

Homework 14B
CS 313H

The important issue is the logic you used to arrive at your answer.

1. Let A be any set and R a symmetric relation on A . Prove that for $n \geq 1$, R^n (the n^{th} order composition of R with itself) is symmetric. (Recall $R^1 = R$ and for $n \geq 1$, $R^{n+1} = R^n \circ R$.)

2. Let A be any set and R a relation on A . Prove that the reflexive closure of R is $R \cup I$. (Remember you must show that $R \cup I$ is reflexive and that, if $R \subseteq \bar{R} \subseteq R \cup I$ and \bar{R} is reflexive, then $\bar{R} = R \cup I$.)

3. Let A be any set and R a relation on A . Prove that the symmetric closure of R is $R \cup R^{-1}$. (Remember you must show that $R \cup R^{-1}$ is symmetric and that, if $R \subseteq \bar{R} \subseteq R \cup R^{-1}$ and \bar{R} is symmetric, then $\bar{R} = R \cup R^{-1}$.)

4. Specify the transitive closure of the following relations:

a. Let $A = \{\text{living people}\}$ and $R = \{(x, y) \in A \times A : x \text{ is a child of } y\}$

b. Let $B = \{x : x \in \mathbb{N} \wedge x \leq 100\}$ and $S = \{(x, y) \in B \times B : y = x + 1\}$

c. Let $C = \{1, 2, 3, 4, 5\}$ and $T = \{(1, 2), (1, 4), (2, 5), (5, 3)\}$