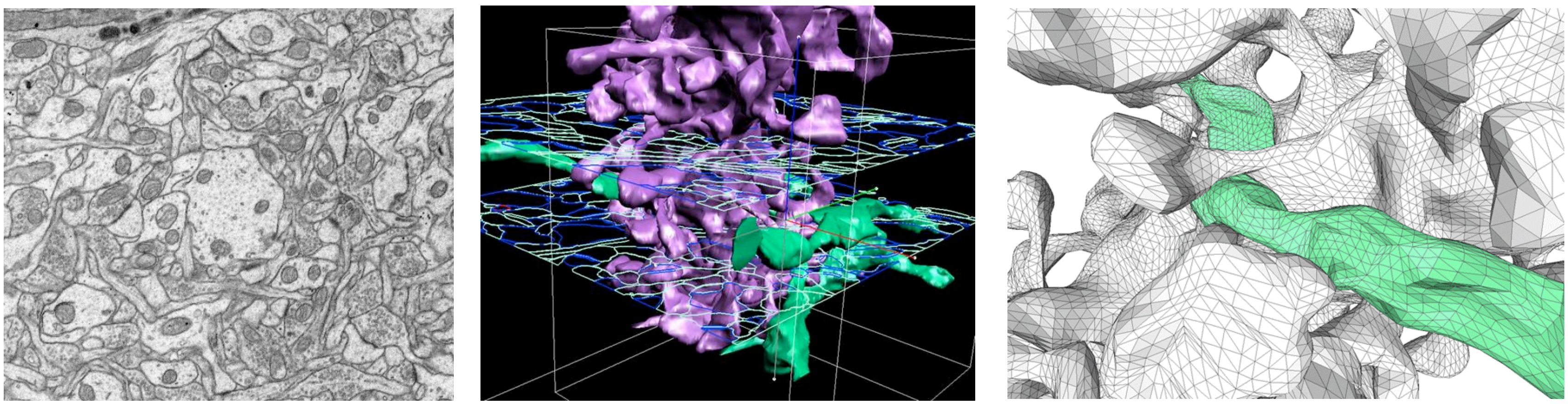
Geometric Modeling and Visualization

http://www.cs.utexas.edu/~bajaj/cs384R08/



(a) (b) (c)

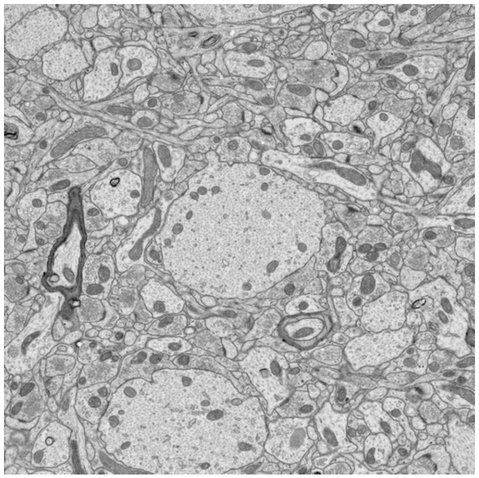
Lecture 8

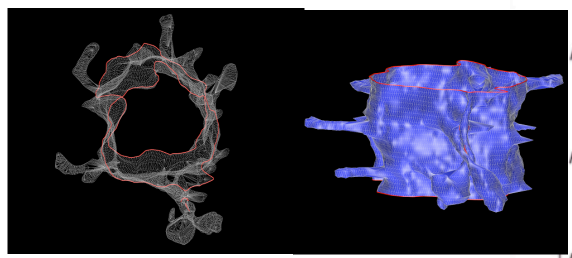
Structure Elucidation: Tiling Cross-Sections or Lofted Finite Elements

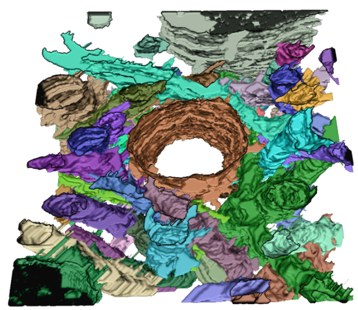
Computational Visualization Center
Institute of Computational and Engineering Sciences
Department of Computer Sciences
University of Texas at Austin

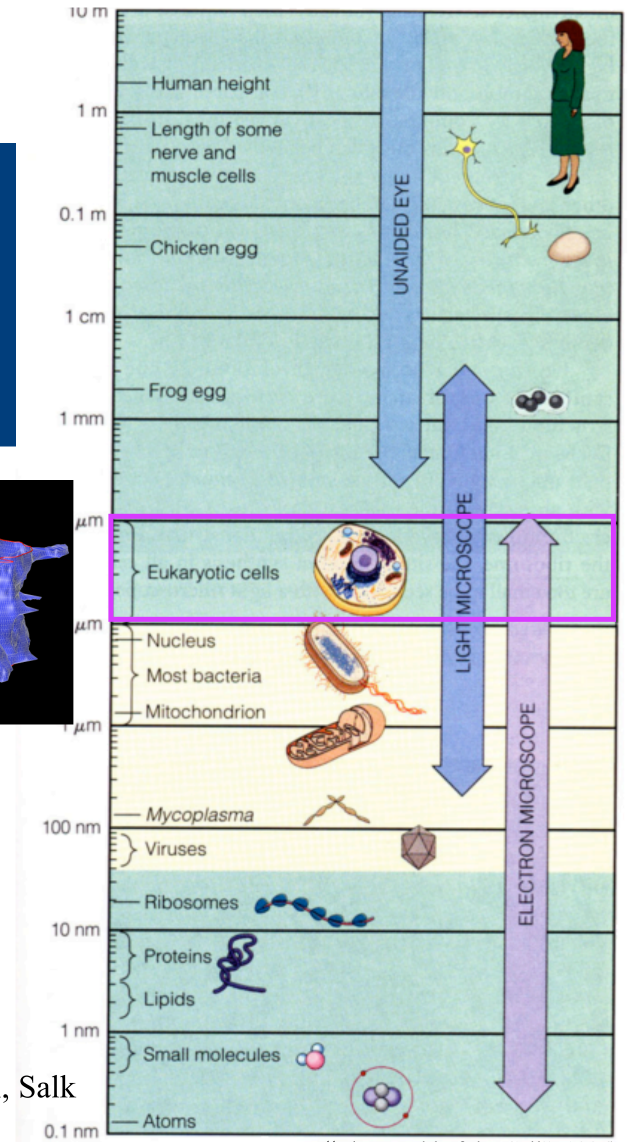
September 2008

Cell Machinery of Life









"The World of the Cell", 1996)

Transmission Electron Microscopy, Thin Sections:

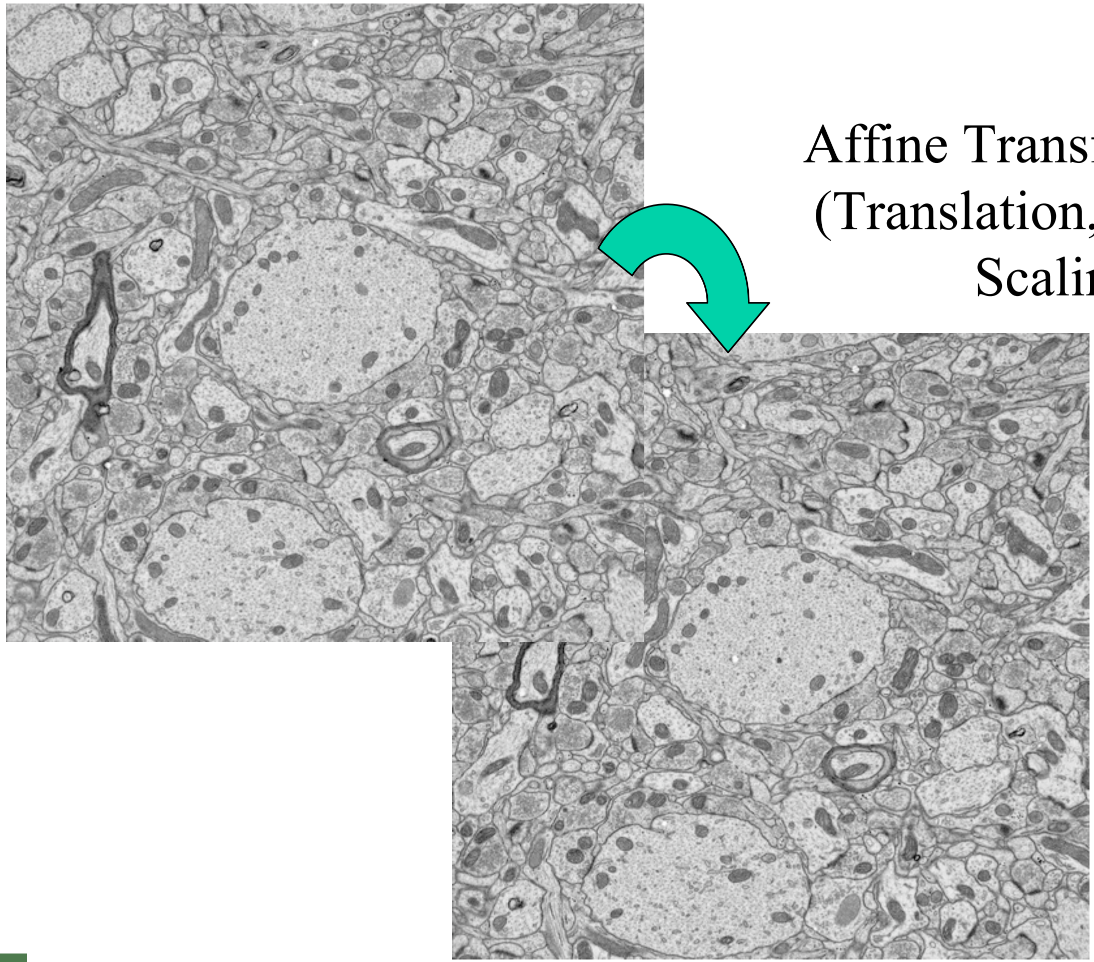
Data Courtesy: Kristen Harris, University of Texas at Austin

