**Examination 1 Solutions**

**CS 313H**

**1. [10]** Using a truth table prove that $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$ is a tautology.

| $p$ | $q$ | $r$ | $p \Rightarrow q$ | $q \Rightarrow r$ | $(p \Rightarrow q) \wedge (q \Rightarrow r)$ | $p \Rightarrow r$ | $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$ |
|---|---|---|---|---|---|---|---|
| F | F | F | T | T | T | T | T |
| F | F | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | T | T | T | T | T | T | T |
| T | F | F | F | T | F | F | T |
| T | F | T | F | T | F | T | T |
| T | T | F | T | F | F | F | T |
| T | T | T | T | T | T | T | T |

**2. [20]** Using sentential calculus (with a four column format), prove $((p \Leftrightarrow (q \vee r)) \wedge r) \Rightarrow p$ holds without premises.

| | | |
|---|---|---|
| $\{Pr_1\}$ | 1. $(p \Leftrightarrow (q \vee r)) \wedge r$ | P (for CP |
| $\{Pr_1\}$ | 2. $p \Leftrightarrow (q \vee r)$ | Simp (1) |
| $\{Pr_1\}$ | 3. $(p \Rightarrow (q \vee r)) \wedge ((q \vee r) \Rightarrow p)$ | Taut (2) |
| $\{Pr_1\}$ | 4. $(q \vee r) \Rightarrow p$ | Res (1), (3) |
| $\{Pr_1\}$ | 5. $r$ | Simp (1) |
| $\{Pr_1\}$ | 6. $q \vee r$ | Add (5) |
| $\{Pr_1\}$ | 7. $p$ | MP (4), (6) |
| | 8. $((p \Leftrightarrow (q \vee r)) \wedge r) \Rightarrow p$ | C (1), (7) |

**3. [20]** Prove the tautology $(p \Rightarrow q) \vee (p \wedge \sim q)$ using resolution.

Negate the conclusion:
$\sim ((p \Rightarrow q) \vee (p \wedge \sim q))$

Convert to CNF:
$\sim ((p \Rightarrow q) \vee (p \wedge \sim q))$
$\sim (p \Rightarrow q) \wedge \sim (p \wedge \sim q)$
$\sim (\sim p \vee q) \wedge (\sim p \vee \sim\sim q)$
$(\sim\sim p \wedge \sim q) \wedge (\sim p \vee q)$
$p \wedge \sim q \wedge (\sim p \vee q)$

| | | |
|---|---|---|
| $\{Pr_1\}$ | 1. $p$ | P |

| | | |
|---|---|---|
| $\{\Pr_2\}$ | 2. $\sim q$ | P |
| $\{\Pr_3\}$ | 3. $\sim p \vee q$ | P |
| $\{\Pr_1, \Pr_3\}$ | 4. $q$ | Res (1), (3) |
| $\{\Pr_1, \Pr_2, \Pr_3\}$ | 5. *false* | ContraPrem. (2), (4) |

**4. [20]** Using sentential calculus (with a four column format), prove that the conclusion "Either Rachel or Sarah has no cat" follows from these premises:

If Mike has no cat then Pat has no cat and if Tom has no cat then Zack has no cat.
If either Mike has a cat or Pat has no cat then Zack has a cat.
If Tom has a cat then it is not true that both Rachel and Sarah have cats.

**Begin by giving them symbolic values to certain sentences and expressing the premises and conclusion in terms of those symbols.**

Let $m$ = "Mike has a cat"
$p$ = "Pat has a cat"
$t$ = "Tom has a cat"
$z$ = "Zack has a cat"
$r$ = "Rachel has a cat"
$s$ = "Sarah has a cat"

| | | |
|---|---|---|
| $\{Pr_1\}$ | (1.) $(\sim m \Rightarrow \sim p) \wedge (\sim t \Rightarrow \sim z)$ | P |
| $\{Pr_2\}$ | (2.) $(m \vee \sim p) \Rightarrow z$ | P |
| $\{Pr_3\}$ | (3.) $t \Rightarrow \sim (r \wedge s)$ | P |
| $\{Pr_1\}$ | (4.) $\sim m \Rightarrow \sim p$ | Simp (1) |
| $\{Pr_1\}$ | (5.) $\sim\sim m \vee \sim p$ | CD (4) |
| $\{Pr_1\}$ | (6.) $m \vee \sim p$ | DN (5) |
| $\{Pr_1, Pr_2\}$ | (7.) $z$ | MP (2), (6) |
| $\{Pr_1, Pr_2\}$ | (8.) $\sim t \Rightarrow \sim z$ | Simp (1) |
| $\{Pr_1, Pr_2\}$ | (9.) $\sim\sim t$ | MT (7), (8) |
| $\{Pr_1, Pr_2\}$ | (10.) $t$ | DN (9) |
| $\{Pr_1, Pr_2, Pr_3\}$ | (11.) $\sim (r \wedge s)$ | MP (10) |
| $\{Pr_1, Pr_2, Pr_3\}$ | (12.) $\sim r \vee \sim s$ | DM (11) |

**5. [20]** Using the predicates defined on the set of UT students:

| | |
|---|---|
| *Exy* | $x$ is equal to $y$, |
| *CSx* | $x$ is a CS major, |
| *Mx* | $x$ is a math major, |

Express in the syntax of Predicate Calculus:

*a. "There is exactly one CS major."*

$$(\exists x)(CSx \wedge (\forall y)(\sim Exy \Rightarrow \sim CSy))$$

*b. "There are at least three students who are both Math majors and CS majors."*

$$(\exists x)(\exists y)(\exists z)(\sim Exy \wedge \sim Exz \wedge \sim Eyz \wedge Mx \wedge My \wedge Mz \wedge CSx \wedge CSy \wedge CSz)$$

*c. "No one is both a Math major and a CS major."*

$$(\forall x)(\sim CSx \vee \sim Mx)$$

*d. "The are at most two Math majors."*

$$\sim (\exists x)(\exists y)(\exists z)(\sim Exy \wedge \sim Exz \wedge \sim Eyz \wedge Mx \wedge My \wedge Mz)$$