Examination 1 Solutions

CS 313H

1. [10] Using a truth table prove that $(p \vee \sim(q \wedge r)) \Rightarrow (q \Rightarrow (r \Rightarrow p))$ is a tautology.

p	q	r	$q \wedge r$	$\sim(q \wedge r)$	$p \vee \sim(q \wedge r)$	$r \Rightarrow p$	$q \Rightarrow (r \Rightarrow p)$	$(p \vee \sim(q \wedge r)) \Rightarrow (q \Rightarrow (r \Rightarrow p$
F	F	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	T	F	F	T	T	T	T	T
F	T	T	T	F	F	F	F	T
T	F	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	T	T	T	F	T	T	T	T

2. [20] Using the predicates defined on the set of people (for x and y) and the set of integers (for n):

Exy	x is equal to y,
Dxn	x will die within n months,
Px	x has the disease polyputrid,
Cxy	x is a child of y,

Express in the syntax of Predicate Calculus:

a. "Exactly one person who has polyputrid has a parent who has it."

$$(\exists x)((\exists y)(Px \wedge Py \wedge Cxy) \wedge (\forall z)(\sim Exz \Rightarrow \sim (\exists w)(Pz \wedge Pw \wedge Czw)))$$

b. "If at least two different people have polyputrid then everyone will die in two to four months."

$$(\exists x)(\exists y)(Px \wedge Py \wedge \sim Exy) \Rightarrow (\forall z)(\sim Dz2 \wedge Dz4)$$

c. "No one who has poluputric will die within the next year."

$$(\forall x)(Px \Rightarrow \sim Dx12)$$

2. [15] Using sentential calculus (with a four column format), prove that the conclusion $\sim p \wedge \sim q$ follows from premises: $(p \Rightarrow r) \wedge (q \Rightarrow s), (s \wedge r) \Rightarrow t, \sim t$ and $p \wedge q$.

$\{Pr_1\}$	(1.) $(p \Rightarrow r) \wedge (q \Rightarrow s)$	P
$\{Pr_2\}$	(2.) $(s \wedge r) \Rightarrow t$	P
$\{Pr_3\}$	(3.) $\sim t$	P
$\{Pr_4\}$	(4.) $p \wedge q$	P
$\{Pr_4\}$	(5.) p	Simp (4)
$\{Pr_1\}$	(6.) $p \Rightarrow r$	Simp (1)
$\{Pr_1, Pr_4\}$	(7.) r	MP (5), (6)
$\{Pr_4\}$	(8.) q	Simp (4)
$\{Pr_1\}$	(9.) $q \Rightarrow s$	Simp (1)
$\{Pr_1, Pr_4\}$	(10.) s	MP (8), (9)
$\{Pr_1, Pr_4\}$	(11.) $s \wedge r$	Add (7), (10)
$\{Pr_1, Pr_2, Pr_4\}$	(12.) t	MP (11), (2)
$\{Pr_1, Pr_2, Pr_3, Pr_4\}$	(13.) $t \wedge \sim t$	Add (12), (3)
$\{Pr_1, Pr_2, Pr_3, Pr_4\}$	(14.) $\sim p \wedge \sim q$	Clav (13)

4. [15] Prove that the conclusion $\sim q \wedge p$ follows from the premises $(p \Rightarrow q) \Rightarrow r$ and $\sim r$. First convert the premise and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

$(p \Rightarrow q) \Rightarrow r$
$\sim (p \Rightarrow q) \vee r$
$\sim (\sim p \vee q) \vee r$
$(\sim\sim p \wedge \sim q) \vee r$
$(p \wedge \sim q) \vee r$
$(p \vee r) \wedge (\sim q \vee r)$

$\sim (\sim q \wedge p)$
$\sim\sim q \vee \sim p$
$q \vee \sim p$

1. $p \vee r$	P
2. $\sim q \vee r$	P
3. $q \vee \sim p$	P
4. $\sim r$	P
5. $q \vee r$	Res (1), (3)
6. r	Res (2), (5)

7. *false* Conj. (4), (6)

5. [20] Using sentential calculus (with a four column format), prove that the conclusion $\sim(\forall z)Rz$ follows from these premises: Pa, $Rc \Rightarrow \sim(Pa \wedge Qb)$, $((\exists y)Py) \Rightarrow ((\forall x)Qx)$.

$\{Pr_3\}$	(1.)	$((\exists y)Py) \Rightarrow ((\forall x)Qx)$	P
$\{Pr_1\}$	(2.)	Pa	P
$\{Pr_1\}$	(3.)	$(\exists y)Py$	EG (2)
$\{Pr_1, Pr_3\}$	(4.)	$(\forall x)Qx$	MP (1), (3)
$\{Pr_1, Pr_3\}$	(5.)	Qb	UI (4)
$\{Pr_1, Pr_3\}$	(6.)	$Pa \wedge Qb$	Add (2), (5)
$\{Pr_2\}$	(7.)	$Rc \Rightarrow \sim(Pa \wedge Qb)$	P
$\{Pr_1, Pr_2, Pr_3\}$	(8.)	$\sim Rc$	MT (6), (7)
$\{Pr_1, Pr_2, Pr_3\}$	(9.)	$(\exists z) \sim Rz$	EG (8)
$\{Pr_1, Pr_2, Pr_3\}$	(10.)	$\sim(\forall z)Rz$	QE (9)