Addtl. Collab: Tom Bartol, Justin Kinney, Terry Sejnowski, Salk

Imaging2Models

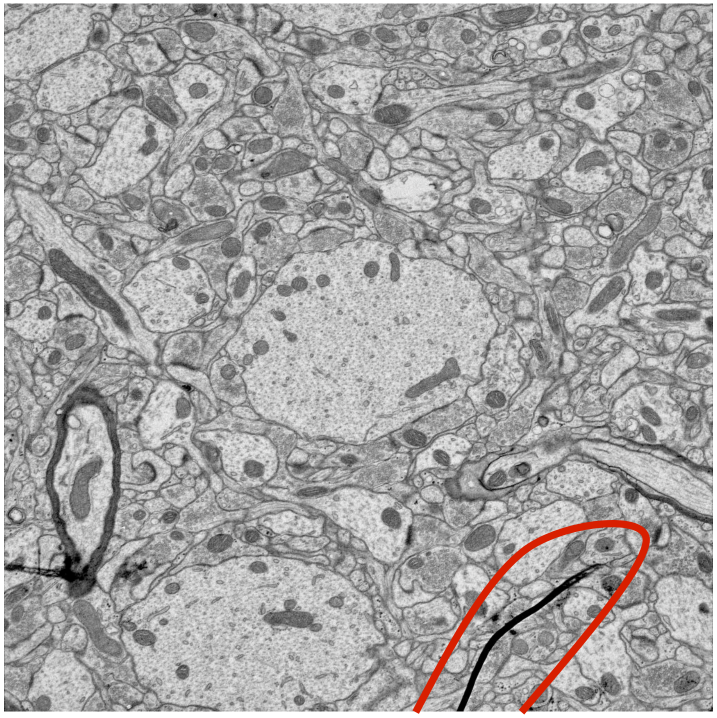
1. X-ray Crystallography → 2D Image Processing → Atomic Centers/Bonds (PDB) → FCC → Surface, Volume Processing → BEM/FEM/Shells
2. Single Particle Cryo-EM → 2D Image Processing → 3D Reconstruction → 3D Image Processing → Symmetry, Surfaces, Volume Processing→ BEM/FEM/Shells
3. Single-section EM/Anisotropic CT/MRI → 2D Image Processing → Planar X-section Contour Stack → BEM/FEM/Shells
4. Tomographic EM/MicroCT/CT/MRI → 3D Image Processing → Higher Order 3D Reconstructions, Surfaces, Skeletons → BEM/FEM/Shells
5. Time Dependent Mesh Maintenance

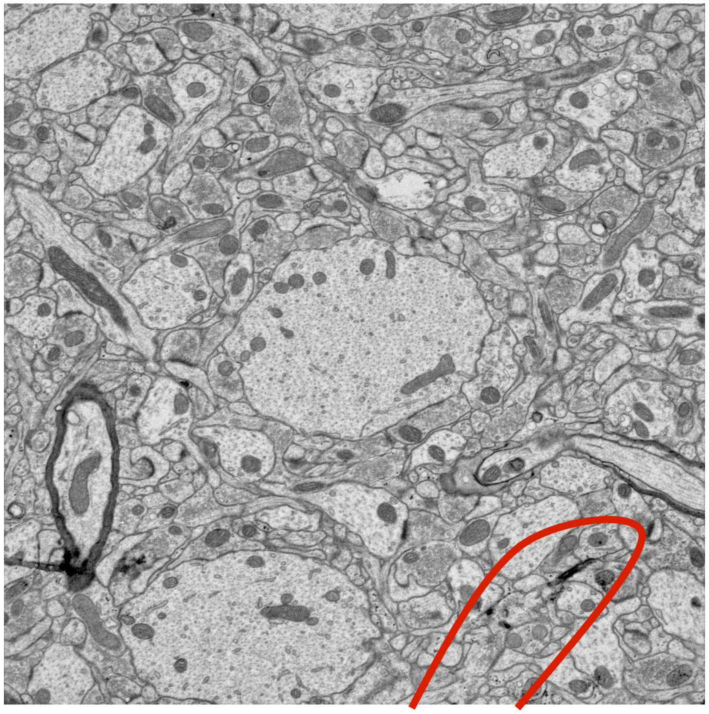
Step #1: Automatic Image Alignment



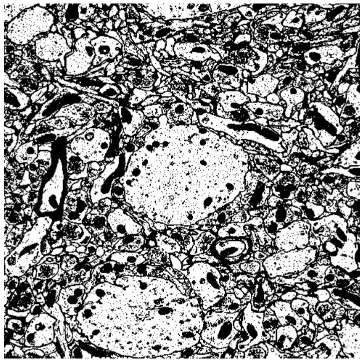
Affine Transformations (Translation, Rotation, Scaling)

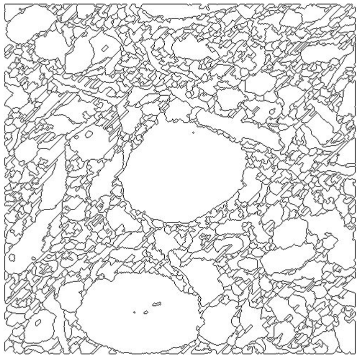
Step #2: Semi-Automatic Image Restoration

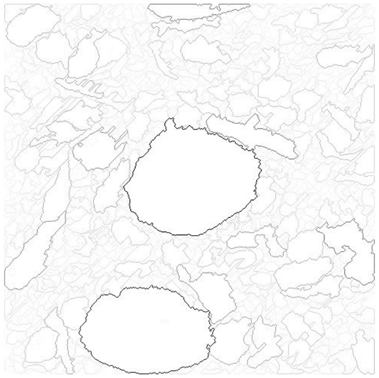


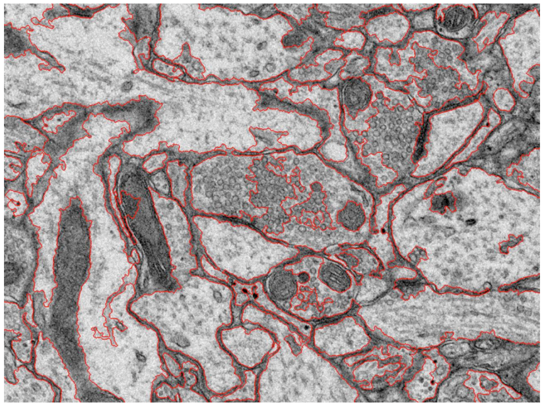


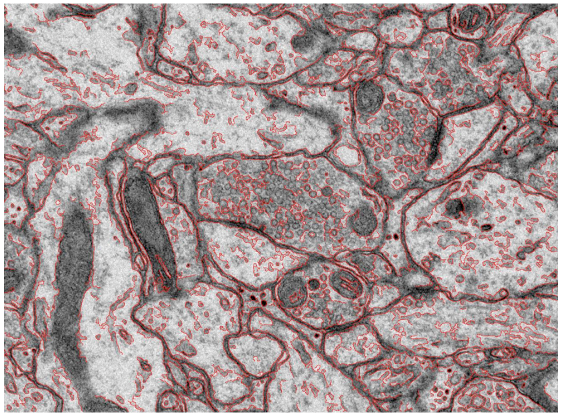
Step #3: Automatic Filtered Segmentation



