Examination 1 Solutions

CS 313H

1. [10] Using a truth table prove that $(p \vee \sim(q \wedge r)) \Rightarrow (q \Rightarrow (r \Rightarrow p))$ is a tautology.

p	q	r	$q \wedge r$	$\sim(q \wedge r)$	$(p \vee \sim(q \wedge r))$	$r \Rightarrow p$	$q \Rightarrow (r \Rightarrow p)$	$(p \vee \sim(q \wedge r)) \Rightarrow (q \Rightarrow (r \Rightarrow p))$
F	F	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	T	F	F	T	T	T	T	T
F	T	T	T	F	F	F	F	T
T	F	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	T	T	T	F	T	T	T	T

2. [20] Using the predicates defined on the set of real numbers:

LTxy x is less than y,
EQxy x is equal to y,
SRxy x is a square root of y,

Express in the syntax of Predicate Calculus:

a. *"All real numbers x, y, and z satisfy if x is less than y and y is less than z then x is less than z."*

$$(\forall x)((\forall y)((\forall z)((LTxy \wedge LTyz) \Rightarrow LTxz)))$$

b. *"For every pair of numbers x and y so that x < y, there is a z strictly between x and y."*

$$(\forall x)((\forall y)(LTxy \Rightarrow (\exists z)(LTxz \wedge LTzy)))$$

c. *"For every pair of numbers x and y so that x < y, there is more than one number strictly between x and y."*

$$(\forall x)((\forall y)(LTxy \Rightarrow (\exists z)((\exists w)(\sim EQzw \wedge LTxz \wedge LTzy \wedge LTxw \wedge LTwy))))$$

d. *"Any positive number has at most one positive square root."*

$$(\forall x)(LT0x \Rightarrow (\forall y)((\forall z)((SRyx \wedge SRzx \wedge LT0y \wedge LT0z) \Rightarrow EQyz)))$$

3. [15] Using sentential calculus (with a four column format), prove $s \Rightarrow r$ follows from the premises $p \Rightarrow (q \Rightarrow r)$, $p \vee \sim s$, and q.

$\{Pr_1\}$	1. $p \Rightarrow (q \Rightarrow r)$	P
$\{Pr_2\}$	2. $p \vee \sim s$	P
$\{Pr_3\}$	3. q	P
$\{Pr_4\}$	4. s	P (for CP)
$\{Pr_2, Pr_4\}$	5. p	DS (2), (4)
$\{Pr_1, Pr_2, Pr_4\}$	6. $q \Rightarrow r$	MP (1), (5)
$\{Pr_1, Pr_2, Pr_3, Pr_4\}$	7. r	MP (3), (6)
$\{Pr_1, Pr_2, Pr_3\}$	8. $s \Rightarrow r$	C (4), (7)

4. [15] Prove $(p \vee \sim q) \Rightarrow (p \vee q)$ follows from the premise $\sim q \Rightarrow p$ using resolution.

Negate the conclusion:
$\sim ((p \vee \sim q) \Rightarrow (p \vee q))$

Convert this and the premise to CNF:
$\sim ((p \vee \sim q) \Rightarrow (p \vee q))$
$\sim (\sim (p \vee \sim q) \vee (p \vee q))$
$(p \vee \sim q) \wedge \sim (p \vee q)$
$(p \vee \sim q) \wedge \sim p \wedge \sim q$

$\sim q \Rightarrow p$
$\sim\sim q \vee p$
$q \vee p$

1. $p \vee \sim q$	P
2. $\sim p$	P
3. $\sim q$	P
4. $q \vee p$	P
5. q	Res (2), (4)
6. *false*	Conj. (3), (5)

5. [20] Prove that $((\exists x)((\forall y)Rxy) \Rightarrow ((\forall u)((\exists v)Rvu)$ holds without premises.

$\{P_1\}$	(1).	$((\exists x)((\forall y)Rxy)$	P (for CP)
$\{P_1\}$	(2).	$(\forall y)Ray$	EI (1)
$\{P_1\}$	(3).	Rab	UI (2)
$\{P_1\}$	(4).	$(\exists v)Rvb$	EG (3)
$\{P_1\}$	(5).	$((\forall u)((\exists v)Rvu)$	UG (4)
	(6).	$((\exists x)((\forall y)Rxy) \Rightarrow ((\forall u)((\exists v)Rvu)$	C (1), (5)