Examination 1 Solutions

CS 313H

1. a [10] Use a truth table to determine for which truth values of $p, q,$ and r $(p \Rightarrow (\sim q \vee r)) \Leftrightarrow (r \wedge q)$ is true.

p	q	r	$\sim q$	$\sim q \vee r$	$p \Rightarrow (\sim q \vee r)$	$r \wedge q$	$(p \Rightarrow (\sim q \vee r)) \Leftrightarrow (r \wedge q)$
T	T	T	F	T	T	T	T
T	T	F	F	F	F	F	T
T	F	T	T	T	T	F	F
T	F	F	T	T	T	F	F
F	T	T	F	F	T	T	T
F	T	F	F	F	T	F	F
F	F	T	T	T	T	F	F
F	F	F	T	T	T	F	F

b. [10] Using the result of the truth table express in an equivalent Disjunctive Normal Form.

In Disjunctive Normal Form the expression is $(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r)$.

2. [20 Using sentential calculus (with a four column format), prove that the conclusion $(\sim (q \wedge s) \wedge (q \vee p)) \Rightarrow ((s \wedge \sim p) \Rightarrow t)$ follows from NO premises. (Hint: Employ Conditionalization more than once.)

$\{Pr_1\}$	1. $\sim (q \wedge s) \wedge (q \vee p)$	P (for CP)
$\{Pr_1\}$	2. $\sim (q \wedge s)$	Simp (1)
$\{Pr_1\}$	3. $q \vee p$	Simp (1)
$\{Pr_2\}$	4. $s \wedge \sim p$	P (for CP)
$\{Pr_2\}$	5 s	Simp (4)
$\{Pr_1\}$	6. $\sim q \vee \sim s$	DeM (2)
$\{Pr_1, Pr_2\}$	7. $\sim q$	DS (5), (6)
$\{Pr_1, Pr_2\}$	8. p	DS (3), (7)
$\{Pr_2\}$	9. $\sim p$	Simp (4)
$\{Pr_1, Pr_2\}$	10. t	ContraPrm (8), (9)
$\{Pr_1\}$	11. $(s \wedge \sim p) \Rightarrow t$	C (4), (10)
$\{\}$	12. $(\sim (q \wedge s) \wedge (q \vee p)) \Rightarrow ((s \wedge \sim p) \Rightarrow t)$	C (1), (11)

3. [20] Prove that the conclusion $\sim p \wedge \sim q$ follows from the premises $(p \Rightarrow r) \wedge (q \Rightarrow s), (s \wedge r) \Rightarrow t, \sim t$ and $\sim(\sim p \vee \sim q)$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

$(p \Rightarrow r) \wedge (q \Rightarrow s)$
$(\sim p \vee r) \wedge (\sim q \vee s)$

$(s \wedge r) \Rightarrow t$
$\sim (s \wedge r) \vee t$
$(\sim s \vee \sim r) \vee t$
$\sim s \vee \sim r \vee t$

$\sim (\sim p \vee \sim q)$
$p \wedge q$

$\sim (\sim p \wedge \sim q)$
$\sim\sim p \vee \sim\sim q$
$p \vee q$

1. $\sim p \vee r$	P
2. $\sim q \vee s$	P
3. $\sim s \vee \sim r \vee t$	P
4. $\sim t$	P
5. p	P
6. q	P
7. $p \vee q$	P
8. r	Res (1), (5)
9. s	Res (2), (6)
10. $\sim s \vee t$	Res (3), (8)
11. t	Res (9), (10)
12. *false*	Conj. (4), (11)

4. a. **[5]** Using the predicate defined on the set of people:

$A(x)$ x will attend the party,

express in the syntax of Predicate Calculus:

"If John will attend the party then exactly one of Mary or Sally will attend the party."

Do not define any other **sets** or **predicates**.

$$A(John) \Rightarrow ((A(Mary) \vee A(Sally)) \wedge \sim (A(Mary) \wedge A(Sally)))$$

b. **[10]** Using the predicates defined for p from the set of people and t from the set of tasks :

$T(t, p_1, p_2)$	task t is completed faster by person p_1 than by person p_2,
$Old(p_1)$	person p_1 is old,
$EQ(p_1, p_2)$	person p_1 is the same as person p_2,

express in the syntax of Predicate Calculus:

"Some task can be performed faster by an old person than any person who is not old."

Do not define any other **sets** or **predicates**.

$$\exists t \exists p_1 (Old(p_1) \wedge \forall p_2\, T(t, p_1, p_2))$$

c. **[15]** Using the same predicates as in part b, express in the syntax of Predicate Calculus:

"Exactly two people can between them (meaning one or the other) complete every task faster than all others and both of them are old."

Do not define any other **sets** or **predicates**.

$$\exists p_1 (\exists p_2 (\forall t (\forall p_3 ((\sim EQ(p_1, p_3) \wedge \sim EQ(p_2, p_3)) \Rightarrow (T(t, p_1, p_3) \vee T(t, p_2, p_3)) \wedge Old(p_1) \wedge Old(p_2))$$

5. [20] Prove that $\exists y(\forall x\, Hxy)$ follows from $\forall x(Fx \Rightarrow \sim Gx)$ and $\exists x(\forall y\, Fy \wedge Gx)$.

$\{P_1\}$	(1). $\forall x(Fx \Rightarrow \sim Gx)$	P
$\{P_2\}$	(2). $\exists x(\forall y\, Fy \wedge Gx)$	P
$\{P_2\}$	(3). $\forall y\, Fy \wedge Ga$	EI (2)
$\{P_2\}$	(4). $Fa \wedge Ga$	UI (3)
$\{P_2\}$	(5). Fa	Simp (4)
$\{P_1\}$	(6). $Fa \Rightarrow \sim Ga$	UI (1)
$\{P_1, P_2\}$	(7). $\sim Ga$	MP (5), (6)
$\{P_2\}$	(8). Ga	Simp (4)
$\{P_1, P_2\}$	(9). $\exists y(\forall x\, Hxy)$	ContraPrm (7), (8)