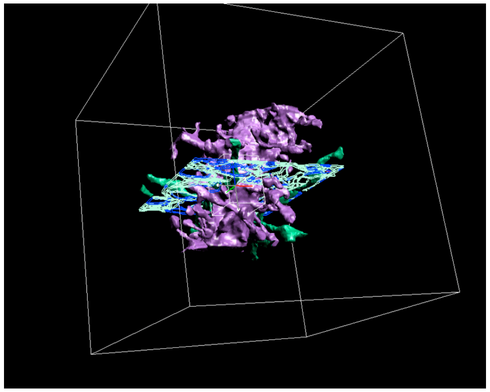


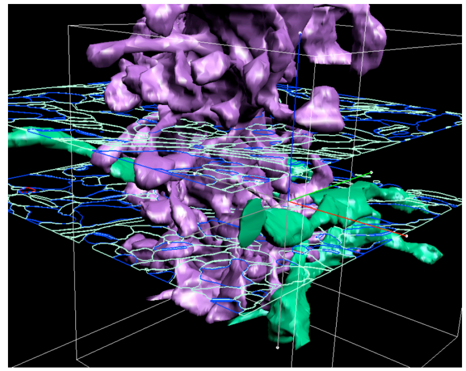


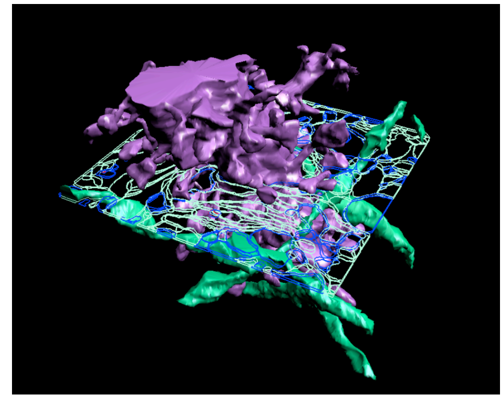


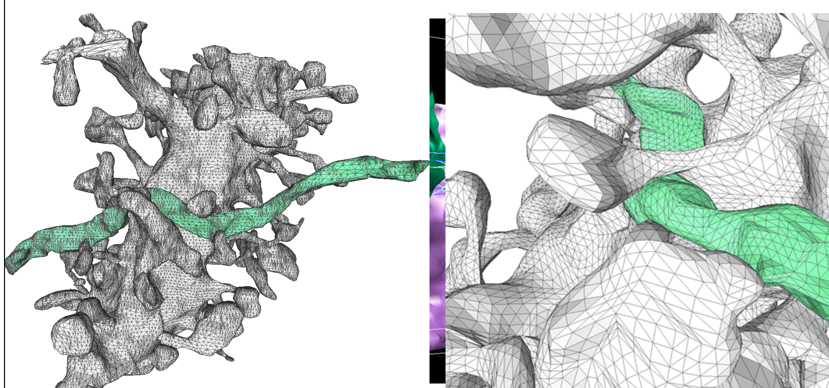


Step #4: Hippocampal Neuron Model Reconstruction

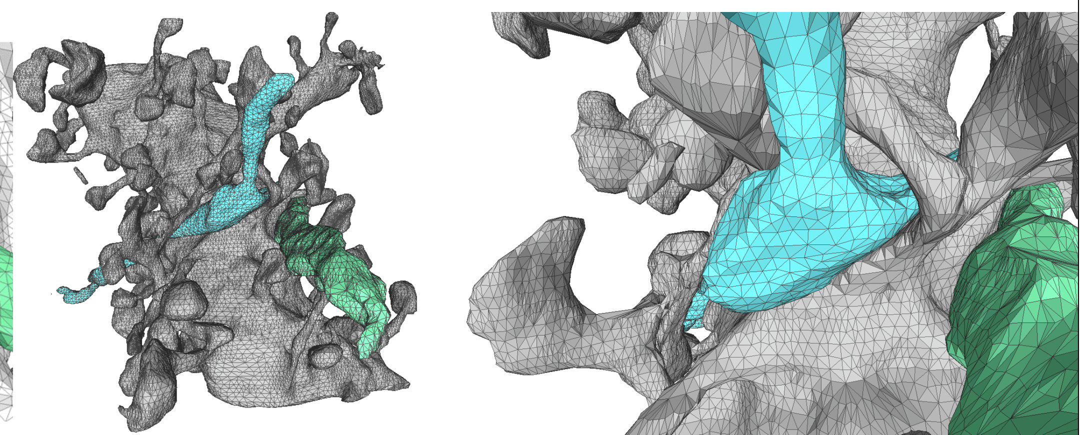








(a) (b)



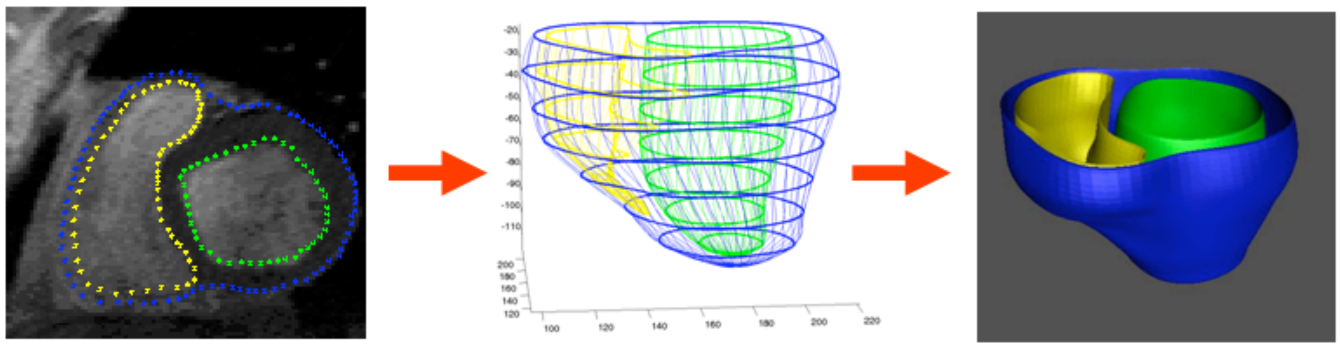
(a) (b)

C.Bajaj, K. Lin, E. Coyle: **Arbitrary Topology Shape Reconstruction from Planar Cross-Sections,** Graphical Models and Image Processing, 58:6, 1996,

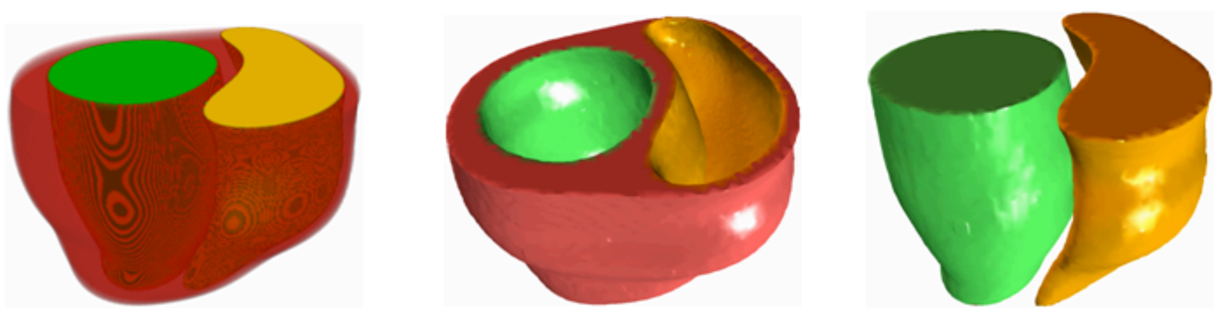
Heart Model via X-section Contour Lofting

First segment the heart into four independent planar contour stacks from MRI data: background (0), heart muscle (81), left ventricle (162), right ventricle (243) and then loft (skin) the planar contour stacks

simulation of the electronic activity of the heart.



Raw MRI data — Manually digitized slices — Continuous model



Volume rendering — Smooth shading

Theoretical Basis - I

Definition

Two algebraic surfaces $f(x, y, z) = 0$ and $g(x, y, z) = 0$ meet with C^k *rescaling continuity* at a point p or along an irreducible algebraic curve C if and only if there exists two polynomials $a(x, y, z)$ and $b(x, y, z)$, not identically zero at p or along C, such that all derivatives of $af - bg$ up to order k vanish at p or along C.

