Examination 1

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. [10] Use a truth table to determine for which truth values of $p, q,$ and r $(\sim(p \wedge (q \vee r))) \Leftrightarrow ((\sim p \vee \sim q) \wedge (\sim p \vee \sim r))$ is true.

2. [20] Using sentential calculus (with a four column format), prove that the conclusion $(\sim p \wedge (\sim q \Rightarrow p)) \Rightarrow q$ is tautologically true (i.e., follows from no premises).

3. [20] Prove that the conclusion $(p \Rightarrow (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$ follows from the premise $p \Rightarrow q$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

4. [10] a. Using the predicates defined on the set of things:

Px	x is a person,
Tt	t is time,
Rrt	r is telling the truth at t,
Lrt	r is lying at t,

express in the syntax of Predicate Calculus (you may use integers as constants):

"Some people tell the truth some of the time and some people never lie."

The universe of discourse is things – which includes, among other things, people and time. Do not define any other sets or predicates.

b. **[10]** Using the predicates defined on the set of real numbers:

Ix	x is an integer,
$Mxyz$	$x \cdot y = z$,
Lxy	$x < y$,

express in the syntax of Predicate Calculus (you may use integers as constants):

"There is a perfect square strictly between 100 and 200."

The universe of discourse is real numbers. Do not define any other sets or predicates. Recall that a prefect square has a square root that is an integer.

5. [20] Prove that $(\forall x)Px \wedge (\exists y)Qy$ follows from $(\forall x)(Px \wedge Qx)$