Formal Verification of x86 Machine-Code Programs

Computer Architecture and Program Analysis

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Software and Reliability

Can we rely on our software systems?

Recent example of a serious bug:

CVE-2016-5195 or "Dirty COW"

- Privilege escalation vulnerability in Linux
- E.g.: allowed a user to write to files intended to be read only
- Copy-on-Write (COW) breakage of private read-only memory mappings
- Existed since around v2.6.22 (**2007**) and was **fixed on Oct 18, 2016**

Formal Verification of Software: Example 1

Software Formal Verification: proving or disproving that the *implementation* of a program meets its *specification* using mathematical techniques

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Suppose you needed to count the number of 1s in the binary representation of a natural number (*population count*).

Specification:

```
popcountSpec(v): [v: natural number]
if v <= 0 then
   return 0
else
   lsb = v & 1
   v   = v >> 1
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endif
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Source: Sean Anderson's Bit-Twiddling Hacks

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Implementation:

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  v = v - ((v >> 1) & 0x55555555);
  v = (v & 0x33333333) + ((v >> 2) & 0x33333333);
  v = ((v + (v >> 4) & 0xF0F0F0F) * 0x1010101) >> 24;
  return(v);
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```

Do the specification and implementation behave the same way for all inputs?

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Formal Verification of Software: Example 2

Suppose you needed to check if a given natural number is a power of 2.

Specification:

```
isPowerOfTwoSpec(x): [x: natural number]
if x == 0 then
   return 0
else
   if x == 1 then
      return 1
   else
      if remainder(x,2) == 0 then
         return isPowerOfTwoSpec(x/2)
      else
         return 0
      endif
   endif
endif
```

Formal Verification of Software: Example 2

Can you trust your specification?

Source: Sean Anderson's Bit-Twiddling Hacks

Formal Verification of Software: Example 2

Can you trust your specification?

***Correctness of* `isPowerOfTwoSpec`:**

1. If `isPowerOfTwoSpec(v)` returns 1, then there exists a natural number `n` such that $v = 2^n$.
2. If $v = 2^n$, where `n` is a natural number, then `isPowerOfTwoSpec(v)` returns 1.

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Implementation:

```
bool powerOfTwo (long unsigned int v) {
  bool f;
  f = v && !(v & (v - 1));
  return f;
}
```

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Formal Verification of Software: Example 2

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Inspection of a Program's Behavior

- **Testing:**
  - × Exhaustive analysis is infeasible
- **Formal Verification:**
  - ✓ Wide variety of techniques
    - Lightweight: e.g., checking if array indices are within bounds
    - Heavyweight: e.g., proving functional correctness

Example: Pop-Count Program

```
popcount_64:
89 fa                mov    %edi,%edx
89 d1                mov    %edx,%ecx
d1 e9                shr    %ecx
81 e1 55 55 55 55    and    $0x55555555,%ecx
29 ca                sub    %ecx,%edx
89 d0                mov    %edx,%eax
c1 ea 02             shr    $0x2,%edx
25 33 33 33 33       and    $0x33333333,%eax
81 e2 33 33 33 33    and    $0x33333333,%edx
01 c2                add    %eax,%edx
89 d0                mov    %edx,%eax
c1 e8 04             shr    $0x4,%eax
01 c2                add    %eax,%edx
48 89 f8             mov    %rdi,%rax
48 c1 e8 20          shr    $0x20,%rax
81 e2 0f 0f 0f 0f    and    $0xf0f0f0f,%edx
89 c1                mov    %eax,%ecx
d1 e9                shr    %ecx
81 e1 55 55 55 55    and    $0x55555555,%ecx
29 c8                sub    %ecx,%eax
89 c1                mov    %eax,%ecx
c1 e8 02             shr    $0x2,%eax
81 e1 33 33 33 33    and    $0x33333333,%ecx
25 33 33 33 33       and    $0x33333333,%eax
01 c8                add    %ecx,%eax
89 c1                mov    %eax,%ecx
c1 e9 04             shr    $0x4,%ecx
01 c8                add    %ecx,%eax
25 0f 0f 0f 0f       and    $0xf0f0f0f,%eax
69 d2 01 01 01 01    imul   $0x1010101,%edx,%edx
69 c0 01 01 01 01    imul   $0x1010101,%eax,%eax
c1 ea 18             shr    $0x18,%edx
c1 e8 18             shr    $0x18,%eax
01 d0                add    %edx,%eax
c3                   retq
```

Functional Correctness:
`RAX = popcountSpec(v)`

specification function

```
popcountSpec(v):
[v: unsigned int]

if v <= 0 then
   return 0
else
   lsb = v & 1
   v   = v >> 1
   return (lsb + popcountSpec(v))
endif
```

Case Study: Pop-Count Program