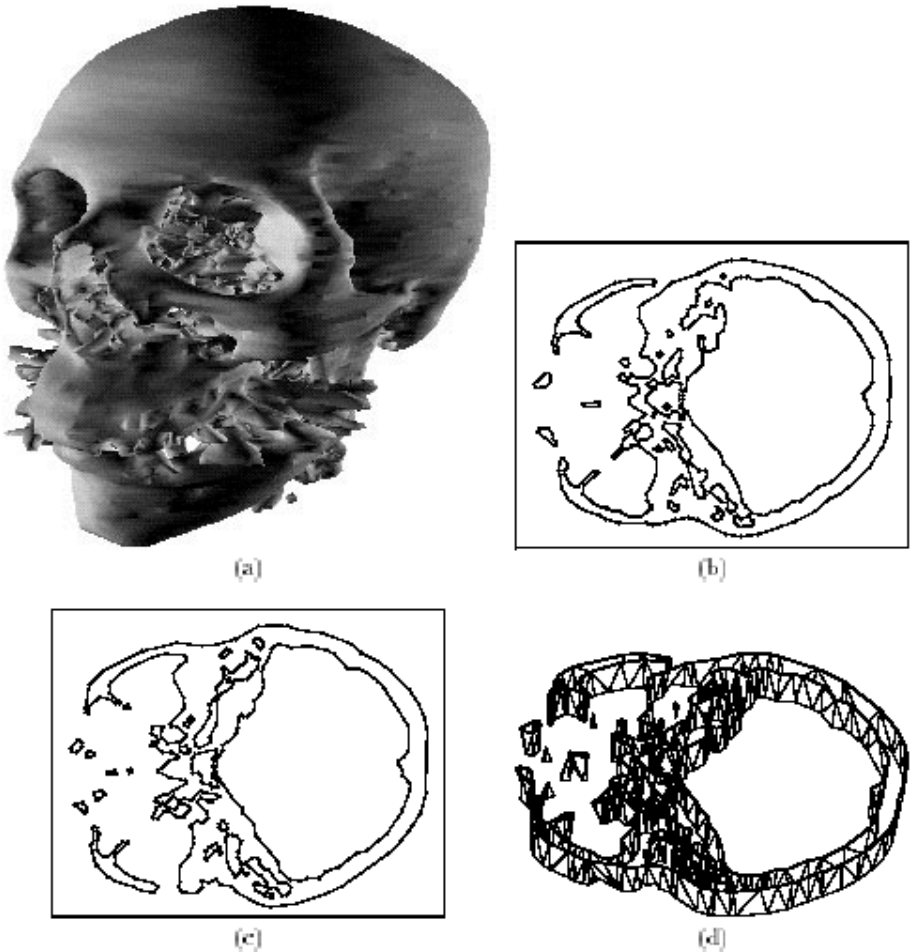
Theoretical Basis - II

Theorem

Let $g(x,y,z)$ and $h(x,y,z)$ be distinct, irreducible polynomials. If the surfaces $g(x,y,z)=0$ and $h(x,y,z)=0$ intersect transversally in a single irreducible curve C, then any algebraic surface $f(x,y,z)=0$ that meets $g(x,y,z)=0$ with C^k rescaling continuity along C must be of the form $f(x,y,z)=\alpha(x,y,z)g(x,y,z)+\beta(x,y,z)h^{k+1}(x,y,z)$. If $g(x,y,z)=0$ and $h(x,y,z)=0$ share no common components at infinity. Furthermore, the degree of $\alpha(x,y,z)g(x,y,z) \leq$ degree of $f(x,y,z)$ and the degree of $\beta(x,y,z)h^{k+1}(x,y,z) \leq$ degree of $f(x,y,z)$.

Higher-Order Interpolation and Least-Squares Approximation Using Implicit Algebraic Surfaces *ACM Transactions on Graphics, (1993)* See Lofting movies

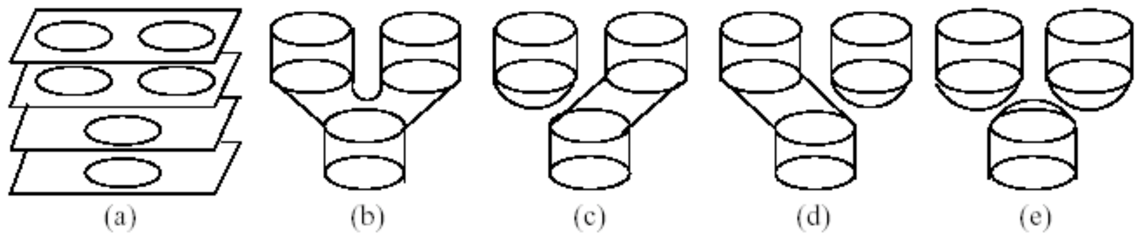
Triangular Meshing



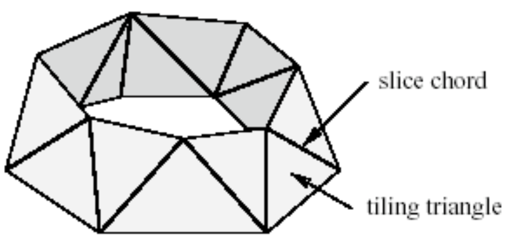
- To generate a boundary element triangular mesh from a stack of cross-sectional polygonal data.
- Subproblems
  - The correspondence problem
  - The tiling problem
  - The branching problem

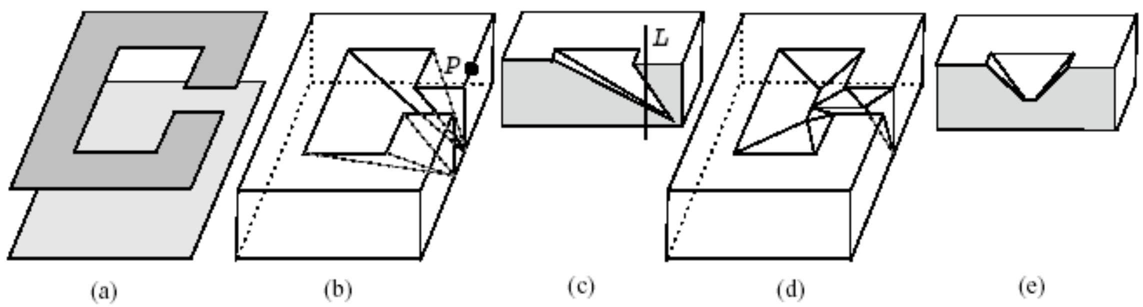
Sub-problems

- Correspondence

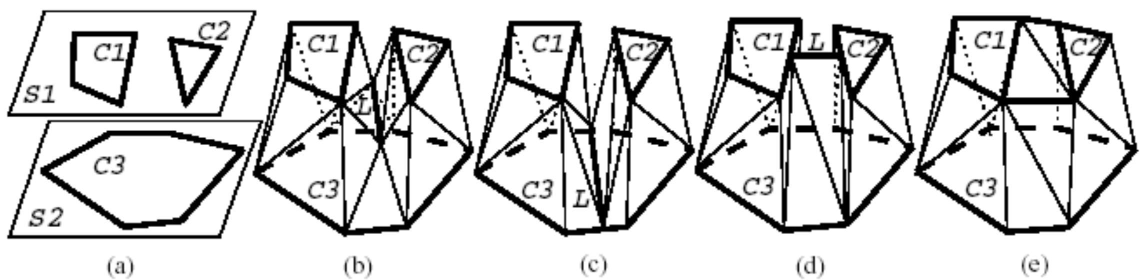


- Tiling





- Branching



Incremental Construction

Algorithm Steps

Step a: Segment closed contours from 2D images

Step b: Create any required augmented contours

Step c: Find correspondences between contours

Step d: Form the tiling region of each vertex

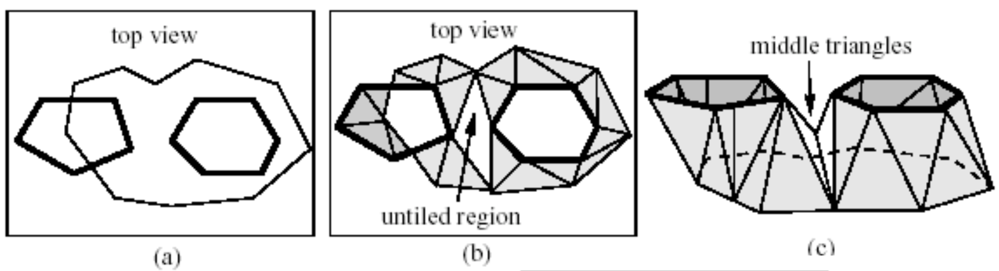
Step e: Construct the tiling

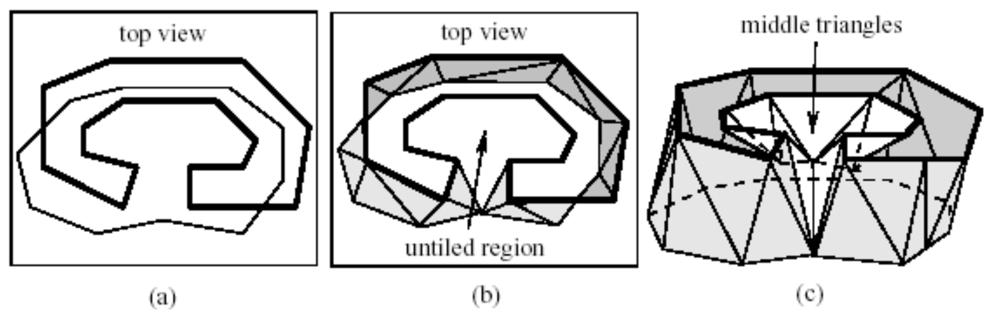
Step f: Collect the boundaries of untiled regions

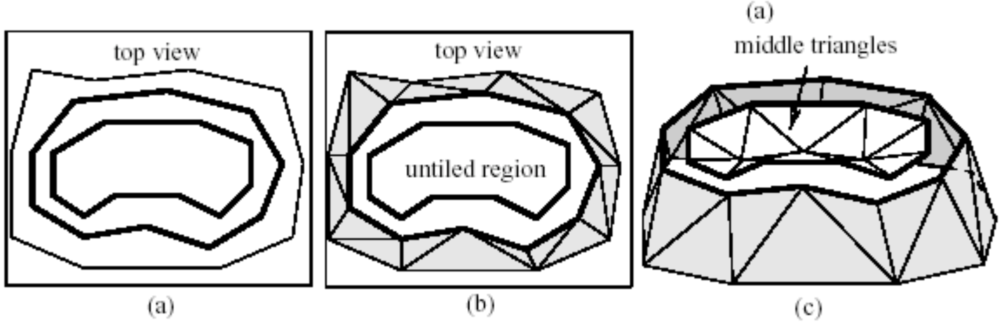
Step g: Form triangles to cover untiled regions based on their edge Voronoi diagram (EVD)

