Illumination Models IV: Global Diffuse (View Independent)

Radiosity

The philosophy of our previous lectures on illumination were based on what we called "quick" methods: efficient approaches that manage to "fool the eye". The *local* illumination model is central to the efficiency of these approaches.

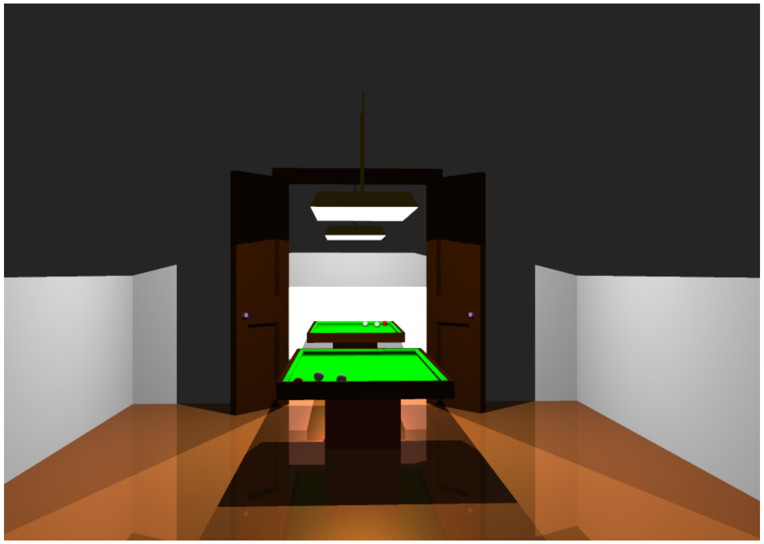
A *global* illumination model is a model which takes into account the fact that light is not just coming from a few point light source, but that light is arriving indirectly from many different directions, and possibly from the objects themselves.

What are the elements of a global illumination model?

The basic idea is viewing each object as being a potential light source. Some objects (light sources) radiate light directly, but others (non-black surfaces) can radiate light indirectly.

Radiosity is another example of a global illumination model.

Example for indirect illumination using RayTracing



Example for indirect illumination using Radiosity

Example for combined illumination

Radiosity Overview

Radiosity: the intensity of each point on the surface of some object in our environment.

This intensity of the point P is a function of

- the emittance of light from this point (if it is a light source),
- the reflection of light coming from other surfaces in the environment.

The second component is quite complicated, because it depends on the radiosity of points on surfaces throughout the environment, whether these points are visible from P, and how reflective the surface is that P lies on.

Sampling

Radiosity computations are quite expensive since for every point we need to know the illumination of all the surface elements that this point can see. A common way is to choose some sampled points in the environment.

How do we sample?

The most common way is based on a generalization of the *finite element method*.

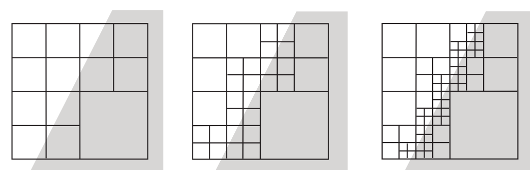
- Subdivide each of the object surface into a number of small polygonal patches (*surface mesh*)
- For each patch, compute an approximation of the radiosity of this patch. For example, this could be done by computing the radiosities at each of its vertices and then averaging these.

How do we construct these patches?

- Small patches can give good accuracy, but expensive.
- Large patches can give speed, but loss of accuracy.

The best method is to use an *adaptive* meshing approach:

- First start with a coarse mesh, and determine in which regions of the mesh the radiosity is varying most rapidly.
- Then refine these mesh regions and iterate.
- When the radiosity values are fairly constant in the neighborhood of a patch of the mesh, or when the patches are deemed to be "small enough" then we do not need to refine further.



More sophisticated methods, i.e. *discontinuity meshing* actually attempt to align the edges of the mesh with sharp changes in radiosity (e.g. as happens along the edges of shadows).

What Comes First?

The radiosity at point A depends on the radiosity from all visible points B, and visa versa.

So how do we compute these radiosities?

There are two general approaches.:

- Define a large linear system of equations, that "encodes" all of the radiosity dependencies. By solving this equation, we can determine all the radiosities at all the points.
  The problem is the size of this linear system is enormous, as for n surface patches (or mesh elements) the matrix size is $\Longrightarrow n^2 \times n^2$, since each variable in the equation involves the transmission of light between each pair of surface patches.
- *Progressive refinement radiosity:* The idea is:
  - start with the brightest light "source" and shoot its radiation to the entire scene.
  - Then we move to the next brightest light "source" and repeat this process.