```
(defthm x86-popcount-64-symbolic-simulation
  (implies
   (and (x86p x86)
        (equal (model-related-error x86) nil)
        (unsigned-byte-p 64 n)
        (equal n (read 'register *rdi* x86))
        (equal *popcount-64-program*
               (read 'memory
                     (address-range
                      (read 'pc x86)
                      (len *popcount-64-program*))
                     x86)))
   (equal (read 'register *rax* (x86-run *num-of-steps* x86))
          (popcountSpec n))))
```

Heavyweight Formal Verification

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- Build a mathematical or ***formal* model** of programs
- Prove ***theorems*** about this model in order to establish program properties

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- Defines the machine language
- Specification of state (registers, memory), machine instructions, instruction encodings, etc.

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- All high-level programs compile down to machine-code programs.
  - A program is just a sequence of machine instructions.

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Instruction Set Architecture: interface between hardware and software

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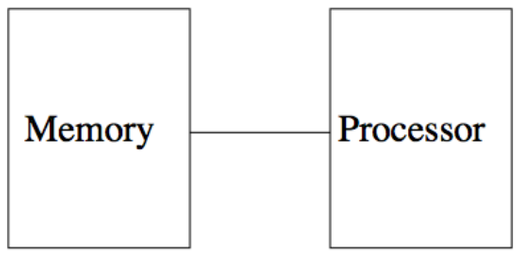
- An ISA model specifies the behavior of each machine instruction in terms of *effects made to the processor state*.
- All high-level programs compile down to machine-code programs.
  - A program is just a sequence of machine instructions.
- We can reason about a program by *inspecting the cumulative effects of its constituent instructions on the machine state*.

Why Not Use Abstract Machine Models?

Abstract vs. Concrete Machine Models

Machine Models

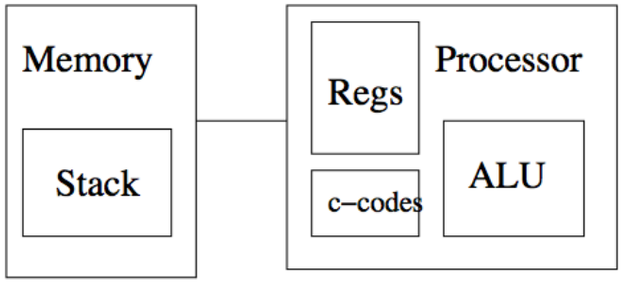
C



Data
1) char
2) int, float
3) double
4) struct, array
5) pointer

Control
1) loops
2) conditionals
3) switch
4) proc. call
5) proc. return

Assembly



1) byte
2) 2-byte word
3) 4-byte long word
4) contiguous byte allocation
5) address of initial byte

1) branch/jump
2) call
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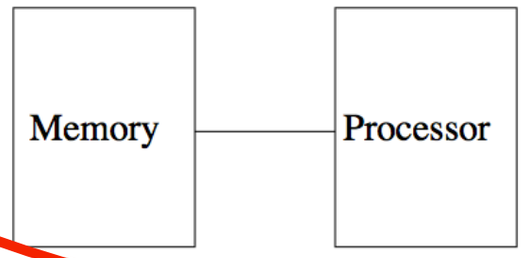

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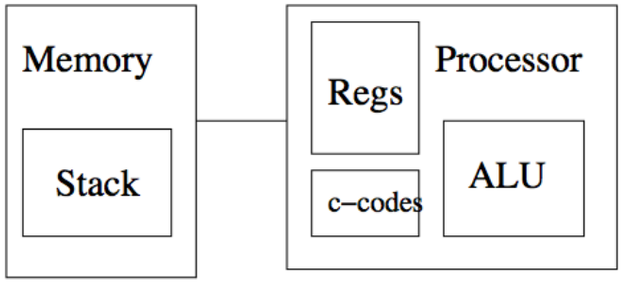

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Why x86 Machine-Code Verification?

- ***Why not high-level code verification?***
  - × Sometimes, high-level code is unavailable (e.g., malware)
  - × High-level verification frameworks do not address compiler bugs
    - ✓ Verified/verifying compilers can help
    - × But these compilers typically generate inefficient code
  - × Need to build verification frameworks for many high-level languages

- ***Why x86?***
  - ✓ x86 is in widespread use

Overview

Goal: *Specify* and *verify* properties of x86 programs

- E.g., correctness w.r.t. behavior, security, resource usage, etc.

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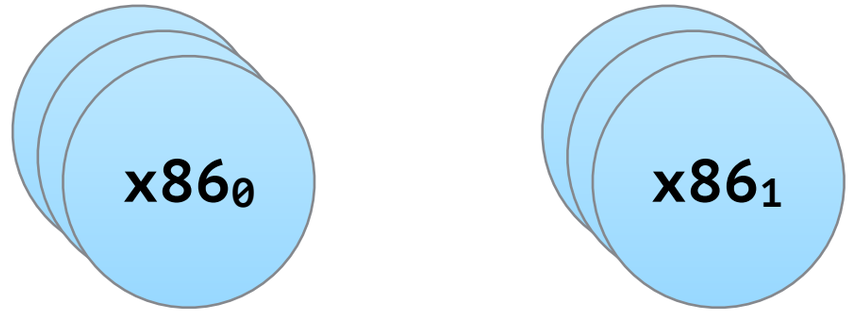

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  - One state, set of states, relationship between a set of final & initial states

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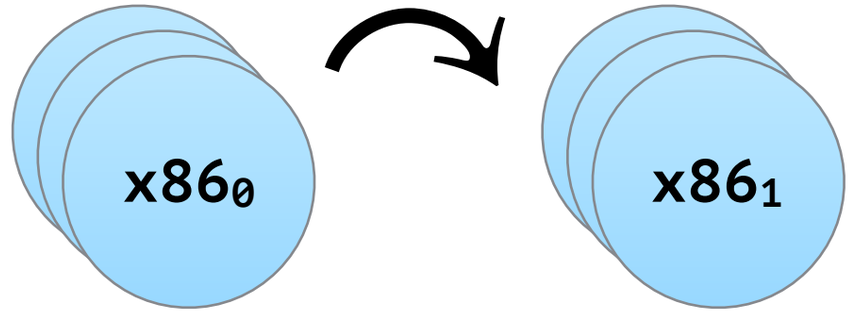

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specify:
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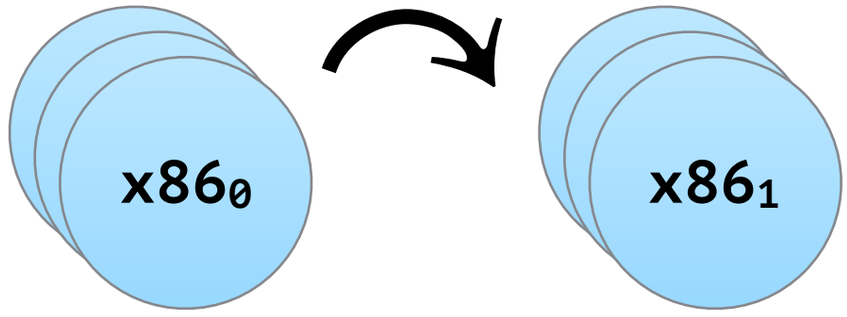
verify:
reason about symbolic executions



- **Program property:** statement about a program's behavior
  - One state, set of states, relationship between a set of final & initial states
- **Program's computation:** how the execution of each instruction transforms one state to another
- **Symbolic Executions:** a final (or next) x86 state is described in terms of *symbolic updates* made to the initial x86 state
  - Allows consideration of many, if not all, possible executions at once

Formal Tool Used: ACL2 Theorem-Proving System

- ACL^2: A Computational Logic for Applicative Common Lisp
  - Programming Language
  - Mathematical Logic
  - Mechanical Theorem Prover

- See [ACL2 Home Page] for more details.
  - Extensive documentation!
  - ACL2 Research Group located at GDC 7S

x86 ISA Model

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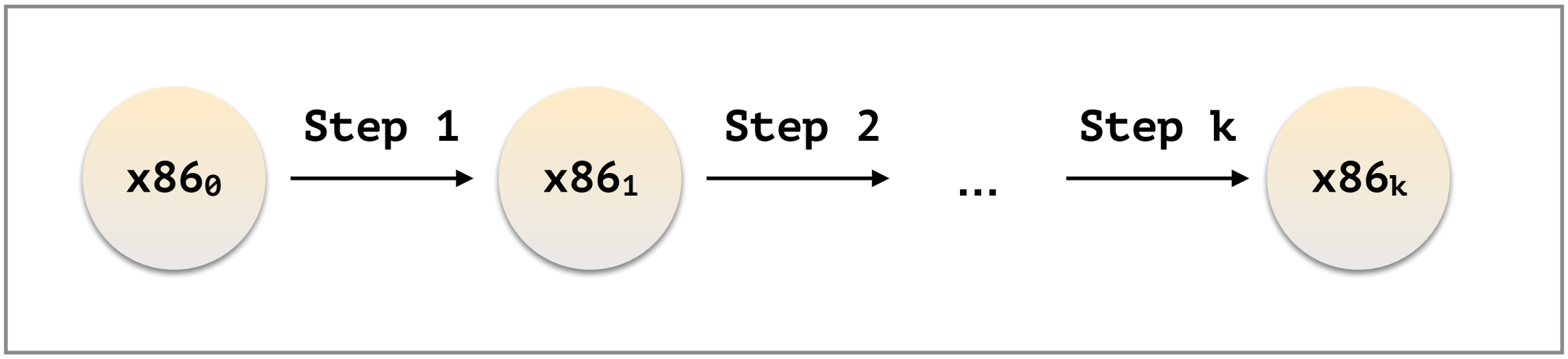


A Run of the x86 Interpreter that executes k instructions

Interpreter-Style Operational Semantics: x86 ISA model is a machine-code interpreter written in ACL2's formal logic

- **x86 State:** specifies the components of the ISA
- **Run Function:** takes n steps or terminates early if an error occurs
- **Step Function:** fetches, decodes, and executes one instruction
- **Instruction Semantic Functions:** specifies instructions' behavior

Run Function

Recursively defined interpreter that specifies the x86 model

```
run(n, x86):
if n == 0 then
    return x86
else
    if model-related error encountered then
        return x86
    else
        run(n - 1, step(x86))
    end if
end if
```

Step Function

State-transition function that corresponds to the execution of a single x86 instruction

```
step(x86):
pc = rip(x86)
[prefixes, opcode, ... , imm] = Fetch-and-Decode(pc, x86)
case opcode:
     #x00 -> add-semantic-fn(prefixes, ... , imm, x86)
     ... ...
     #xFF -> inc-semantic-fn(prefixes, ... , imm, x86)
```

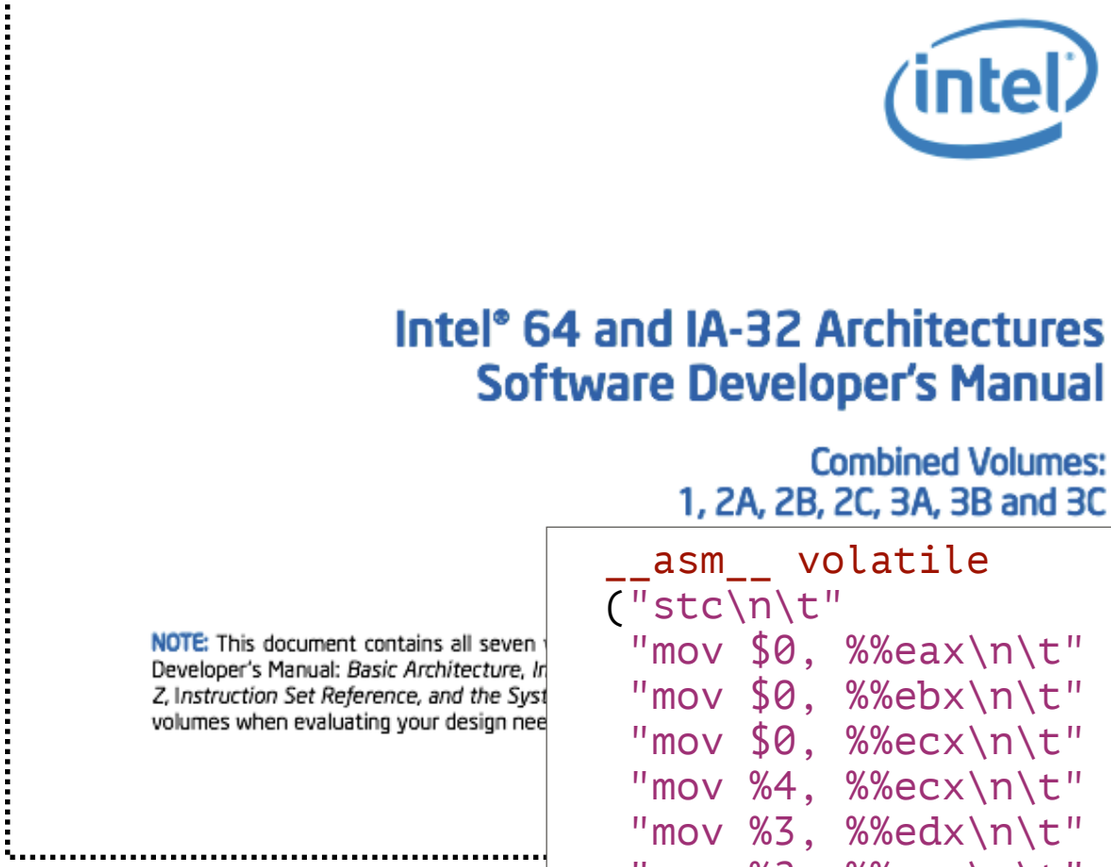
Instruction Semantic Functions

- A semantic function describes the effects of executing an instruction.
  - Input: x86 state and decoded parts of the instruction
  - Output: next x86 state
- Every instruction has its own semantic function.