Algorithmic Subtleties

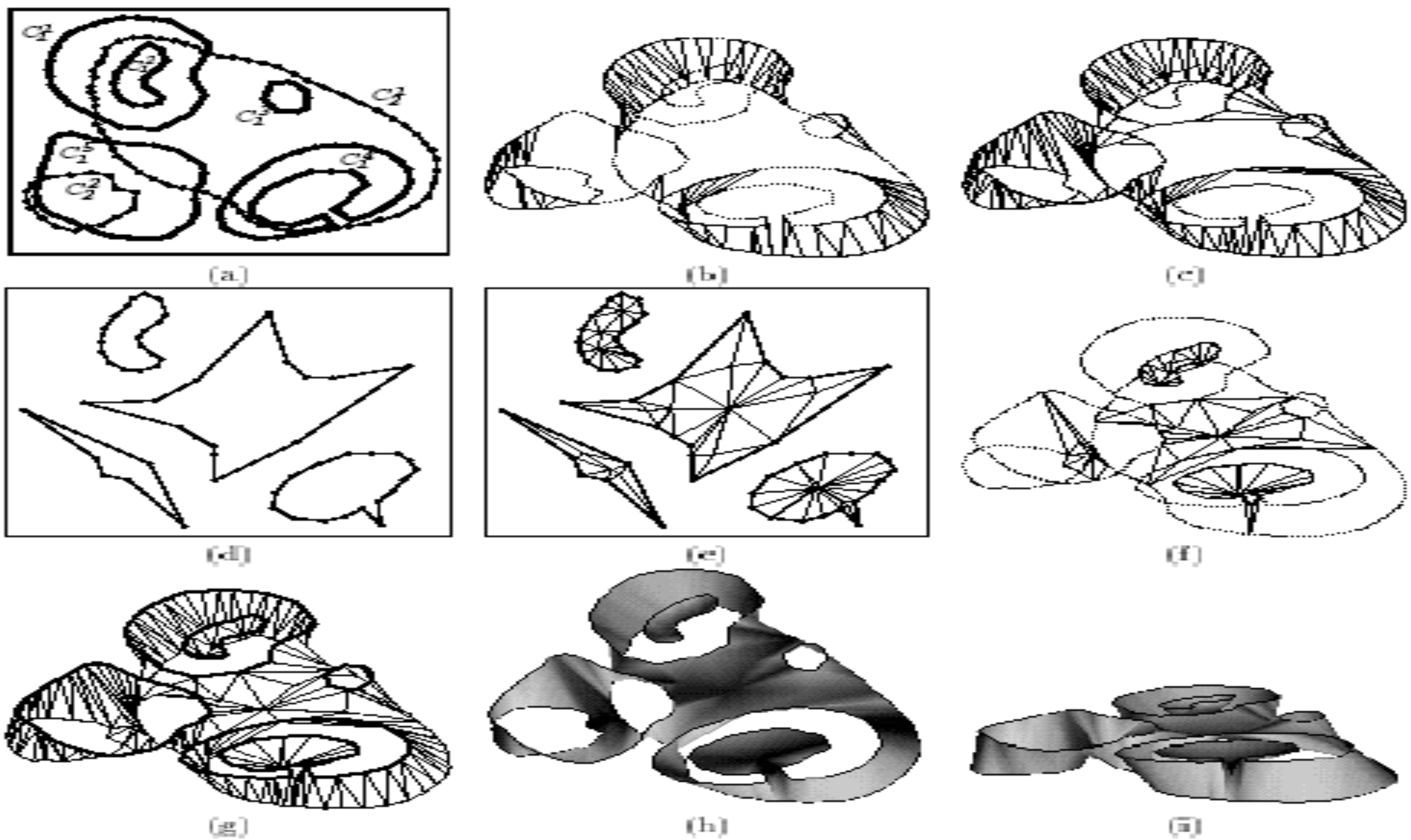
- A multi-pass tiling approach followed by the postprocessing of untiled regions



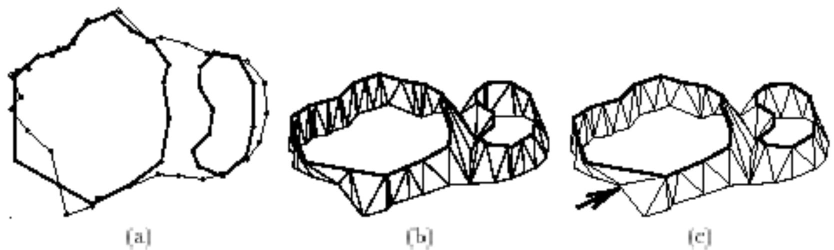


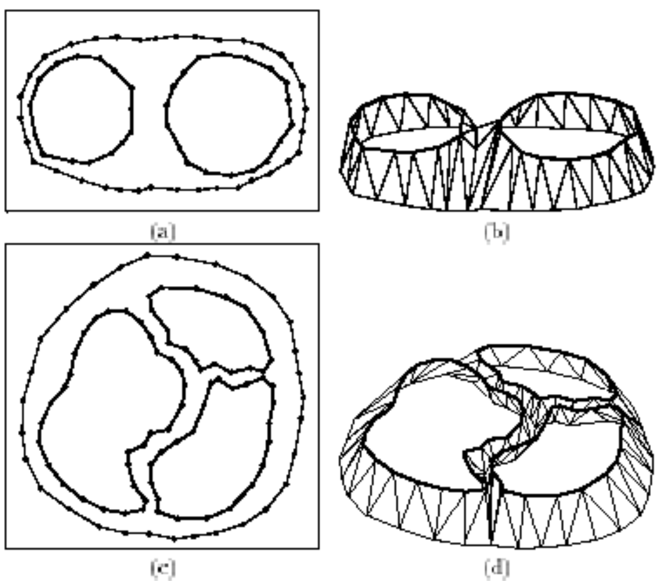


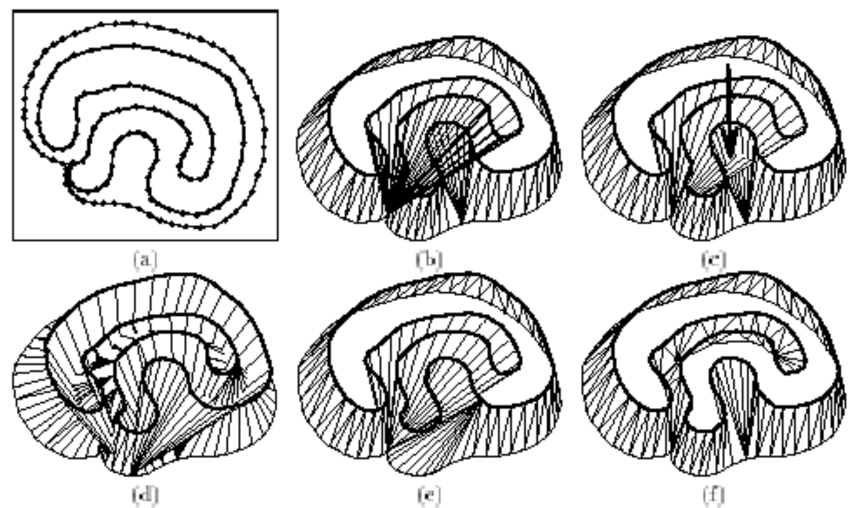
Algorithm Steps on actual data



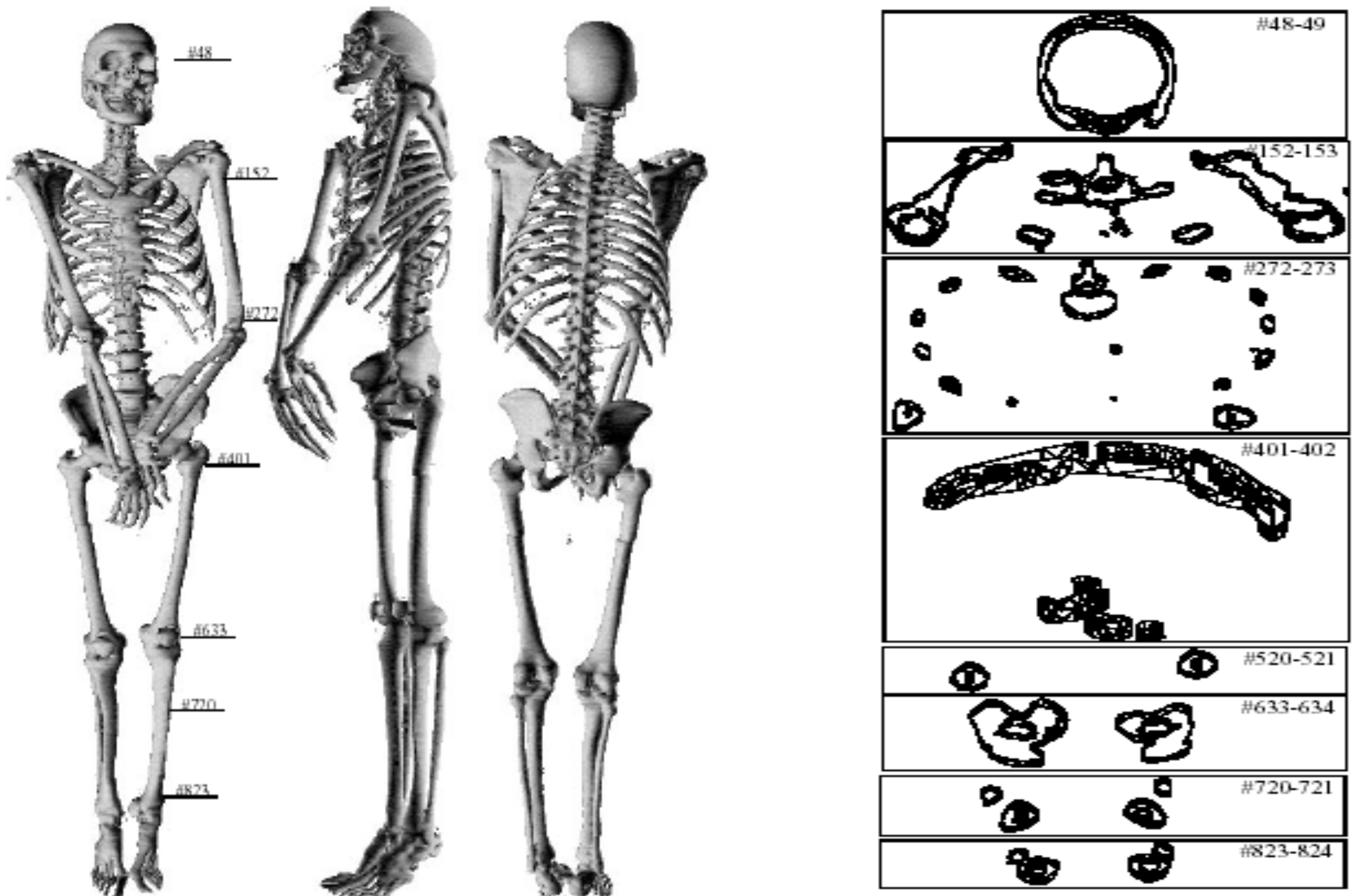
Using the Edge Voronoi Diagram as Ridges



