Examination 1 Solutions

CS 313H

1. [10] Use a truth table to determine for which truth values of $p, q,$ and r $(\sim(p \wedge (q \vee r))) \Leftrightarrow ((\sim p \vee \sim q) \wedge (\sim p \vee \sim r))$ is true.

Let E = $((\sim(p \wedge (q \vee r))) \Leftrightarrow ((\sim p \vee \sim q) \wedge (\sim p \vee \sim r)))$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$\sim(p \wedge (q \vee r))$	$\sim p$	$\sim q$	$\sim r$	$\sim p \vee \sim q$	$\sim p \vee \sim r$	$(\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$	E
T	T	T	T	T	F	F	F	F	F	F	F	T
T	T	F	T	T	F	F	F	T	F	T	F	T
T	F	T	T	T	F	F	T	F	T	F	F	T
T	F	F	F	F	T	F	T	T	T	T	T	T
F	T	T	T	F	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	F	T	T	T	T	T
F	F	T	T	F	T	T	T	F	T	T	T	T
F	F	F	F	F	T	T	T	T	T	T	T	T

So the expression is true for all truth values of $p, q,$ and r.

2. [20] Using sentential calculus (with a four column format), prove that the conclusion $(\sim p \wedge (\sim q \Rightarrow p)) \Rightarrow q$ is tautologically true (i.e., follows from no premises).

$\{Pr_1\}$	(1.) $\sim p \wedge (\sim q \Rightarrow p)$	P (for C)
$\{Pr_1\}$	(2.) $\sim p$	Simp (1)
$\{Pr_1\}$	(3.) $\sim q \Rightarrow p$	Simp (1)
$\{Pr_1\}$	(4.) $\sim\sim q$	MT (2), (3)
$\{Pr_1\}$	(5.) q	DN (4)
$\{\}$	(6.) $(\sim p \wedge (\sim q \Rightarrow p)) \Rightarrow q$	C (1), (5)

3. [20] Prove that the conclusion $(p \Rightarrow (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$ follows from the premise $p \Rightarrow q$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

$p \Rightarrow q$
$\sim p \vee q$

$\sim(p \Rightarrow (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$
$\sim(\sim(p \Rightarrow (q \Rightarrow r)) \vee (p \Rightarrow r))$
$\sim\sim(p \Rightarrow (q \Rightarrow r)) \wedge \sim(p \Rightarrow r)$
$(\sim p \vee (\sim q \vee r)) \wedge \sim(\sim p \vee r)$
$(\sim p \vee (\sim q \vee r)) \wedge (\sim\sim p \wedge \sim r)$
$(\sim p \vee \sim q \vee r) \wedge p \wedge \sim r)$

1. $\sim p \vee q$ P

2. $\sim p \vee \sim q \vee r$ P
3. p P
4. $\sim r$ P
5. q Res (2), (3)
6. $\sim q \vee r$ Res (1), (2)
7. r Res (5), (6)
8. *false* Conj. (6), (7)

4. [10] a. Using the predicates defined on the set of things:

Px	x is a person,
Tt	t is time,
Rrt	r is telling the truth at t,
Lrt	r is lying at t,

express in the syntax of Predicate Calculus (you may use integers as constants):

"Some people tell the truth some of the time and some people never lie."

The universe of discourse is things – which includes, among other things, people and time. Do not define any other sets or predicates.

$$(\exists x)(\exists t)(Px \wedge Tt \wedge Rxt) \wedge (\exists y)(Py \wedge (\forall t)(Tt \Rightarrow \sim Lyt))$$

b. **[10]** Using the predicates defined on the set of real numbers:

Ix	x is an integer,
$Mxyz$	$x \cdot y = z$,
Lxy	$x < y$,

express in the syntax of Predicate Calculus (you may use integers as constants):

"There is a perfect square strictly between 100 and 200."

The universe of discourse is real numbers. Do not define any other sets or predicates. Recall that a prefect square has a square root that is an integer.

$$(\exists x)(L100x \wedge Lx200 \wedge (\exists z)(Iz \wedge Mzzx))$$

5. [20] Prove that $(\forall x)Px \wedge (\exists y)Qy$ follows from $(\forall x)(Px \wedge Qx)$.

$\{P_1\}$	(1).	$(\forall x)(Px \wedge Qx)$	P
$\{P_1\}$	(2).	$Pa \wedge Qa$	UI (1)
$\{P_1\}$	(3).	Pa	Simp (2)
$\{P_1\}$	(4).	$(\forall x)Px$	UG (3)
$\{P_1\}$	(5).	Qa	Simp (2)
$\{P_1\}$	(6).	$(\exists y)Qy$	EG (3)
$\{P_1\}$	(7).	$(\forall x)Px \wedge (\exists y)Qy$	Conj (6)