```
add-semantic-fn(prefixes, ... , imm, x86):
operand1    = getOperand1(prefixes, ... , imm, x86)
operand2    = getOperand2(prefixes, ... , imm, x86)
resultSum   = fix(operand1 + operand2, ...)
resultFlags = computeFlags(operand1, operand2, result, x86)
x86         = updateState(resultSum, dst, resultFlags)
return x86
```

Obtaining the x86 ISA Specification

~3400 pages



Intel® 64 and IA-32 Architectures
Software Developer's Manual

Combined Volumes:
1, 2A, 2B, 2C, 3A, 3B and 3C

NOTE: This document contains all seven
Developer's Manual: *Basic Architecture, In*
Z, Instruction Set Reference, and the Syst
volumes when evaluating your design nee

~3000 pages

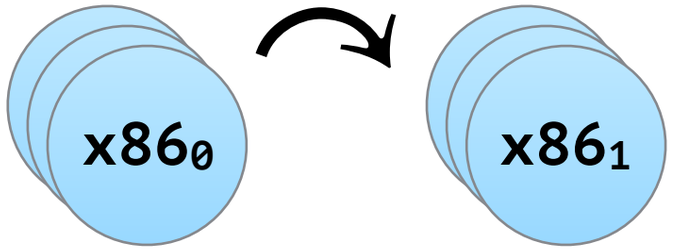
AMD64 Technology

AMD64 Architecture
Programmer's Manual

```
__asm__ volatile
("stc\n\t"                      // Set CF.
 "mov $0, %%eax\n\t"            // Set EAX = 0.
 "mov $0, %%ebx\n\t"            // Set EBX = 0.
 "mov $0, %%ecx\n\t"            // Set ECX = 0.
 "mov %4, %%ecx\n\t"            // Set CL = rotate_by.
 "mov %3, %%edx\n\t"            // Set EDX = old_cf = 1.
 "mov %2, %%eax\n\t"            // Set EAX = num.
 "rcl %%cl, %%al\n\t"           // Rotate AL by CL.
 "cmovb %%edx, %%ebx\n\t"       // Set EBX = old_cf if CF = 1.
                                // Otherwise, EBX = 0.
 "mov %%eax, %0\n\t"            // Set res = EAX.
 "mov %%ebx, %1\n\t"            // Set cf  = EBX.

