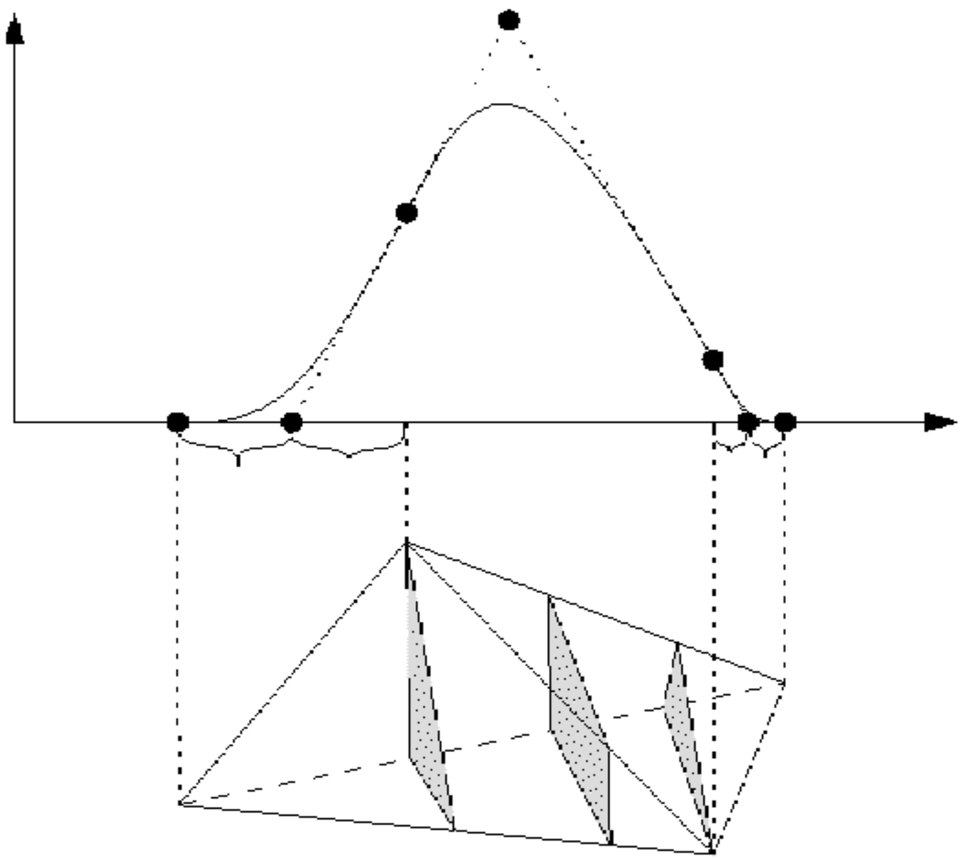


– **The length of each contour is a C^0 spline function.**

The area inside/outside each isocontour is a C^1 spline function.

Spectral Analysis (signature computation)

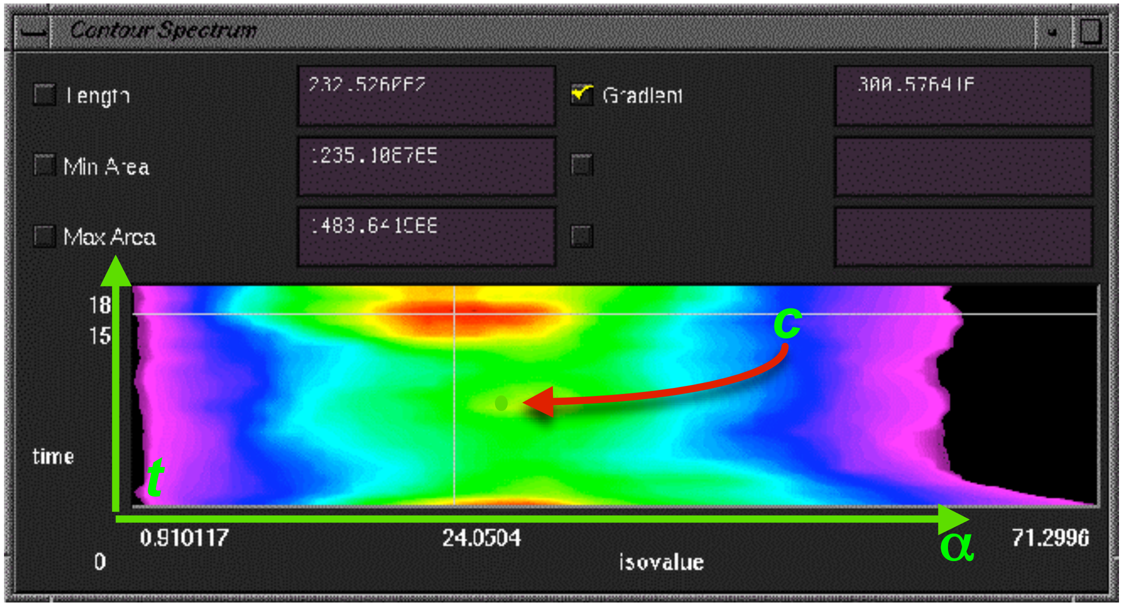
- In general the size of each isocontour of a scalar field of dimension d is a spline function of d-2 continuity.
- The size of the region inside/outside is given by a spline function of d-1 continuity



Center for Comput
Institute of Computational and Engineering Sciences
Department of Computer Sciences
University of Texas at Austin

Spectral Analysis

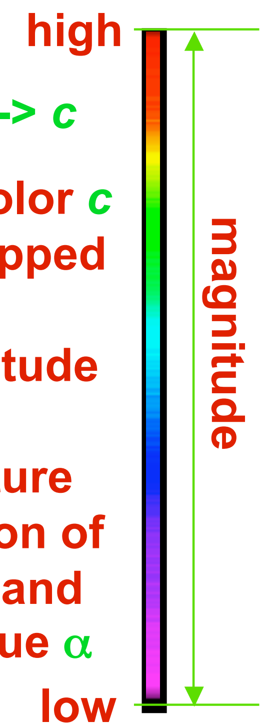
Graphical User Interface for time varying data



The horizontal axis spans the scalar value dimension α
The vertical axis spans the time dimension t

$(\alpha, t) \dashrightarrow c$

The color c is mapped to the magnitude of a signature function of time t and isovalue α



Further Reading

- C. Bajaj (ed) “DataVisualization Techniques”, John Wiley & Sons 1998
- C. Bajaj, V. Pascucci, D. Schikore, “Contour Spectrum” IEEE Viz,1997
- M. van Kreveld, van Oostrum, C. Bajaj,V. Pascucci, D. Schikore “Contour Trees & Small Seed Sets” ACM SoCG 1997, also book chap in 2004
- B. Sohn, C. Bajaj. “Topology Preserving Tetrahedral Decomposition of Trilinear Cell”, CS/ICES Tech. Rep. TR2004.
- S.Goswami, A. Gillette, C. Bajaj “Efficient Delaunay Mesh Generation from Sampled Scalar Functions”, 16h IMR, 2007
- J. Bloomenthal, C. Bajaj, J. Blinn, M. Gascuel, A. Rockwood, B. Wyvill, G. Wyvill **Introduction to Implicit Surfaces** *Morgan Kaufman Publishers Inc., (1997).*

. A. Lopes and K. Brodlie Improving the Robustness and accuracy of marching cubes algorithm for isosurfacing, IEEE Trans. on Vis and Computer Graphics, vol 9, page 16 - 29, 2003