  Note that as we do this, surface patches that were initially black start picking up more and more intensity. Eventually a non-emitting light "source" accumulates more and more intensity, until it becomes the brightest light source, and then it shoots its intensity to the surrounding scene.

Energy Balance Equation

$$L^o(x, \theta_x^o, \phi_x^o, \lambda^o) = L^e(x, \theta_x^o, \phi_x^o, \lambda^o) + \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \int_{\lambda_{\min}}^{\lambda_{\max}} \rho_{bd}(x, \theta_x^i, \phi_x^i, \lambda^i, \theta_x^o, \phi_x^o, \lambda^o) \cos(\theta_x^i) L^i(x, \theta_x^i, \phi_x^i, \lambda^i) d\lambda^i \sin(\theta_x^i) d\phi_x^i d\theta_x^i$$

- $L^o(x, \theta_x^o, \phi_x^o, \lambda^o)$ is the total radiance
  - at wavelength λ^o
  - leaving point x
  - in direction θ_x^o, ϕ_x^o
- $L^e(x, \theta_x^o, \phi_x^o, \lambda^o)$ is the (*exitance*) radiance emitted by the surface from the point
- $L^i(x, \theta_x^i, \phi_x^i, \lambda^i)$ is the incident radiance impinging on the surface point
- $\rho_{bd}(x, \theta_x^i, \phi_x^i, \lambda^i, \theta_x^o, \phi_x^o, \lambda^o)$ in short BRDF, describes the directional reflective properties at the surface point.

  - describes the surface's interaction with light at the point
- the integration is over the hemisphere above the point
- the standard property used to describe light sources is *exitance*, or radiant emitted flux density, defined as energy radiated per unit time and unit area (similar to radiancy).

Radiosity Equation (Lambertian Surfaces)

In practice, we cannot expect to be able to solve this energy balance integral equation. As mentioned before, most radiosity methods are based on subdividing space into small patches, and assuming that the radiosity is constant for each path. Using some additional simplifications of radiance and with symmetry arguments and assuming that $B(y)$ is constant for all points y in a surface patch we obtain a simplified radiosity equation (see the last portion of this lecture for details),

The radiosity equation becomes

$$B(x) = E(x) + \rho_d(x) \sum_{j=1}^{N} \int_{y \in P_j} B(y) \frac{\cos\theta \cos\theta'}{\pi r^2} V(x,y) dy.$$

where

$$V(x,y) = \begin{cases} 1, \text{ if } x \text{ can see } y \\ 0, \text{ otherwise} \end{cases}$$

is the *visibility function*.

Since $B(y) = B_j, \forall y \in P_j$,

$$B(x) = E(x) + \rho_d(x) \sum_{j=1}^{N} B_j \int_{y \in P_j} \frac{\cos\theta \cos\theta'}{\pi r^2} V(x,y)dy.$$

Note: Even though $B(x)$ varies over a patch P_i, we consider B as constant over a patch only for purpose of illuminating other surfaces. The constant radiosity value is an area-weighted average of the pointwise radiosities $B_i = \frac{1}{A_i} \int_{x \in P_i} B(x)dx$; similarly exitance $E_i = \frac{1}{A_i} \int_{x \in P_i} E(x)dx$ and $P_d(x) = P_i$ (constant).

Hence

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j \frac{1}{A_i} \int_{x \in P_i} \int_{y \in P_j} \frac{\cos\theta \cos\theta'}{\pi r^2} V(x,y)dxdy$$

or more compactly,

$$B_i = E_i + \rho_i \sum_{j=1}^{N} F_{ij} B_j$$

or

$$E_i = B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

where *Form factor* $F_{i,j}$*:* the fraction of light energy leaving P_i that arrives at patch P_j:

$$F_{i,j} = \frac{1}{A_j} \int_{x \in P_i} \int_{y \in P_j} \frac{\cos\theta \cos\theta'}{\pi r^2} V(x,y) dy dx$$

$F_{i,j}$ is a dimensionless quantity. If patches are close, large, and facing one another, $F_{i,j}$ will be large.

Here B_i is the radiosity of patch i (the amount of light reflected per unit area), E_i is the amount of light emitted from this patch per unit area, ρ_i is the reflectivity of patch i ($\rho \approx 0$ means a dark non-reflecting object and $\rho \approx 1$ means a bright highly reflecting object). A_i and A_j are the areas of patches P_i and P_j, respectively.

