1. (20 points) TEXTURE COORDINATES: Recall cylindrical texture coordinate generation, in which a texture is “wrapped” around an object in a cylindrical fashion to generate texture coordinates for each vertex in the object. Write a function that, given the coordinates of a vertex \( <v_x, v_y, v_z> \), will compute the cylindrical texture coordinates \((s, t)\) of that vertex. You may assume that the bottom center of the object rests at the origin, the top of the object extends along the \(y\) axis up to \(y = 1\), and the \(Z\) axis points out of the page (like OpenGL’s default coordinate system).

\[
\begin{align*}
\text{top view:} & \\
\theta &= \arctan 2(x, z) + \pi \\
\text{cyl-coord}(x, y, z) & \\
\theta &= \arctan 2(x, z) + \pi \\
S &= \frac{\theta}{2\pi} \\
T &= y \\
\text{return } (S, T)
\end{align*}
\]
2. (30 points) RAY TRACING: Refer to the scene below. The object \( \text{A} \) has specular, reflective, and refractive surface properties, while objects \( \text{B} \) and \( \text{C} \) have simple diffuse surface properties. There are three light sources in the scene: \( \text{L0} \), \( \text{L1} \), and \( \text{L2} \). A ray \( \text{P} \) is cast into the scene from the eye point \( \text{E} \), and intersects object \( \text{A} \) at point \( \text{I} \). Draw all of the rays necessary to evaluate the surface color at point \( \text{I} \), clearly indicating the direction of each ray and what object or light it hits (if any). (You may ignore internal reflections inside \( \text{A} \).) Label the rays with \( \text{R} \) for reflected rays, \( \text{T} \) for transmitted (refracted) rays, and \( \text{S} \) for shadow rays. Do any objects cast shadows on point \( \text{I} \)? Indicate any such objects and the lights that they block.

B blocks the light from \( \text{L2} \) from hitting \( \text{I} \).
3. (20 points) BARYCENTRIC COORDINATES: Refer the triangle below, with vertices A, B, and C. Answer the following questions about a point p with barycentric coordinates \((u, v, w) = (0, 0.6, 0.4)\). (Note: You may ignore any perspective effects in this question.)

\[ C = \langle 0, 10, 5 \rangle \]

\[ p = \langle 6, 4, -1 \rangle \]

\[ A = \langle 0, 0, 0 \rangle \]

\[ B = \langle 10, 0, -5 \rangle \]

a. (10 pts) Draw and label the point described above on the diagram, and compute its position in 3D.

\[
p = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} 0 + \begin{bmatrix} 10 \\ 0 \\ -5 \end{bmatrix} 0.6 + \begin{bmatrix} 0 \\ 10 \\ 5 \end{bmatrix} 0.4 = \langle 6, 4, -1 \rangle
\]

b. (4 pts) If the texture coordinates of A, B, and C are \((0, 0), (2, 1), \) and \((-2, 1)\) respectively, what are the interpolated texture coordinates at \(p\)?

\[
(0, 0) 0 + (2, 1) 0.6 + (-2, 1) 0.4 = (0.4, 1)
\]

c. (6 pts) If the vertex normals at A, B, and C are \(<0, 0, 1>, <1, 0, 0>, \) and \(<0, 1, 0>\) respectively, use Phong interpolation to compute the normal at \(p\).

\[
N' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} 0 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} 0.6 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} 0.4 = \begin{bmatrix} 0.6 \\ 0.4 \\ 0 \end{bmatrix}
\]

re-normalize: \[
\frac{1}{\|N'\|} \approx 1.38675... \Rightarrow \text{normalized } N \approx \langle 0.832, 0.555, 0 \rangle
\]
4. (20 points) LIGHTING: Consider the two-dimensional projection of a simple scene below, containing two spheres, a point light, and a vector that points towards either a directional light source at infinity or a far-away viewpoint (depending on the subproblem).

![Diagram of two spheres with light sources](image)

**a. (4 pts)** If the two spheres have matte surfaces, indicate the brightest spots on the spheres if only the directional light is turned on. Label the points with "d".

**b. (4 pts)** If the two spheres have matte surfaces, indicate the brightest spots on the spheres if only the point light is turned on. Label the points with "p".

**c. (4 pts)** Shade the portion of the spheres, if any, that is completely unlit if both lights are on but there is no ambient light.

**d. (8 pts)** Now assume that the point light is on, and the direction vector points to a far-away viewpoint or camera. If the spheres are specular, where will the center of the highlight be on each sphere? Label these points with "s", and also provide a written explanation of why you think that is the right location.

![Handwritten note](image)
5. (20 points) KD-TREES: Given the arrangement of objects below, build a KD-tree by drawing split planes on the diagram. Label each split plane with a letter, and draw a tree diagram of your nodes and leaves below the picture of the scene. Terminate your tree-building algorithm when each leaf node contains a single object. The first split plane A has been drawn for you. In addition, list the order that nodes and objects would be visited for the ray R cast in from the side.

Traversal order: ACEG342BD65