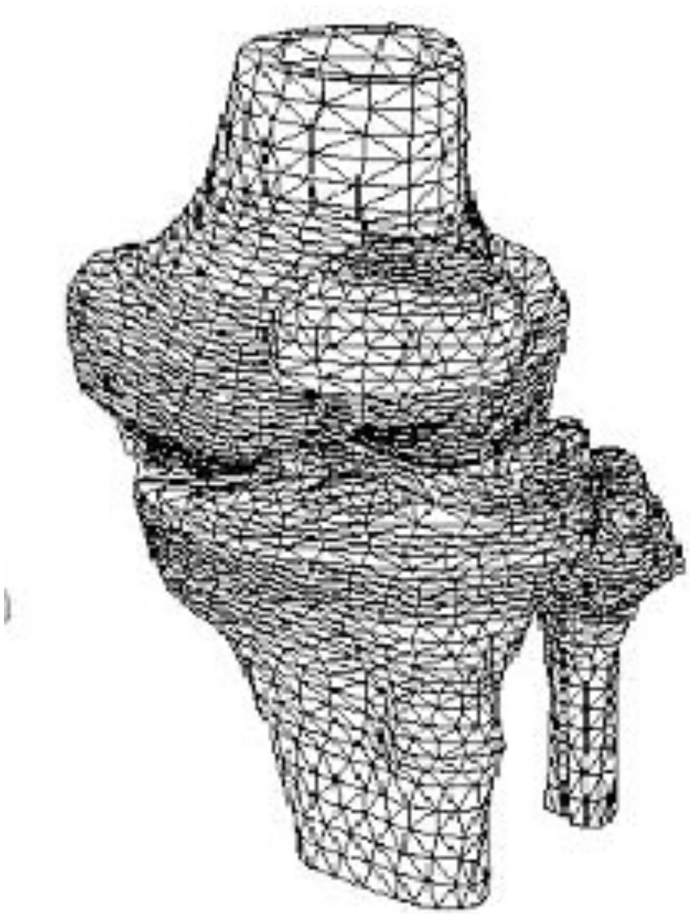




Boundary Element Triangular Mesh



Tetrahedral Meshing

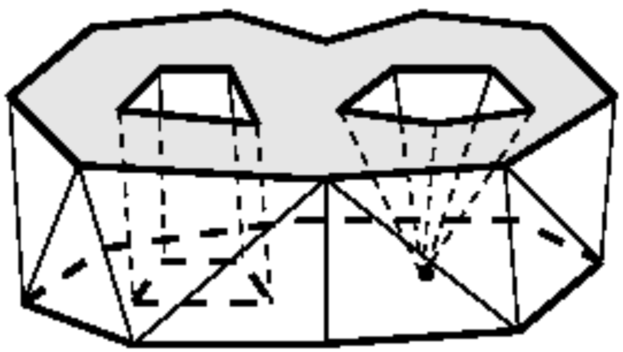


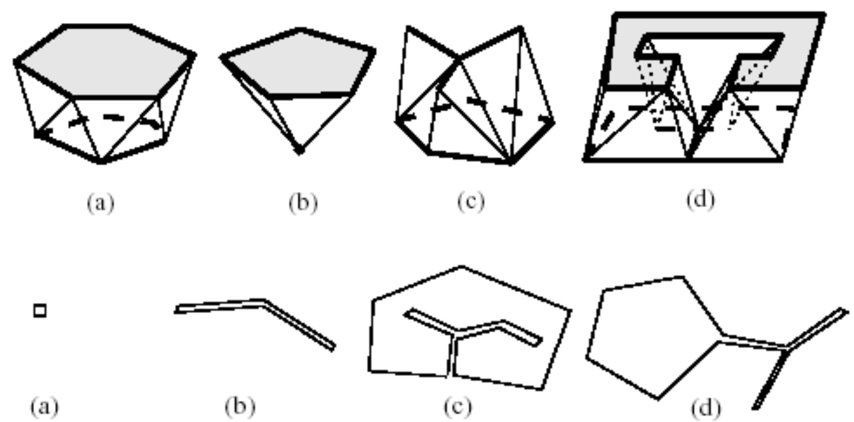
- To generate a 3D finite element tetrahedral mesh of the simplicial polyhedron obtained via the BEM construction of cross-section polygonal slice data.

- Subproblems
  - The shelling of tetrahedra to reduce polyhedron to prismatoids
  - The tetrahedralization of prismatoids

What is prismatoid?

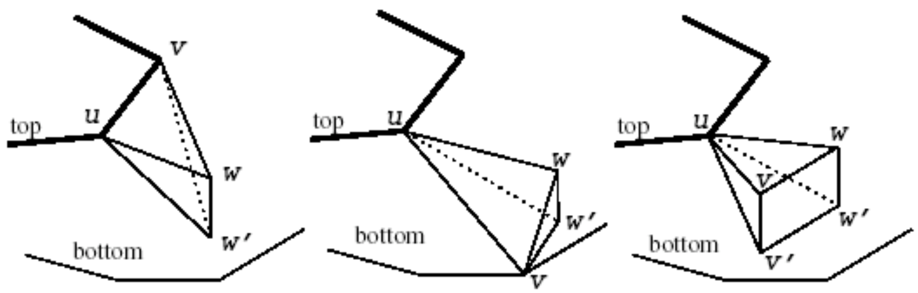
A prismatoid is a polyhedron having for bases two simple polygons (possibly degenerate) in parallel planes, and for lateral faces triangles or trapezoids having one vertex or side lying in one base (or plane), and the opposite vertex or side lying in the other base (or plane).





The Shelling Step

- Shell tetrahedra from the polyhedron, so the remaining part is a prismatoid or can be divided into prismatoids.



(a) (b) (c)

(a) (b) (c)

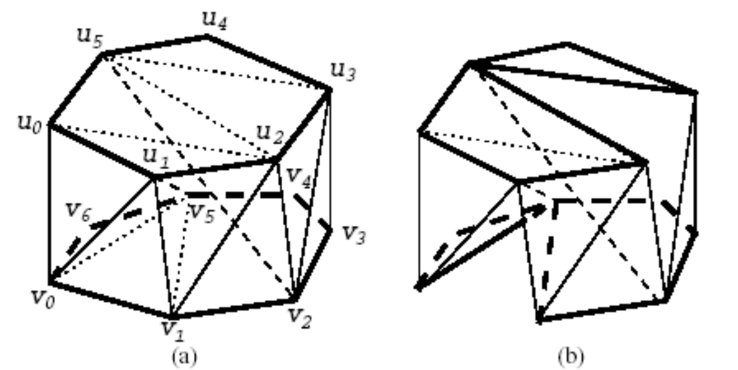
Prismatoid → Tetrahedra

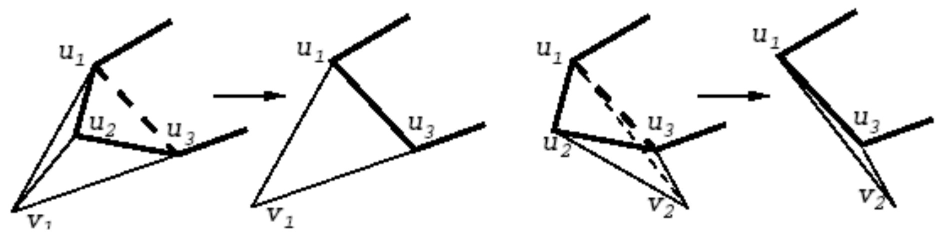
- To tetrahedralize a non-nested prismatoid without Steiner points.
  1. For each boundary triangle on both slices, calculate its metric.
  2. Pick up the boundary triangle with the best metric and form one set of tetrahedra.
  3. Update the advancing front and go to Step 1.
  4. If the remaining part is non-tetrahedralizable, postprocess it.