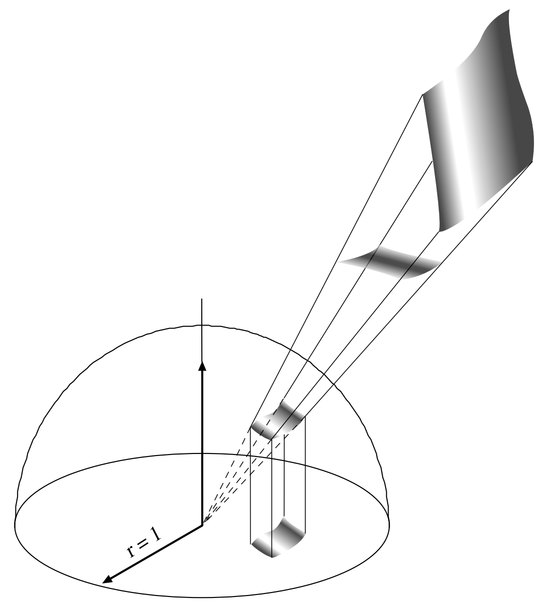
Form Factors

Its matrix form is

$$\begin{pmatrix} 1-\rho_1 F_{1,1} & -\rho_1 F_{1,2} & \cdots & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & \cdots & -\rho_2 F_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ -\rho_n F_{n,1} & \rho_n F_{n,2} & \cdots & 1-\rho_n F_{n,n} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix}$$

The values ρ_i are dependent on the surface types. The hard thing to compute are the values of $F_{i,j}$. The linear system is sparse. Iterative techniques from numerical analysis, such as Gauss-Seidel, can be used to solve this type of system.

It can be shown that there is a fairly simple geometric interpretation of $F_{i,j}$.



1. Break the i-th patch into small mesh elements.
2. For each mesh element consider a hemisphere surrounding this element, and project patch j onto this hemisphere through its center.
3. Then project this hemispherical projection orthographically onto the base circle of the hemisphere.
4. The value of $F_{i,j}$ is the area of this projection, divided by the area of the circle.

Thus intuitively, patches that occupy a larger field of view contribute more to $F_{i,j}$ and patches that are nearly orthogonal to the hemisphere contribute less.

Computing this orthogonal projection of a hemi-spherical projection is somewhat tricky (considering that it must be repeated for every tiny element of every patch), so it is important of speed this computation up, at the cost of the introduction of approximation errors. On thus approximates the hemisphere by a hemicube, and discretizes the surface of the hemicube into square (pixel-like) elements. We project all the surrounding patches on to each of the faces of the hemicube. (Note that this is essentially a visible surface elimination task, which can be solved with hardware assistance, e.g. using a z-buffer algorithm. More about this in subsequent lectures) Each cell of the hemicube is now associated with a patch, and we apply a weighting factor that depends on the square of the hemicube, and sum these up.

Needless to say, this process is extremely computationally intensive. We are basically solving a visible surface determination problem at *every* point on the surface of our objects. Much of the research in radiosity is devoted to mechanisms to save computations, without sacrificing realism.

Basics of Radiance

The most basic concept of radiosity is radiance.

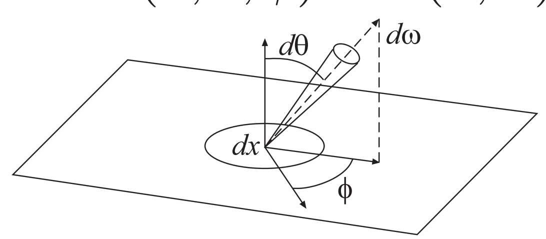
Radiance L: the amount of energy per unit time (or equivalently power) emitted from a point x in a given direction per unit area (perpendicular to direction) per unit solid angle.

Define:

- θ: the angle with respect to the surface normal,
- ϕ: the angle of the projection onto the surface.
- ω: the resulting directional vector.

Thus

$$L = L(x, \theta, \phi) = L(x, \omega)$$



The energy radiating during time interval dt from a small patch dx^2 in some small solid angle dw can be expressed as:

$$L(x, \theta, \phi) \underbrace{dx \cos\theta}_{\text{projectedarea}} d\omega dt$$

Radiance is measured in watts per square meter per steradian.