 : "=g"(res), "=g"(cf)
 : "g"(num), "g"(old_cf), "g"(rotate_by)
 : "rax", "rbx", "rcx", "rdx");
```

Running tests on x86 machines



x86 State

Focus: Intel's 64-bit mode

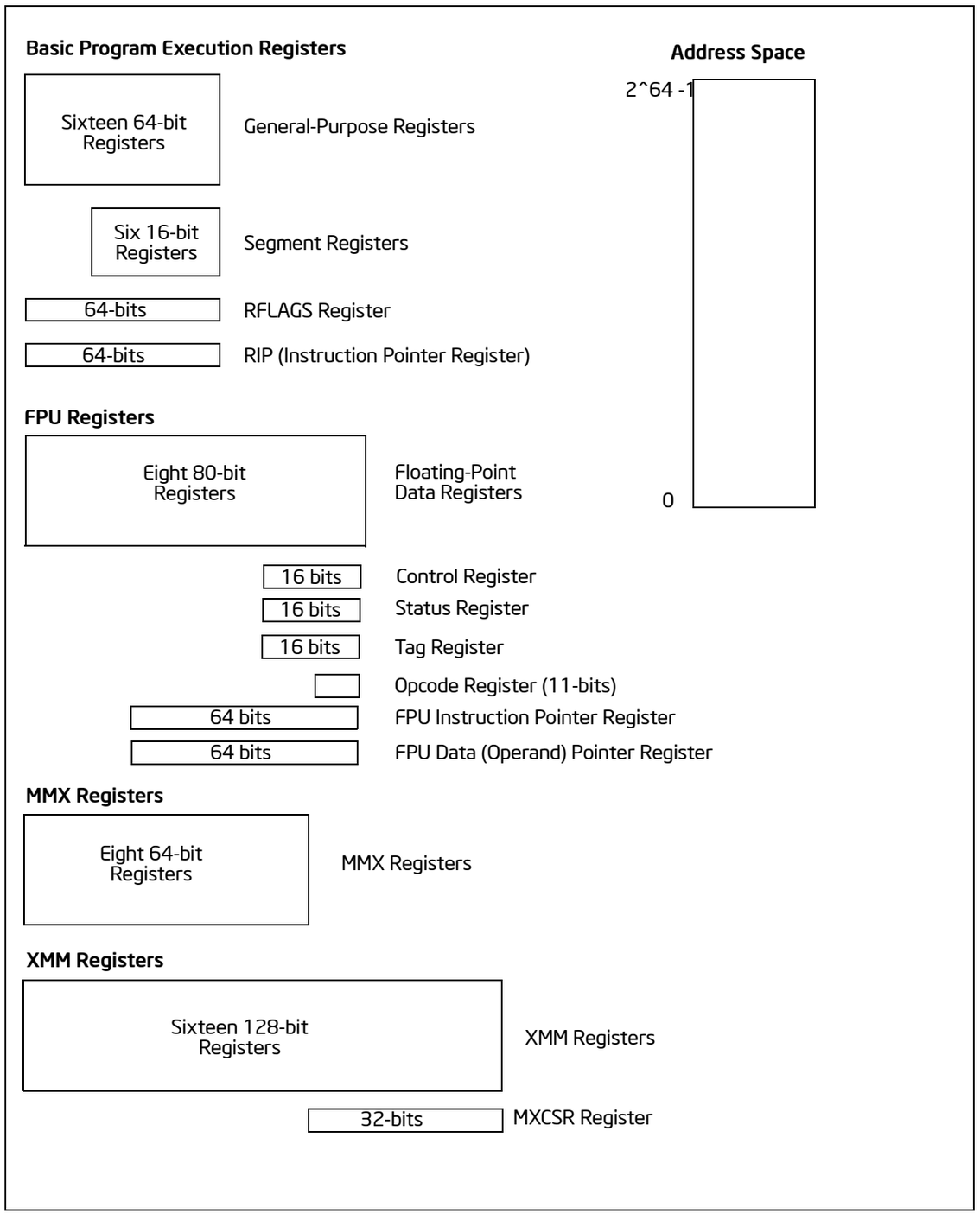
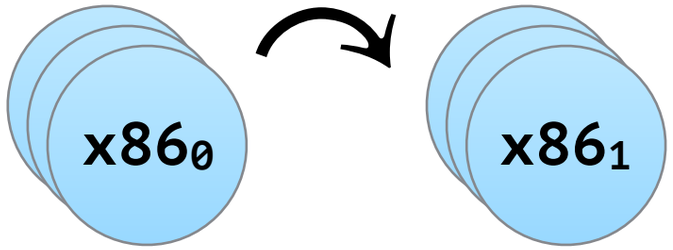


Figure 3-2. 64-Bit Mode Execution Environment

Source: Intel Manuals



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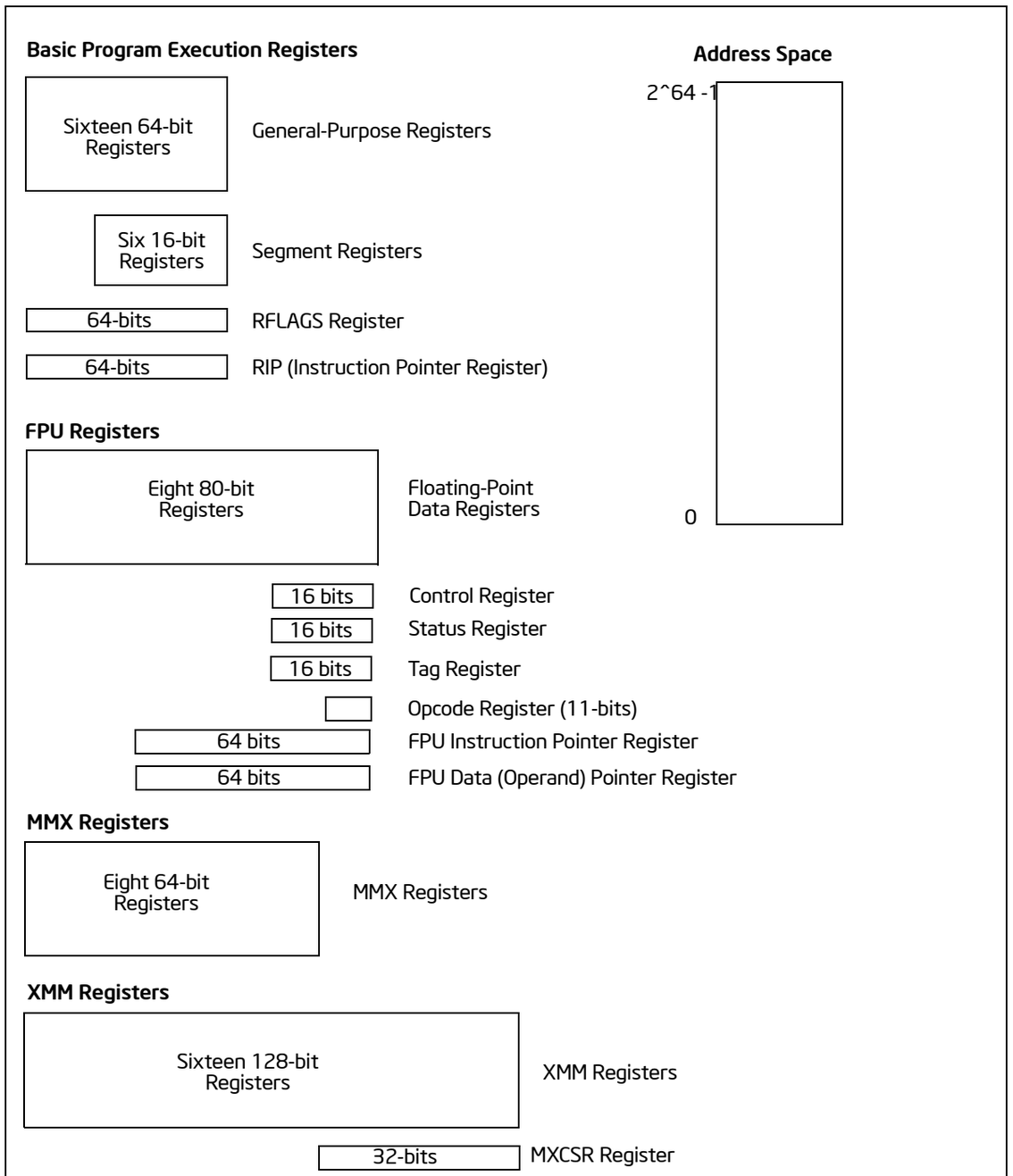

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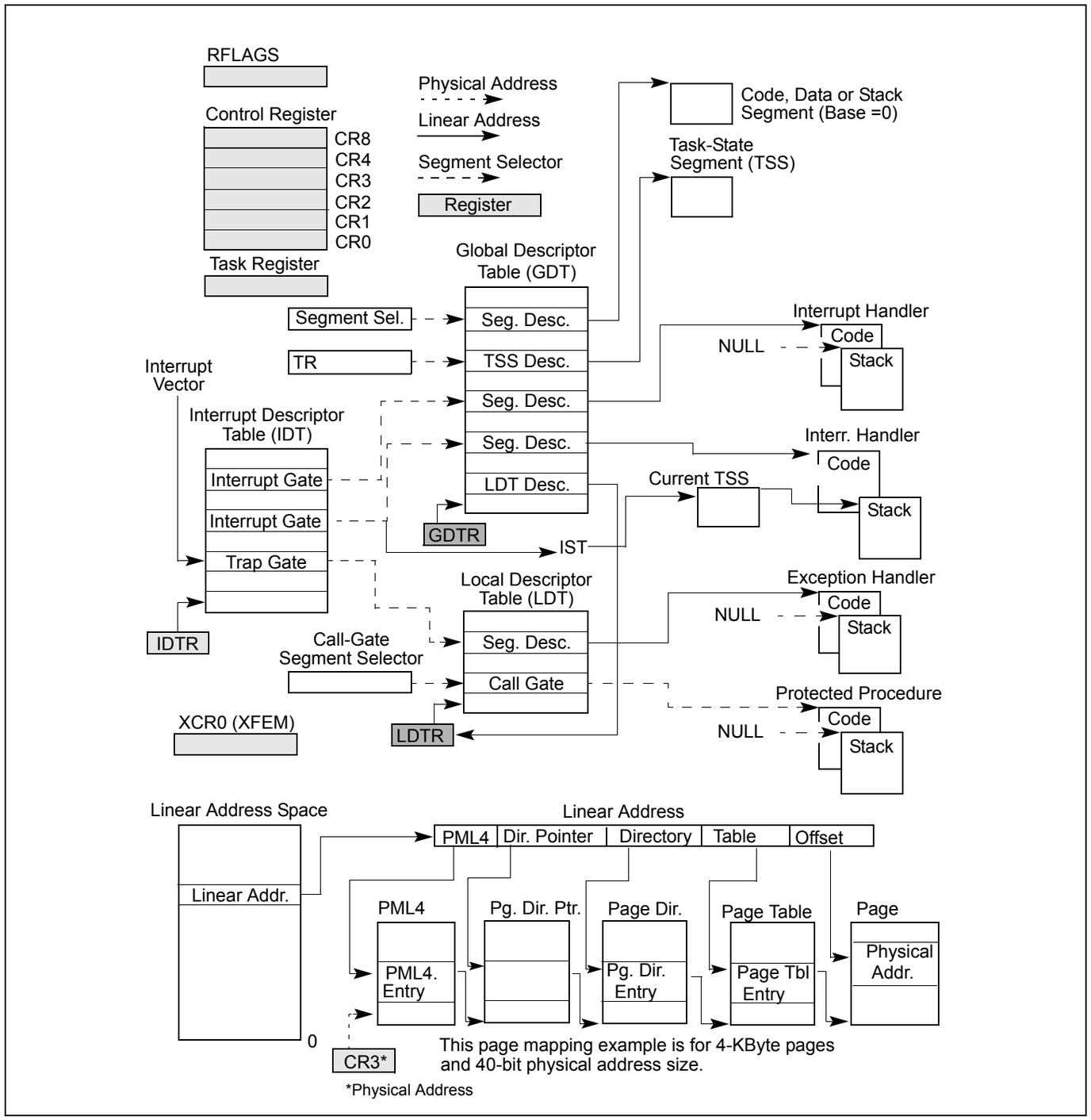


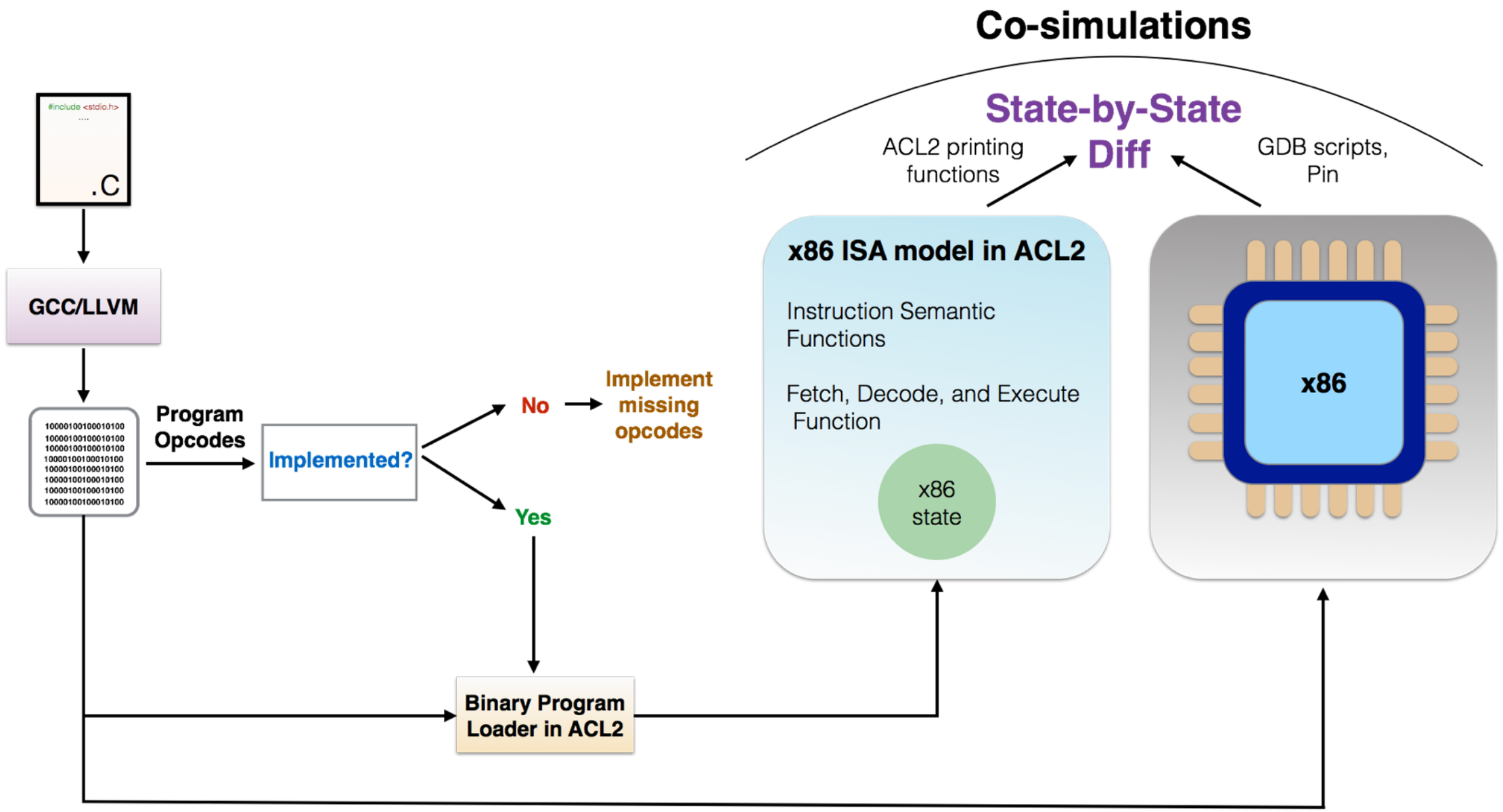
Figure 2-2. System-Level Registers and Data Structures in IA-32e Mode

Source: Intel Manuals

Model Validation

How can we know that our model faithfully represents the x86 ISA?

