Name ________________________________

Row 1 2 3 4 5

Homework 18
CS 313H

The important issue is the logic you used to arrive at your answer.

1. On the set $\mathbb{Z}^+$ of positive integers, consider the division relation $D = \{(x, y) : x \text{ evenly divides } y\}$. Prove that D is a partial order on $\mathbb{Z}^+$.

2. Prove or disprove (with a simple counterexample): the division relation on $\mathbb{Z}^+$ is a total order.

3. Given a non-empty set A, on its power set $P(A)$ consider the subset relation $S = \{(x, y) : x \subseteq y\}$. Prove that S is a partial order on $P(A)$.

4. Prove or disprove (with a simple counterexample): For all non-empty sets A, the subset relation on $P(A)$ is a total order.

5. On the set $\mathbb{Z}\times\mathbb{Z}$ of ordered pairs of integers, consider the lexicographical (or "dictionary") ordering relation
$LE=\{((x, y),(w,z)):(x\leq w)\wedge((x=w)\Rightarrow(y\leq z))\}$. Prove that LE is a partial order on $\mathbb{Z}\times\mathbb{Z}$. (**Note:** Be very careful here. Since $\mathbb{Z}\times\mathbb{Z}$ has ordered pairs as elements, LE has pairs of pairs as elements.)

6. Prove or disprove (with a simple counterexample): the lexicographical ordering relation on $\mathbb{Z}\times\mathbb{Z}$ is a total order.