Then the *radiosity*, denoted by B, the total power (energy per unit time) leaving a point on a surface, per unit area on the surface is as follows

$$B(x) = \int_{\Omega} L(x, \theta, \phi) \cos\theta d\omega$$

where Ω is the hemisphere's surface lying on the above the surface.

A corresponding quantity called *irradiance* is expressed in the same units as B, represents the total incident <u>flux</u> density at a point and is measured in $Watts/m^2$.

Simple Radiosity Equation

If surfaces are Lambertian (idea diffuse surface), then we can simplify $L(x, \theta, \phi)$ and just write $L(x)$. The radiosity at the point x is given by

$$
\begin{aligned}
B(x) &= \int_\Omega L(x, \theta, \phi) \cos\theta d\omega \\
&= L(x) \int_\Omega \cos\theta d\omega \qquad\qquad \text{where} \;\; d\omega = \sin\theta d\theta d\phi \\
&= L(x) \int_0^\pi \int_0^{2\pi} \cos\theta \sin\theta d\theta d\phi \\
&= \pi L(x).
\end{aligned}
$$

This means simply that depends only on the radiance, the light power, at the point.

Radiosity equation (for Lambertian reflectors):

$$L(x) = L_e(x) + \frac{\rho_d(x)}{\pi} \int_\Omega L_i(x, \theta, \phi) \cos\theta d\omega$$

where L_e denotes emitted radiance and L_i denotes the incoming irradiance, $\rho_d(x)$ denotes the coefficient of diffuse reflection (earlier we had written this as k_d).

We cannot eliminate the directional component from the L_i term, because we still need to consider Lambert's law for incoming radiation.

If we define

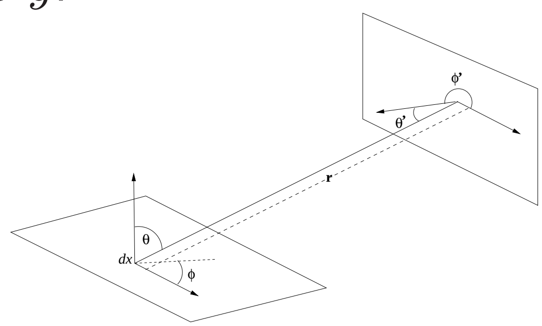
$$H(x) = \int_\Omega L_i(x, \theta, \phi) \cos\theta d\omega$$

and let $E(x)$ denote the emitted radiosity $\pi L_e(x)$, and recall that $B(x) = \pi L(x)$ then we can write this as

$$B(x) = E(x) + \rho_d(x) H(x)$$

The term $H(x)$ essentially describes how much illumination energy is arriving from all other points in the scene.

To simplify $H(x)$ we can use the Lambertian assumption. Rather than integrating over the angular space surrounding x, instead we will integrate over the set of points on the surface, denoted S. Let $y \in S$ be such a surface point visible from x in direction $\omega =< \theta, \phi >=< \theta', \phi' >$. Let θ' denote the angle between the surface normal at y and the line-of-sight vector from y to $x(-\omega)$, and let ϕ' be defined similar to ϕ but for y. Let r denote the distance from x to y.



By symmetry of radiance, we have $L(x, \theta, \phi) = L(y, \theta', \phi')$.

Since all surfaces are Lambertian, we have

$$L(y, \theta', \phi') = \frac{B(y)}{\pi}.$$

And

$$d\omega = \frac{\cos\theta' dy}{r^2}.$$

Putting these together, we can define $H(x)$ in terms of an integral over surface points:

$$H(x) = \int_{y \in S} B(y) \frac{\cos\theta \cos\theta'}{\pi r^2} V(x, y) dy.$$

where

$$V(x, y) = \begin{cases} 1, \text{ if } x \text{ can see } y \\ 0, \text{ otherwise} \end{cases}$$

is the *visibility function*.

Hence the radiosity equation (Lambertian)

$$B(x) = E(x) + \rho_d(x) \int_{y \in S} B(y) \frac{\cos\theta \cos\theta'}{\pi r^2} V(x, y) dy.$$

Reading Assignment and News

Please review the appropriate sections related to this lecture in chapter 7, and associated exercises, of the recommended text.

(Recommended Text: Interactive Computer Graphics, by Edward Angel, Dave Shreiner, 6th edition, Addison-Wesley)

Please track Blackboard for the most recent Announcements and Project postings related to this course.

(http://www.cs.utexas.edu/users/bajaj/graphics2012/cs354/)