Metric, Weight Factor, Grouping

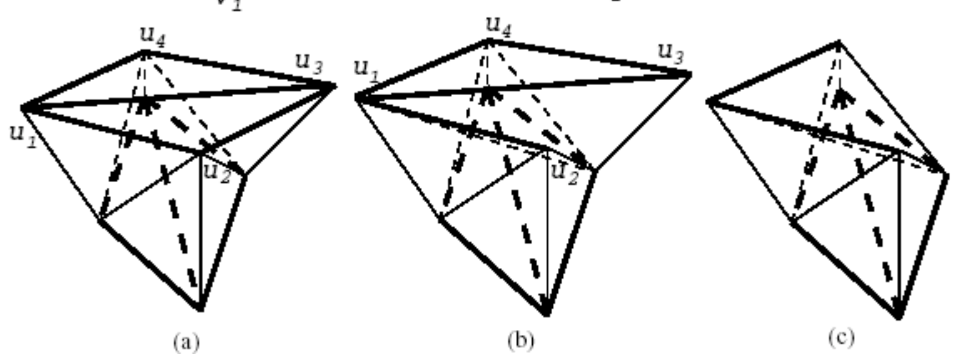
- Metric = volume/(edge)^3
- Weight factor

$$w = \begin{cases} 2(1-\frac{d}{h}) & \text{if } d \le 0.5h \\ 1 & \text{if } 0.5h < d < h \\ \frac{h}{d} & \text{if } d \ge h \end{cases}$$





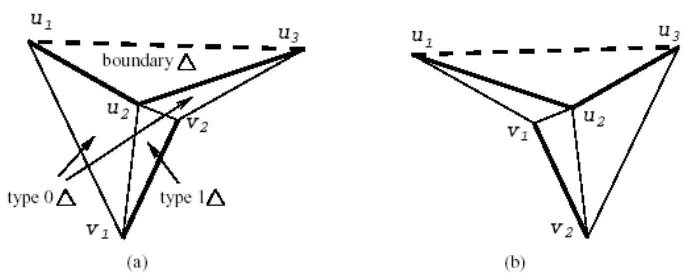
- Grouping can avoid irregular remaining part

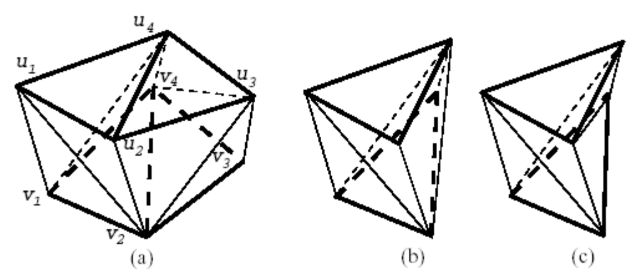


Protection Rule

Lemma 1: Suppose a top boundary triangle $\Delta u_1u_2u_3$ is under the constraint that no more than one type 1 triangle is between the two type 0 triangles containing the contour segments u_1u_2 and u_2 u_3. Furthermore, let the bottom vertices of the two type 0 triangles be v_1 and v_2. Our grouping operation cannot apply to $\Delta u_1u_2u_3$ to form a set of tetrahedra, if and only if all the following conditions are satisfied.

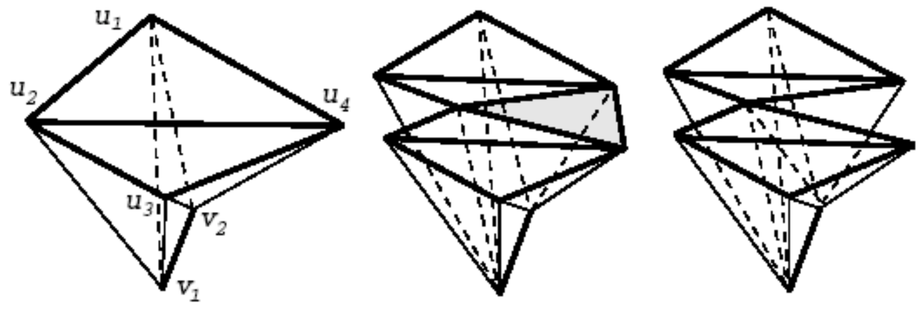
1. v_1v_2 is exactly one contour segment.
2. One of the slice chords u_2v_1 and u_2v_2 is reflex and the other is convex.
3. Both u_1v_2 and u_3v_1 are not inside the prismatoid.





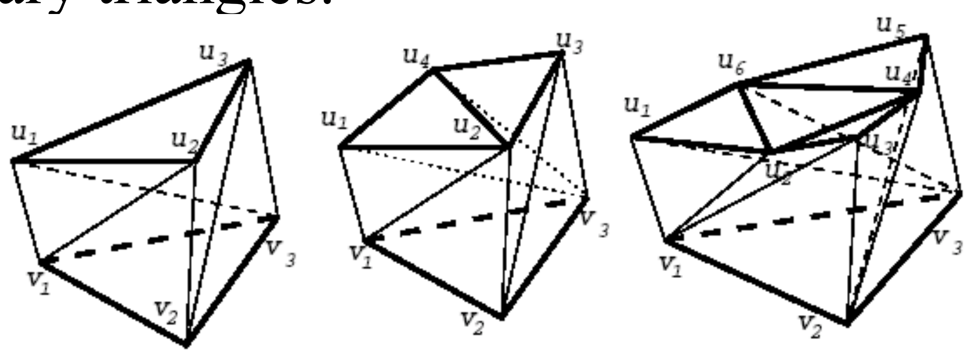
Classification of Untetrahedralizable Prismatoids

1. Has two boundary triangles on the top face and one line segment on the bottom face.



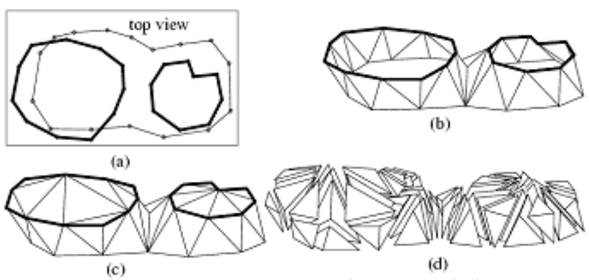
(a) (b) (c)

2. Has one bottom triangle which is treated as three boundary triangles.

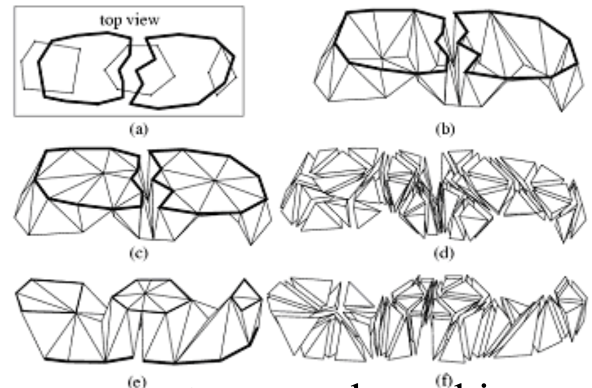


(a) (b) (c)

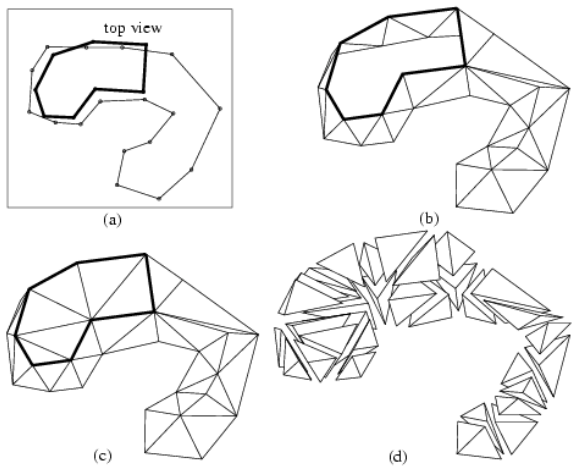
Multiple Tetrahedralizable Cases



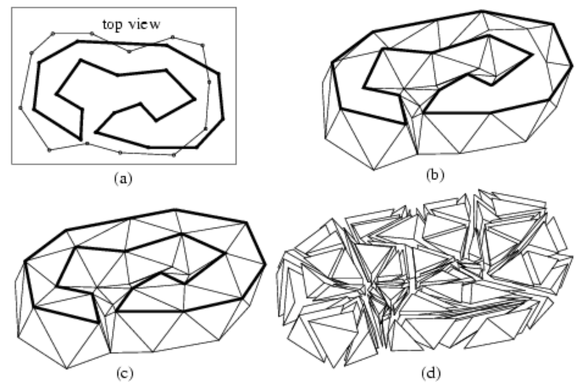
One-to-many branching



many-to-many branching

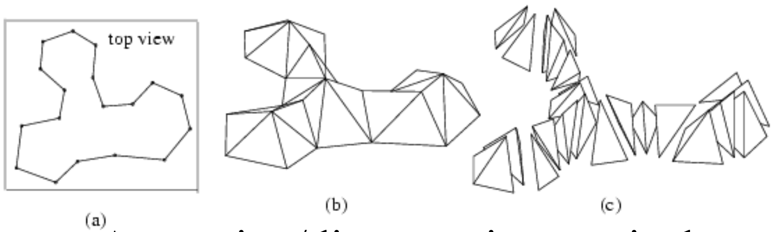


Dissimilar region (the right bottom portion of the bottom contour)