Validate the model to increase trust in the applicability of formal analysis



Symbolic Execution

Supporting Symbolic Execution

1. **read** instruction from `mem`
2. **read** `flags`
3. **write** new value to `pc`

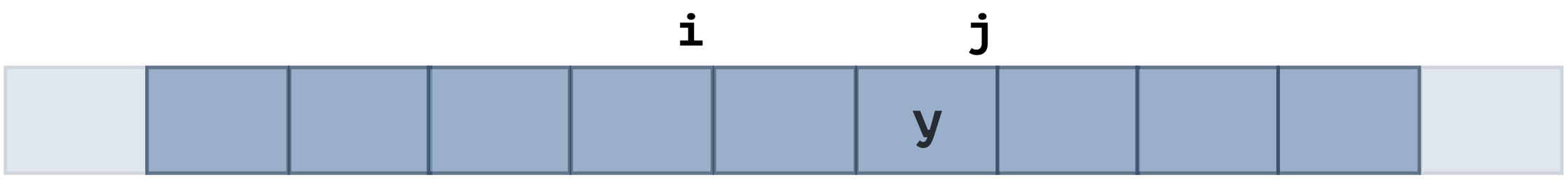
```
add %edi, %eax
je  0x400304
```

1. **read** instruction from `mem`
2. **read** operands
3. **write** sum to `eax`
4. **write** new value to `flags`
5. **write** new value to `pc`

Rules (theorems) describing interactions between these reads and writes to the x86 state enable *symbolic execution* of programs.

Read-over-Write Theorem #1

non-interference



Program Order

Read-over-Write Theorem #1

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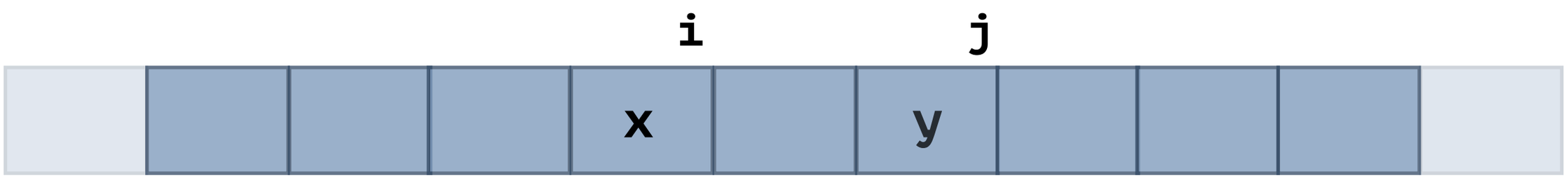


Program Order

