

## Examination 1

## CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes

1. a [10] Use a truth table to determine for which truth values of  $p, q,$  and  $r$   $(p \Rightarrow (\sim q \vee r)) \Leftrightarrow (r \wedge q)$  is true.
- b. [10] Using the result of the truth table express  $(p \Rightarrow (\sim q \vee r)) \Leftrightarrow (r \wedge q)$  in an equivalent Disjunctive Normal Form.
2. [20] Using sentential calculus (with a four column format), prove that the conclusion  $(\sim (q \wedge s) \wedge (q \vee p)) \Rightarrow ((s \wedge \sim p) \Rightarrow t)$  follows from NO premises. (Hint: Employ Conditionalization more than once.)
3. [20] Prove that the conclusion  $\sim p \wedge \sim q$  follows from the premises  $(p \Rightarrow r) \wedge (q \Rightarrow s), (s \wedge r) \Rightarrow t, \sim t$  and  $\sim (\sim p \vee \sim q)$ . First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

4. a. [5] Using the predicate defined on the set of people:

$A(x)$   $x$  will attend the party,

express in the syntax of Predicate Calculus:

*“If John will attend the party then exactly one of Mary or Sally will attend the party.”*

Do not define any other **sets** or **predicates**.

b. [10] Using the predicates defined for  $p$  from the set of people and  $t$  from the set of tasks :

$T(t, p_1, p_2)$  task  $t$  is completed faster by person  $p_1$  than by person  $p_2$ ,

$Old(p_1)$  person  $p_1$  is old,

$EQ(p_1, p_2)$  person  $p_1$  is the same as person  $p_2$ ,

express in the syntax of Predicate Calculus:

*“Some task can be performed faster by an old person than any person who is not old.”*

Do not define any other **sets** or **predicates**.

c. [15] Using the same predicates as in part b, express in the syntax of Predicate Calculus:

*“Exactly two people can between them (meaning one or the other) complete every task faster than all others and both of them are old.”*

Do not define any other **sets** or **predicates**.

5. [20] Prove that  $\exists y(\forall x Hxy)$  follows from  $\forall x(Fx \Rightarrow \sim Gx)$  and  $\exists x(\forall y Fy \wedge Gx)$ .