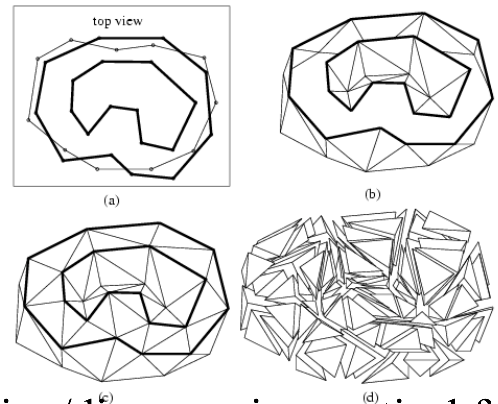


Dissimilar region (the inner portion of the top contour)

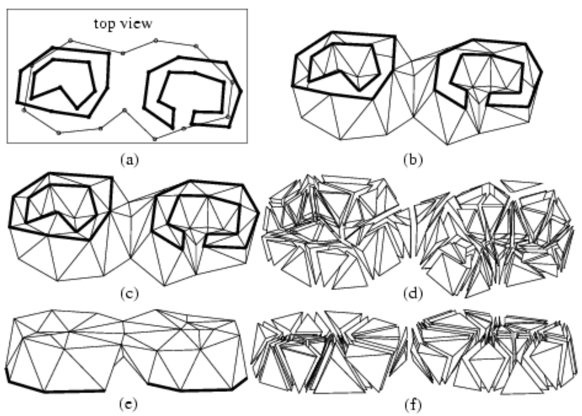
Multiple Tetrahedralizable Cases



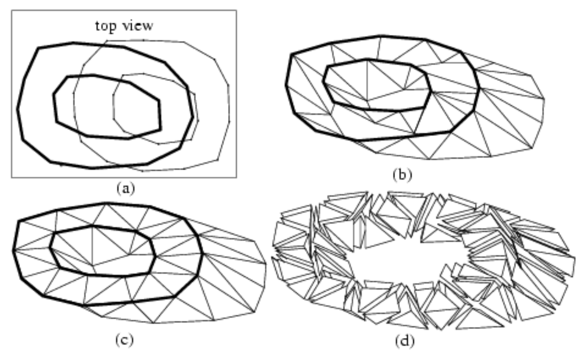
Appearing/disappearing vertical feature of a solid interior



Appearing/disappearing vertical feature (the top inner contour) of a void interior

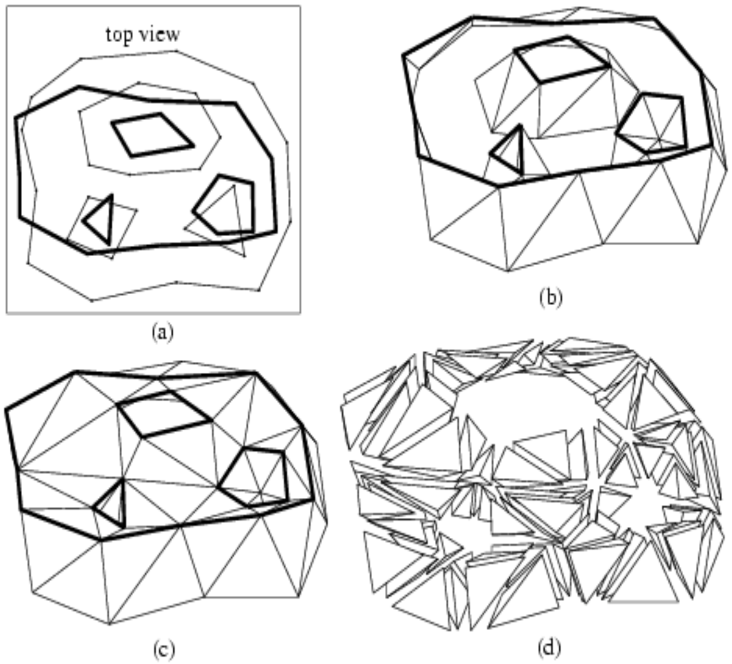


A branching, a dissimilar portion (the inner portion of the top right contour), and an appearing/disappearing vertical feature (the inner contour at the left of the top slice)

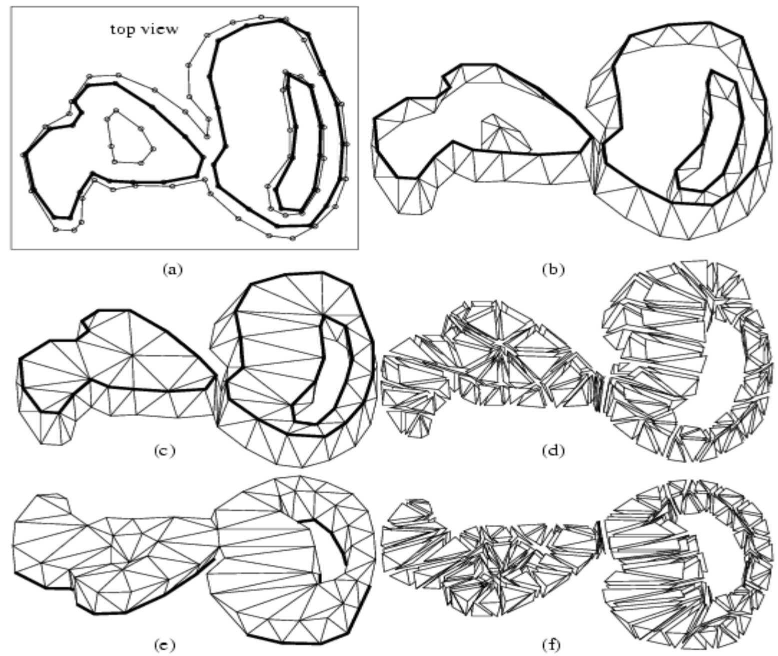


Nested prismatoids

Multiple Tetrahedralizable Cases

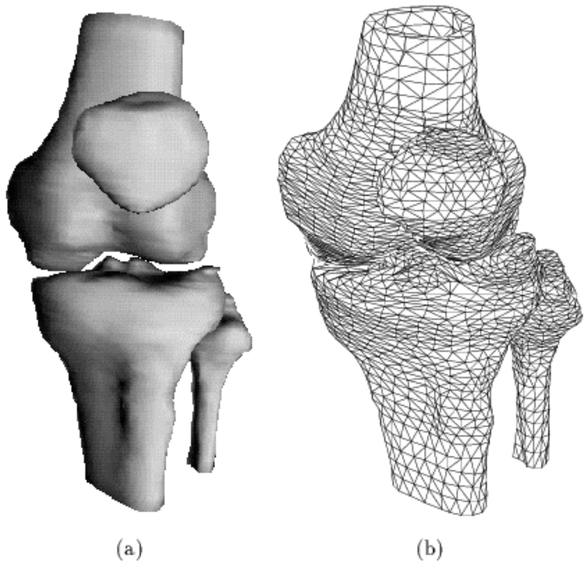


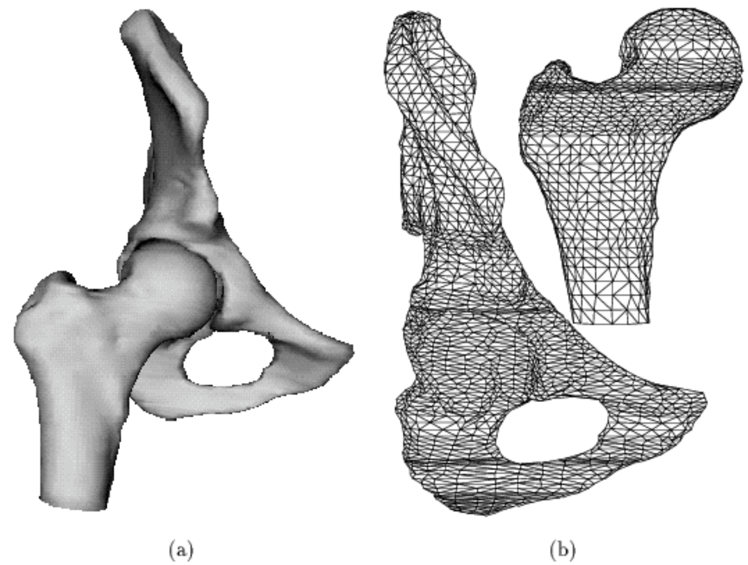
Multiply-nested prismatoid



Solid region between two slices of a human tibia

Examples





Knee joint (the lower femur, the pper tibia and fibula and the patella)

(a) Gouraud shaded

(b) The tetrahedralization

Hip joint (the upper femur and the pelvic joint)

(a) Gouraud shaded

(b) The tetrahedralization

Mini-summary

- The characterization, avoidance of non-tetrahedralizable polyhedra is one of the main challenges
- The mix of numerical precision and topological decision making needs precise rules so errors don't propagate.

Further Reading

[1] C. Bajaj, E. Coyle, K. Lin. Arbitrary topology shape reconstruction from planar cross sections. *Graphical Models and Image Processing*, 58(6):524-543, Nov.1996.

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