$W_i(x)$

memory

Read-over-Write Theorem #1

non-interference

Program Order

$W_i(x)$

R_j: y

Read-over-Write Theorem #2

overlap

`i`

Program Order

memory

Read-over-Write Theorem #2

overlap

Program Order

$W_i(x)$

memory

Read-over-Write Theorem #2

overlap

Program Order

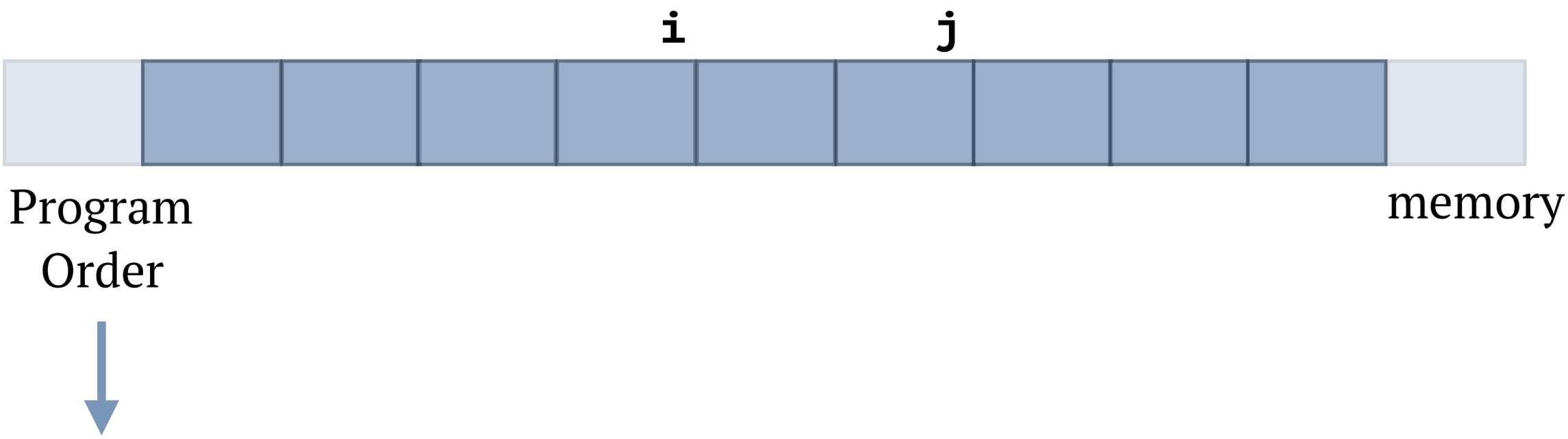
$W_i(x)$

R_i: x

memory

Write-over-Write Theorem #1

independent writes commute safely



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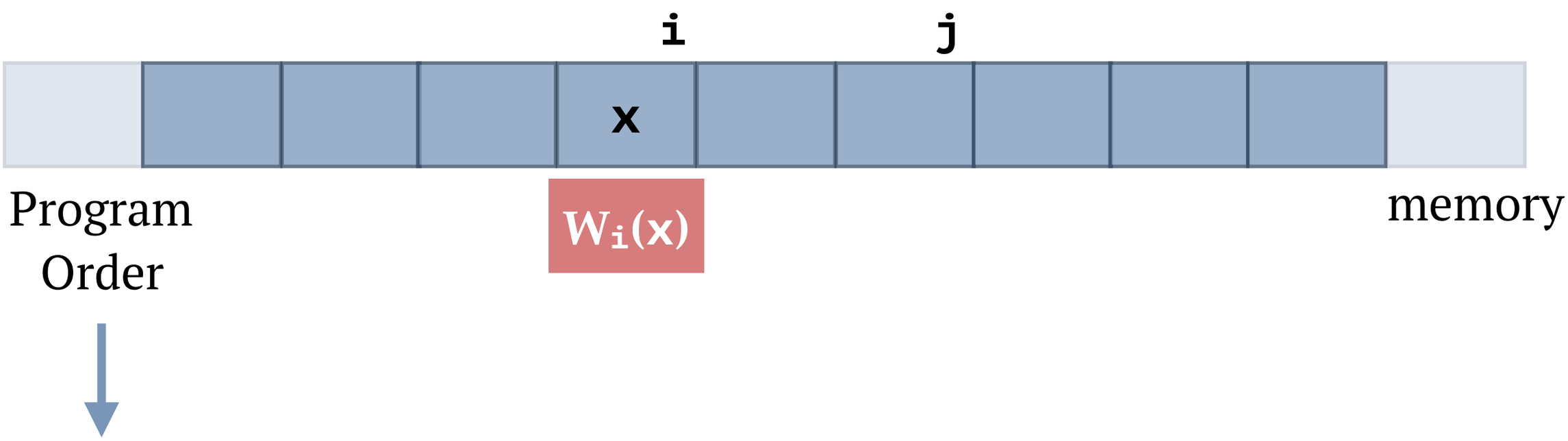

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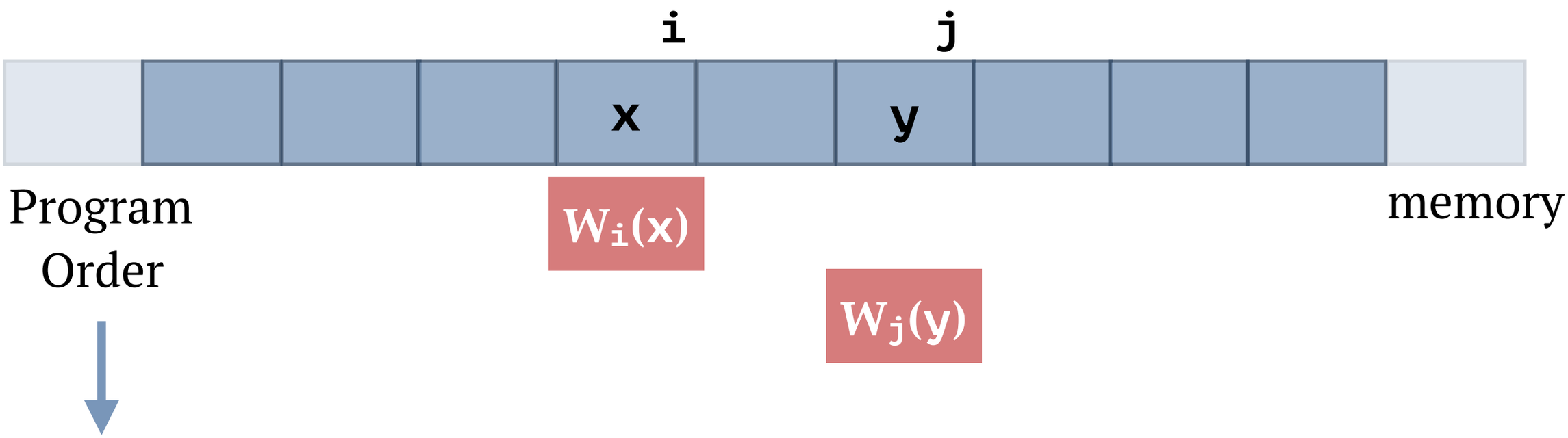
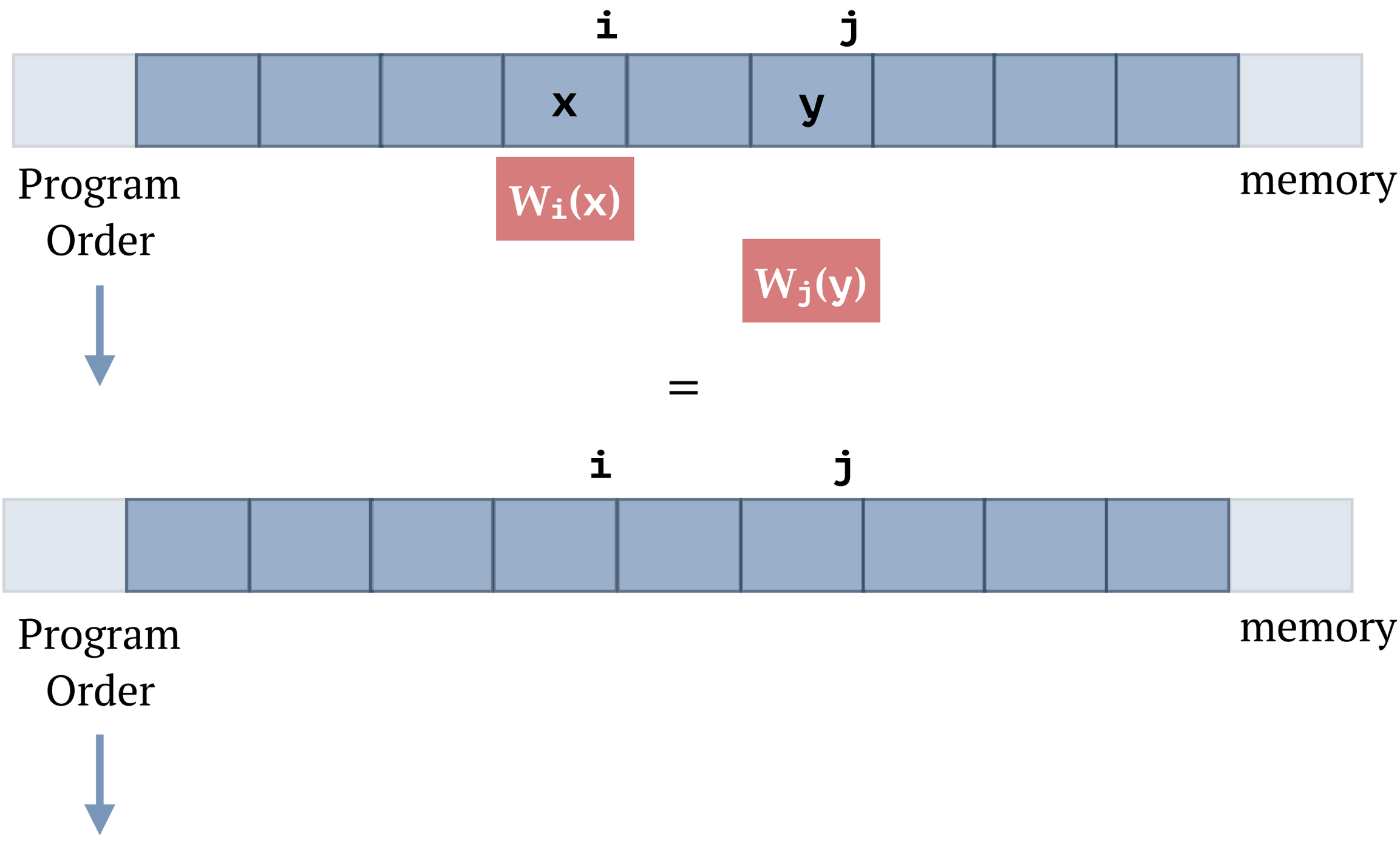


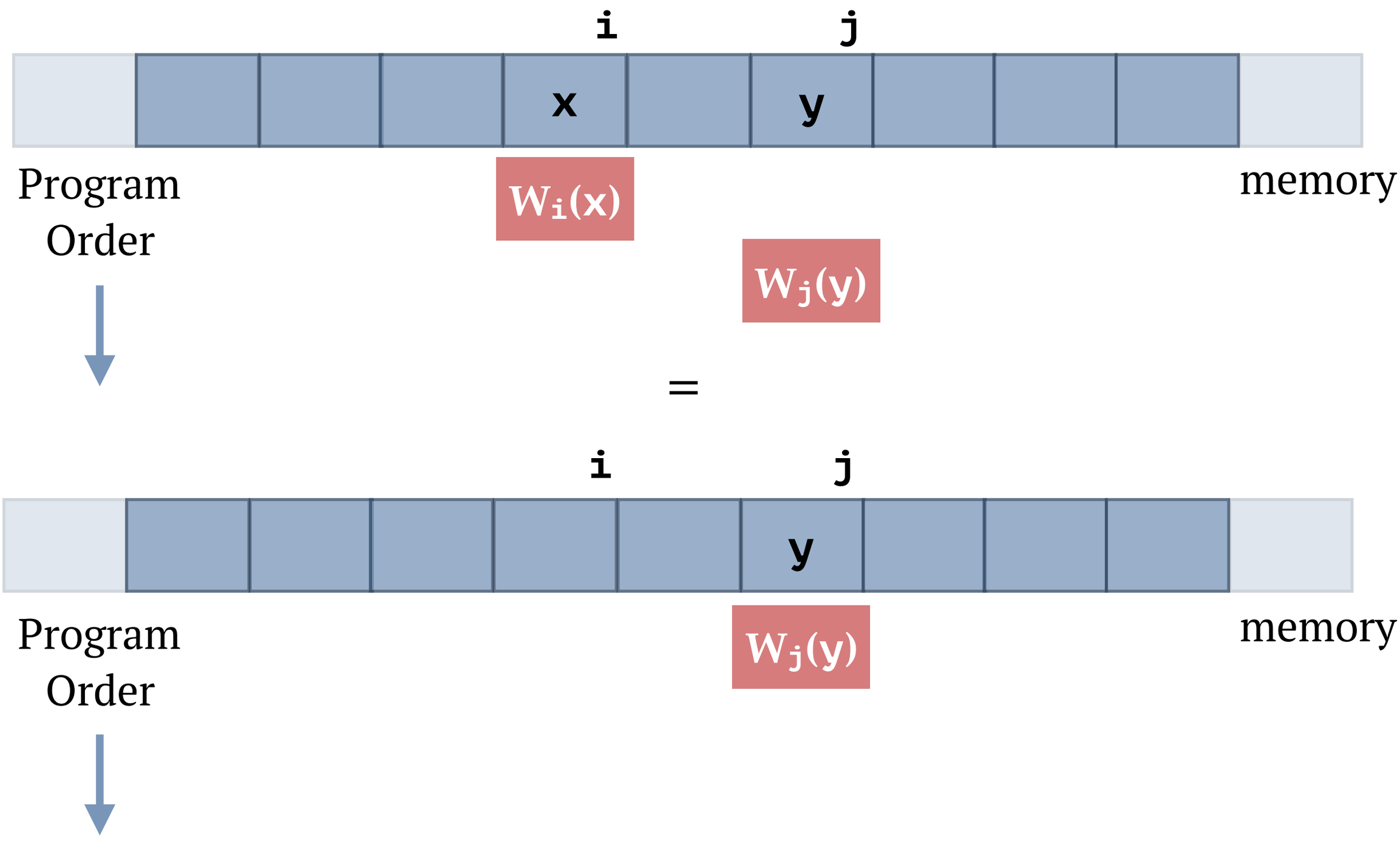
Write-over-Write Theorem #1

independent writes commute safely



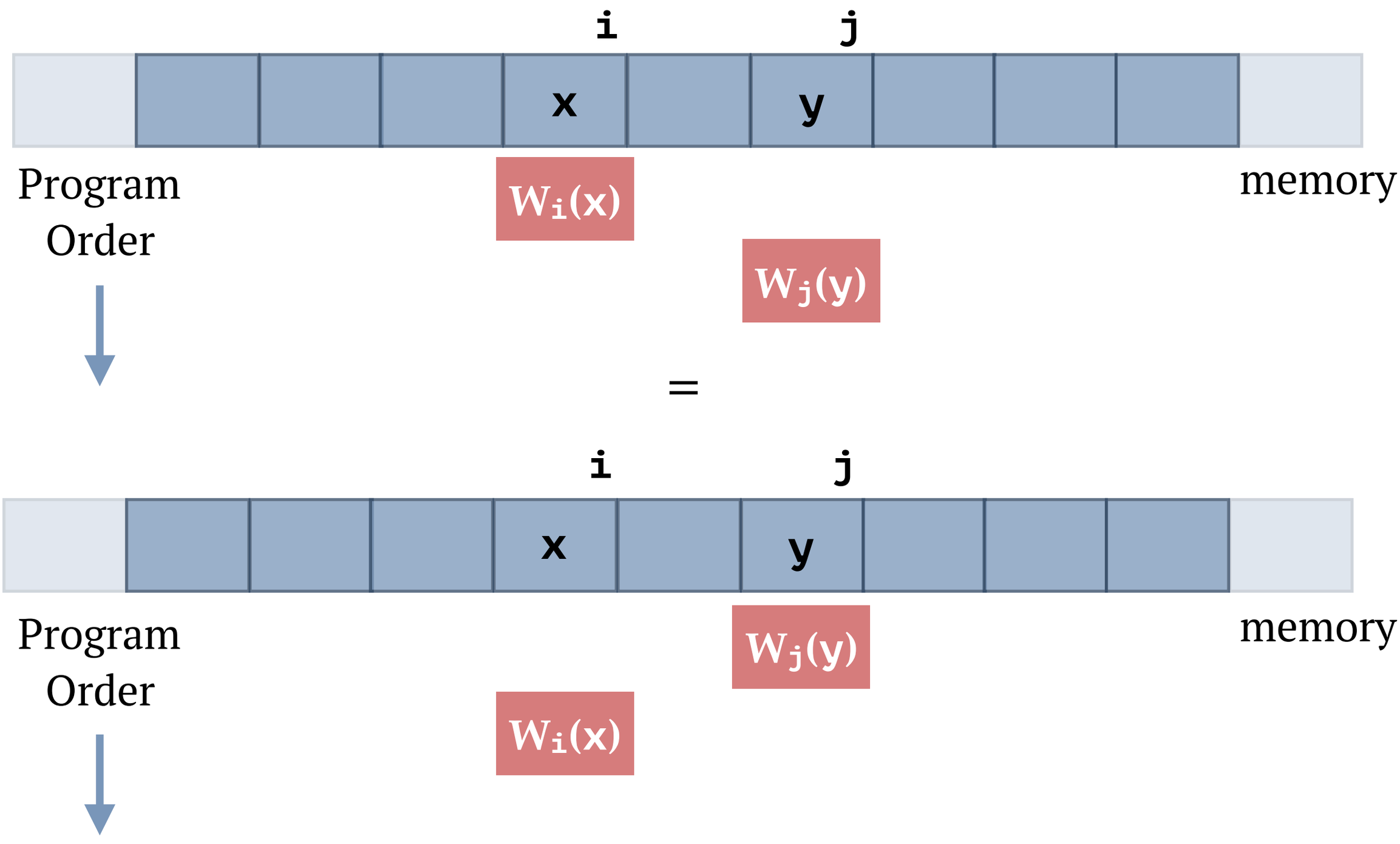
Write-over-Write Theorem #1

independent writes commute safely



Write-over-Write Theorem #1

independent writes commute safely



Write-over-Write Theorem #2

visibility of writes

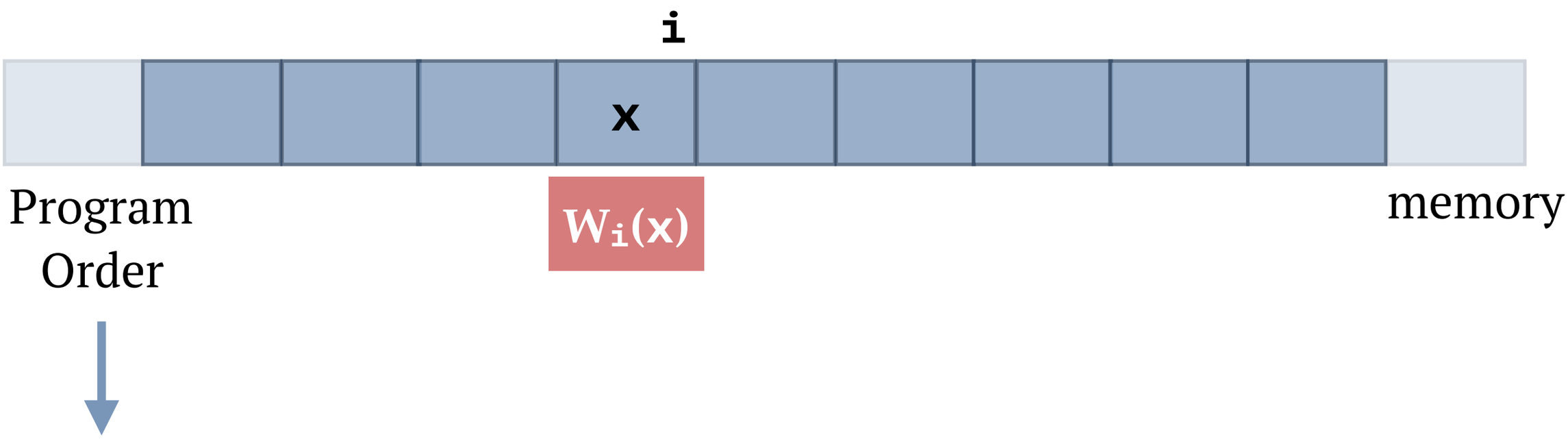
`i`

Program Order

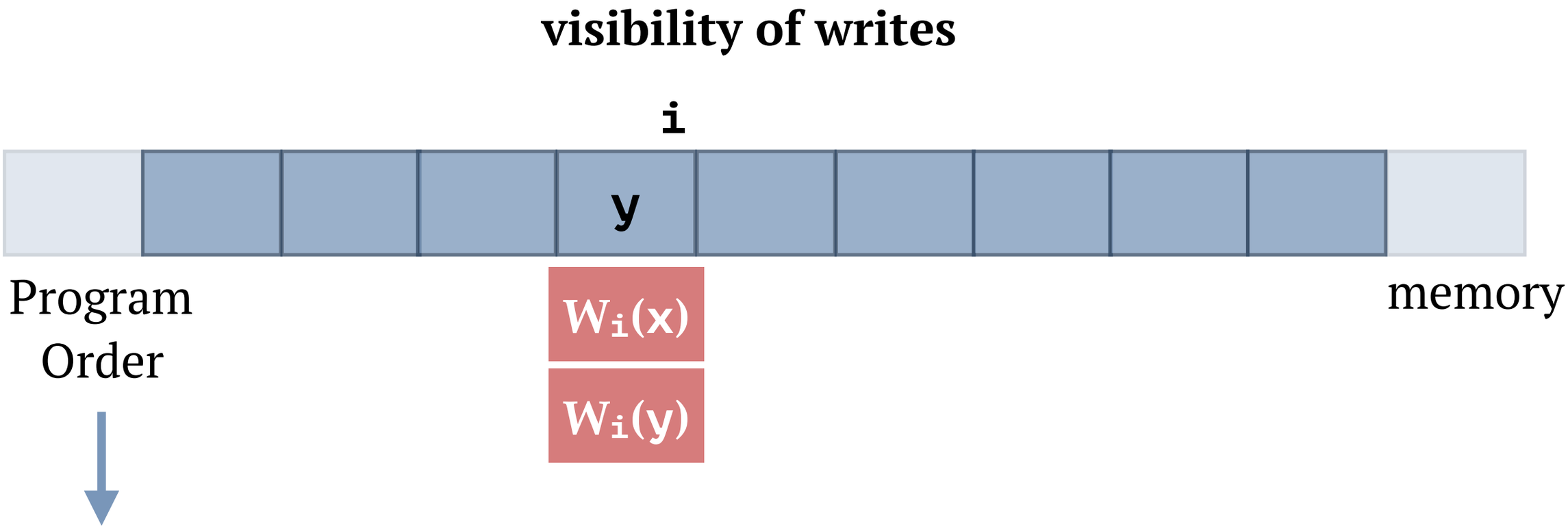
memory

Write-over-Write Theorem #2

visibility of writes

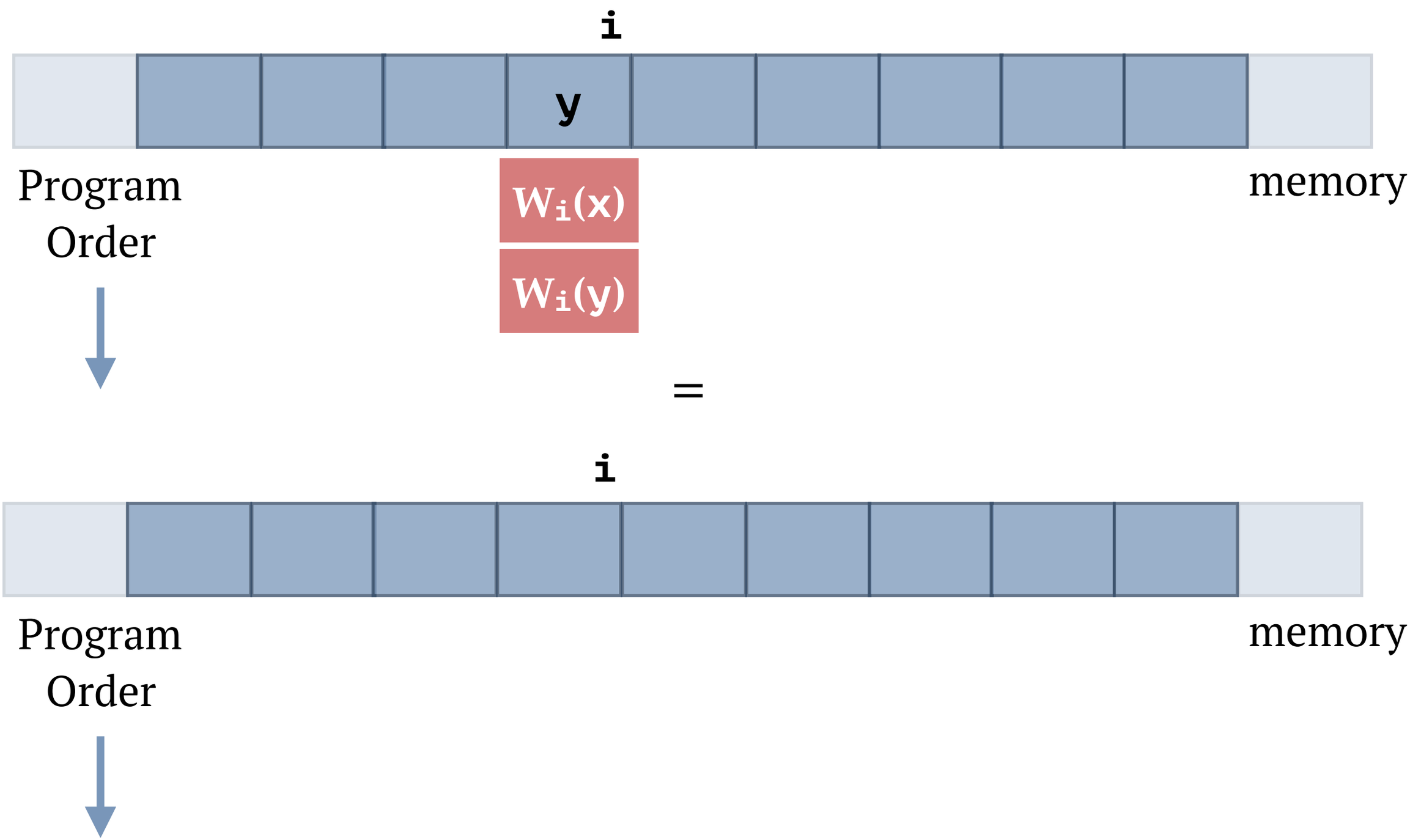


Write-over-Write Theorem #2



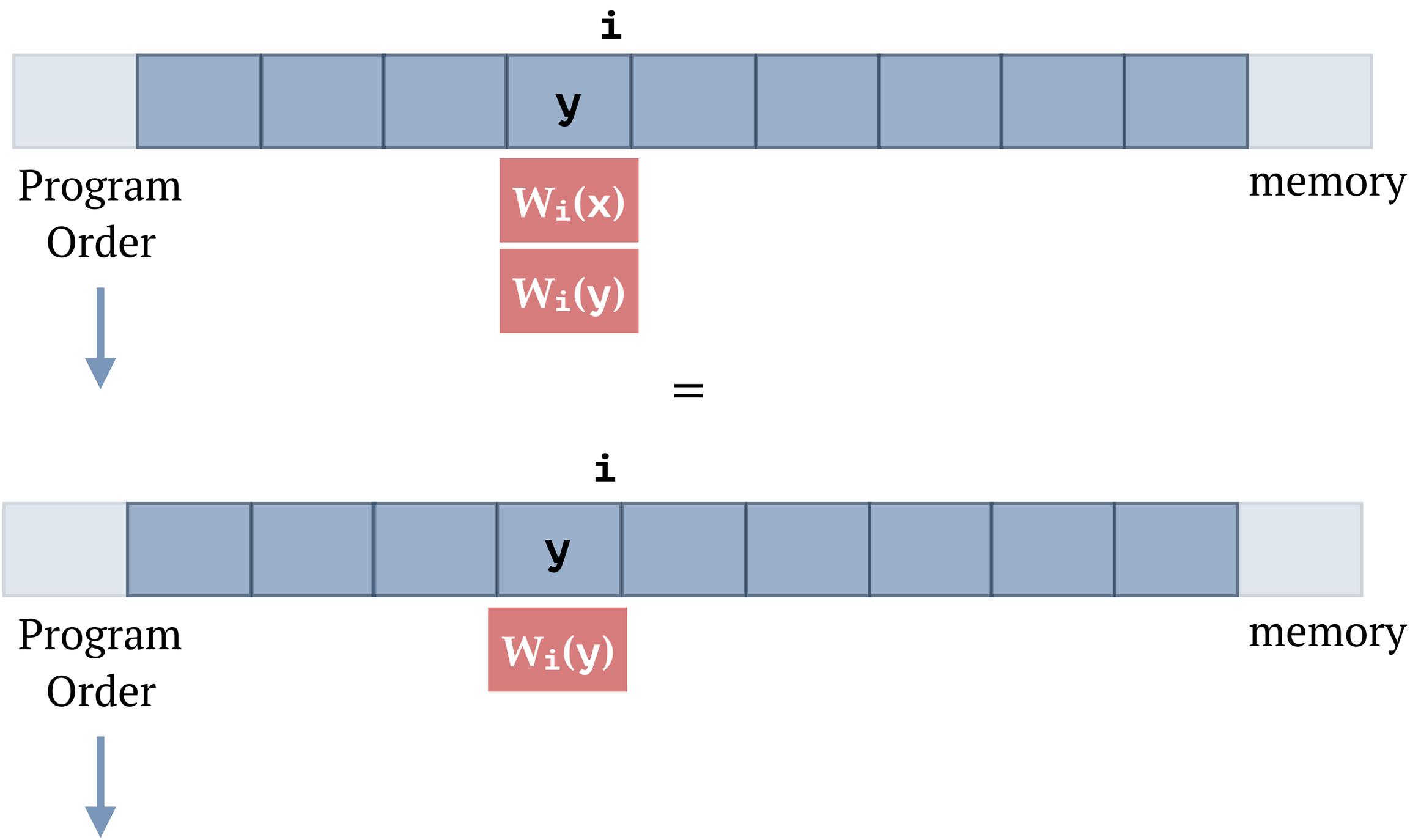
Write-over-Write Theorem #2

visibility of writes



Write-over-Write Theorem #2

visibility of writes



Symbolic Execution

These read-over-write and write-over-write lemmas operate on **symbolic expressions** that describe the program's behavior.

```
(implies
 (preconditions loc val x86)
 (let ((old-rbx (read 'register *rbx* x86))
       (old-pc  (read 'pc x86)))
   (equal
    (x86-run (clk) x86)
    (write 'register *rax* old-rbx
      (write 'pc (+ 18 old-pc)
        (write 'memory loc val x86))))))
```

Also, we can project out *relevant* parts of the resulting state.

Conclusions

What I Haven't Talked About Today…

1. **How to prove theorems using a mechanical theorem prover**
   - Useful to reason about both hardware and software
   - Fall'16 Grad-level Course: *Programming Languages*
   - Spring'17 Grad-level Course: *Recursion and Induction*

2. **Supervisor-mode features of the x86 ISA**
   - Useful for developing and analyzing kernel programs
   - An advanced architecture class
   - An OS class

Opportunities for Future Research

Operating System Verification

detect reliance on non-portable or undefined behaviors

User-friendly Program Analysis

automate the discovery of preconditions

Multi-process/threaded Program Verification

reason about concurrency-related issues

Reasoning about the Memory System

determine if caches are (mostly) transparent, as intended

Firmware Verification

formally specify software/hardware interfaces

Micro-architecture Verification

x86 ISA model serves as a build-to specification

Resources

We have exciting research and engineering projects in this area!
Please feel free to email if you want to know more.

- See ACL2 Home Page
- Talk to people on GDC 7S
- See some publications

Publications

Shilpi Goel, Warren A. Hunt, Jr., and Matt Kaufmann. *Abstract Stobjs and Their Application to ISA Modeling*. In Proceedings of the ACL2 Workshop 2013, EPTCS 114, pp. 54-69, 2013

Shilpi Goel and Warren A. Hunt, Jr. *Automated Code Proofs on a Formal Model of the x86*. In Verified Software: Theories, Tools, Experiments (VSTTE'13), volume 8164 of Lecture Notes in Computer Science, pages 222– 241. Springer Berlin Heidelberg, 2014

Shilpi Goel, Warren A. Hunt, Jr., Matt Kaufmann, and Soumava Ghosh. *Simulation and Formal Verification of x86 Machine-Code Programs That Make System Calls*. In Proceedings of the 14th Conference on Formal Methods in Computer-Aided Design (FMCAD'14), pages 18:91–98, 2014

Shilpi Goel, Warren A. Hunt, Jr., and Matt Kaufmann. *Engineering a Formal, Executable x86 ISA Simulator for Software Verification*. In Provably Correct Systems (ProCoS), 2015

Shilpi Goel. *Formal Verification of Application and System Programs Based on a Validated x86 ISA Model*. Ph.D. Dissertation, The University of Texas at Austin, 2016

[Source Code]

Github

[Documentation]

x86isa in the ACL2+Community Books Manual